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# Rent Sharing in an Equilibrium Model of Matching and Turnover

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This article characterizes labor markets in which the heterogeneity of workers and firms results in thin markets and rents. Neoclassical marginal analysis and matching are blended into a computable general equilibrium model of trade in efficiency units of labor. Although workers' bargaining problems are interrelated, a simple wage contract generates wage flexibility and efficient matching in the model's equilibrium. Equilibrium wages are predicted to vary with the diversity of firms, the scarcity of skills, and the costliness of search. The model is applied to superstar markets, union bargaining in sports, interindustry wage differentials, and the relationship between pay and profit.

Are you paid what you are worth? If the labor market is as thin as it reveals itself to the casual observer, then the answer is almost certainly no. You, like Michael Jordan and your child's baby-sitter, are both over- and underpaid. You are paid more than you require to do your job, but less than you are worth to your employer. Your wage is not determined uniquely by the forces of supply and demand. Despite the opportunity for you and

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your employer to bargain over the division of these rents, the neoclassical model of the labor market—with a downward-sloping labor demand schedule and an upward-sloping labor supply schedule—characterizes the labor market you share with Michael Jordan and everyone else. These are the messages that emerge from the analysis that follows.

How is your wage determined? Theoretical and applied economists have developed a variety of models of wage and employment determination that reflect thinness in the labor market. Some models forgo marginal analysis and endogenous firm size; others introduce nonconvexities, trade frictions, contractual restrictions, or asymmetric information; but each deviates from the neoclassical representation.

Begin by applying Koopmans and Beckmann's (1957) classic model of efficient matching to the labor market. As a monogamous marriage market,  $n$  heterogeneous workers match with  $n$  heterogeneous firms. Heterogeneity implies that there is not a unique division of each employment pair's match value into wage and profit: rents are associated with each optimal match. Determining unique wage offers involves a complex bargaining problem (Rochford 1984). To solve a typical bargaining problem, one needs to know threat points. In the employment context, each bargaining pair must know the wages the worker would receive with other firms and the profits the firm would receive from employing other workers. But wages and profits in alternative matches are also the result of bargaining. Since the threat points in one bargaining pair depend on the solutions to bargaining in other pairs, the bargaining problems are interrelated. Rochford (1984) and Bennett (1988) provide solutions to the problem of interrelated bargaining in monogamous marriages, but in general the solutions do not produce unique wages and profits.

A second approach to pinning down precise payoffs has evolved from Gale and Shapley's (1962) analysis of the matching of students to colleges. This literature emphasizes mechanisms or algorithms to solve the matching game (Crawford and Knoer 1981; Kelso and Crawford 1982; Roth 1984a, 1984b; Roth and Sotomayor 1988). Although these rules for making, rejecting, and accepting wage offers are somewhat artificial, the proposed algorithms lead to efficient matching with the division of the rents determined by the bidding rules. There is a precise sense in which the side that "proposes" captures all the rents. Given the bidding rules, the bargaining problem vanishes.

In more applied models of the labor market, economists have recognized that search costs as trading frictions would generate rents to firm-worker pairs even if all firms and all workers were identical. Nash-bargained wages are typically employed in such search models (e.g., Diamond 1982; Pissarides 1990) and some insider-outsider models (e.g., Solow 1985). Other insider-outsider models (e.g., Lindbeck and Snower 1988) allocate wage-setting power to individual workers. Although bargaining problems are

substantially less complex in these models, search externalities and insider power produce inefficient matching and perhaps involuntary unemployment.

Contracting models with match capital or investment in firm-specific human capital also confront the bargaining problem. From learning (e.g., Jovanovic 1979) or investment in firm-specific human capital (e.g., Becker 1962), *ex ante* identical individuals become heterogeneous workers *ex post*. Thus, *ex ante* competition in contracts can limit opportunistic wage bargaining *ex post*. For instance, a common solution is to set the wage or wage profile in advance (e.g., Becker 1962; Hashimoto and Yu 1980; Hall and Lazear 1984; Malcolmson 1984). One alternative allows the firm to set the *ex post* wage, with the proceeds of the opportunism transferred to the worker *ex ante*; the proceeds are transferred back to the firm if the worker sets the *ex post* wage (e.g., Hall and Lazear 1984). Consequently, in this literature, the bargaining problem is contracted away *ex ante*.

With unions, collective bargaining allows identical workers to capture some producer surplus that results from the average product of labor exceeding its marginal product. A common approach is to include a parameter that summarizes union bargaining power (e.g., McDonald and Solow 1981; Abowd 1991). Alternatively, the literature on union strike activity sets bargaining in a noncooperative environment with private information. Wage offers evolve over the course of the strike to reveal the private information (e.g., Tracy 1987). Consequently, the accepted wage offer is the solution to a noncooperative bargaining game.

