Part 1: Theory

Suppose f is a function defined on a certain interval being at least $(n+1)$ times differentiable. For x_1, x_0 out of this interval and for suitable $0 < \theta < 1$ the **Taylor's formula** about the (inner) point x_0 is

$$f\left(x\right) = f\left(x_0\right) + \sum_{r=1}^{n} \frac{f^{(r)}\left(x_0\right)}{r!}\left(x - x_0\right)^r + f^{(n+1)}\left(x_0 + \theta\left(x - x_0\right)\right)\frac{\left(x - x_0\right)^{n+1}}{(n+1)!},$$

where $f^{(r)}\left(x\right) = \frac{d^r f(x)}{dx^r}$ denotes the r-th derivation of f at x.

(a) Find Taylor's formula for

i. $exp(x)$ for $x_0 = 0$ and $n = 1, 2, \ldots$.

ii. $log(1+x)$ with $|x| < 1$ at $x_0 = 0$ at $n = 1, 2, \ldots$.

(b) Consider the following Taylor series:

$$f(x - hz) = f(x) - hzf'(x) + \frac{1}{2}(hz)^2 f''(x) + o(h^2)$$

where $o(h^2)$ represents terms that converge to zero faster than h^2 as h approaches zero (see lecture notes, p. 13).

i. Approximate the pdf of the exponential distribution

$$f(x) = \lambda e^{-\lambda x}, x \geq 0$$

with its Taylor series representation (n=2).

ii. Plot the pdf of the exponential distribution and the approximation with **R**.

iii. Show (graphically with **R**) that $o(h^2)$ really converges to zero faster than h^2 as h approaches zero.

Part 2: Practical

(a) Familiarize yourself with the functions:

   i. `dnorm( )`, `pnorm( )`, `qnorm( )`, `rnorm( )`,
   ii. `dexp( )`, `pexp( )`, `qexp( )`, `rexp( )`,
   iii. `dpois( )`, `ppois( )`, `qpois( )`, `rpois( )`,
   iv. `approxfun( )`.

(b) Read the "credit"–data with
`credit<-read.csv("D:/kursdaten_IntAktDat/Kredit-part.csv ",header=T)`
and divide values by 1000 (units are now 1000 DM). Seperate the good (`c1`) and the problem customers (`c0`).

   i. Estimate the density for `c0`, plot a histogram and the estimated density
   ii. Consider the estimated density in the interval $[0, 20]$. Estimate the area unter the density with the function `approxfun( )`. Reapproximate the density in the interval $[0, 20]$.
   iii. Given the reapproximated density, construct
      - the (probability) distribution function (pdf)
      - the inverse of the pdf
      - a random number generator

      Verify your results!