Assignment 3.

1. Consider the simple linear regression model

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

where α and β are scalars and $\varepsilon_i \sim N\left(0, \sigma^2\right)$. For $H_0 : \beta = 0$,

a) Derive the LR statistic and show that it can be written as $n \ln\left(1/\left(1 - r^2\right)\right)$ where r^2 is the square of the correlation coefficient between y and X.

b) Derive the Wald statistic for testing $H_0 : \beta = 0$. Show that it can be written as $nr^2/\left(1 - r^2\right)$.

c) Derive the LM statistics for testing $H_0 : \beta = 0$. Show that it can be written as nr^2.

2. A consumption function that allows for different short and long run propensity to consume can be written as

$$\ln c_t = \alpha + \beta_1 \ln c_{t-1} + \beta_2 \ln y_t + \varepsilon_t$$

where c_t is consumption and y_t is income at time t. β_2 can be interpreted as the marginal short term propensity to consume and the non-linear function $\gamma = \frac{\beta_2}{1-\beta_1}$ can be interpreted as the long-run marginal propensity to consume.

The following estimates were obtained when this model was estimated on annual Swedish data between 1950 and 1975:

$$\ln c_t = 1 + 0.98 \ln c_{t-1} + 0.015 \ln y_t + \varepsilon_t$$

$$Cov(\mathbf{b}) = s^2(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 0.100 & 0.001 & 0.002 \\ & 0.030 & -0.009 \\ & & 0.003 \end{bmatrix}$$

where $\mathbf{b} = (a, b_1, b_2)$ and $s^2 = \mathbf{e}'\mathbf{e}/22 = 1$.

a) Test $H_0 : \beta_2 = 1 - \beta_1$ against $H_1 : \beta_2 < 1 - \beta_1$

b) Test $H_0 : \gamma = 0$ against $H_1 : \gamma \neq 0$ on the 5 % level.

3. Explain the concept "encompassing principle" and how it relates to conventional tests of linear hypotheses and to J-tests respectively.

4. The data

X	Y
2	4
3	7
1	3
5	9
9	17

is generated by a linear relation. The variance matrix for the disturbance underlying the data is

$$V(\varepsilon) = \sigma^2 diag\{0.10, 0.05, 0.20, 0.30, 0.15\}.$$

Estimate the model by OLS and use the information on the variance matrix to calculate the correct standard errors for the OLS estimates and compare with those obtained from the conventional formula.

5. Do exercise 1 b to Chapter 12 in Greene.

6. Write down an asymptotic test for structural change based on the Wald statistic. Use the assumption that the two samples are independent and that the variance-covariance matrix of the first and second sub-samples are Ω_1 and Ω_2 respectively.