These diverse and valuable literatures illustrate a variety of ways to confront the problem of determining wages in thin markets with rents. Although these literatures provide insightful perspectives on wage bargaining and the effects of thin markets, research on wage bargaining and rent sharing is incomplete. In particular, the literature is missing a model that looks strikingly neoclassical despite the presence of rents. Imagine a model with endogenous firm size and well-defined marginal products determined by the usual marginal analysis. For a particular worker, let these marginal products vary across firms. And incorporate a genuine bargaining problem, one neither resolved by an imposed bidding structure nor contracted away *ex ante*. Such a model would include rich interactions. A worker's productivity in one firm would depend on the other workers the firm employs. And wage bargains would be interrelated: a worker's wage offer from one firm would depend on his or her outside wage offers. Thus, bargaining would add an extra level of complexity to the already complex problem of sorting resources by comparative advantage. Would such a model be tractable to analyze? Can the market solve such a problem efficiently?

I present an equilibrium model of matching and turnover that blends these neoclassical and bargaining features. In Section I, I develop a matching model that employs neoclassical marginal analysis to generate well-defined productivity values for each worker. Firms and workers adopt a wage

contract that divides the difference between productivity within the firm and the best outside wage offer. This implies unique wage and profit offers for each potential match. Although a worker is almost always paid less than his productivity value, wages are flexible, and both matching and turnover are efficient and invariant to bargaining strength. In the market equilibrium, a worker matches with the firm in which his productivity value is highest.

Applications of the model are explored in Section II. The rent-sharing model is most applicable where firms are relatively diverse, skills relatively scarce, and search relatively costly. And rent sharing is likely to be quantitatively more important in markets for superstars. From this background, I study three applications in more detail: union bargaining, interindustry wage differentials, and the relationship between pay and profit. The application to union bargaining captures a peculiar feature of collective bargaining in sports: players unions do not bargain over wages.

## I. Model

Roy's (1951) celebrated model of selection across markets (e.g., geographic, occupational, or industrial) illustrates how competition among firms within a market for heterogeneous workers retains the principal neoclassical features of supply and demand. In Roy's model, the heterogeneous talents of workers lead to a rising supply price of labor to each market, and competition among identical firms within each market forces wages to equal marginal products worker by worker. However, if firms *within* each market are discretely heterogeneous, competition fails to generate unique wages. From this perspective, the following model extends Roy's model to a "thinner" environment.

### Environment

Firm  $j$  is characterized by a neoclassical production function  $F_j$  that maps efficiency units of labor  $L_j$  and physical capital  $K_j$  into output  $q_j$ :

$$q_j = F_j(L_j, K_j). \quad (1)$$

Following the matching tradition, I take each firm's production technology as given and hold the number of firms fixed at  $J$ . To allow for unemployment, some "firms" might be in the home sector.

With heterogeneous workers, the firm's labor input  $L_j$  emerges from valuing individual workers' skills and aggregating these valuations across all employees. Each worker  $i$  is a collection of  $n$  skills summarized by the vector  $s_i = (s_{i1}, \dots, s_{in})$ . Since some skills are more valuable in one firm than in another, the valuations of skills must be firm-specific. Let  $X_j(s_i)$  denote a firm-specific function that maps worker  $i$ 's vector of skills into

efficiency units of labor, a scalar value  $x_{ij}$ .<sup>1</sup> The  $x_{ij}$  are summed over all employees to produce firm  $j$ 's labor input  $L_j$ :

$$L_j \equiv \sum_{i=1}^I X_j(s_{i1}, \dots, s_{in}) \cdot E_{ij} \equiv \sum_{i=1}^I x_{ij} \cdot E_{ij}, \quad (2)$$

where each indicator variable  $E_{ij}$  equals unity if worker  $i$  is employed by firm  $j$ , and equals zero otherwise.

If each firm faced a market price of labor, the standard marginal analysis would derive demands for firm-specific efficiency units of labor. Rather than imposing price-taking behavior, I use the marginal productivity schedule to define  $\omega_j$ , the shadow price of labor in firm  $j$ :

$$\omega_j = P_j \cdot \frac{\partial F_j}{\partial L_j}, \quad (3)$$

which is a function of labor, capital, and the price of  $j$ 's product.

The marginal value of worker  $i$  to firm  $j$  is the product of the shadow price of labor in firm  $j$  and the amount of labor worker  $i$  delivers to firm  $j$ :

$$\begin{aligned} M_{ij} &\equiv P_j \cdot \frac{\partial F_j}{\partial L_j} \cdot x_{ij} \\ &\equiv \omega_j \cdot x_{ij}, \end{aligned} \quad (4)$$

where  $M_{ij}$  denotes worker  $i$ 's productivity value in firm  $j$ . In general, worker  $i$ 's productivity values vary across firms.

Before developing the operation of a decentralized labor market, it is useful to establish the properties of an efficient allocation of workers to firms in this environment.

### Efficient Matching

In the optimal match, is worker  $i$  assigned to the firm in which his productivity value is greatest? Yes. By definition, the optimal match maximizes the value of output *in the market*. Consequently, if the “maximal productivity” match were suboptimal, then it would be possible to reassign workers and thereby increase the value of output in the market. Since every possible reassignment involves a transfer of labor between firms, it is suf-

<sup>1</sup> This feature of the model has as antecedents the work of Mandelbrot (1962) and Heckman and Sedlacek (1985). In this literature, bundles of worker skills are transformed into “tasks,” which is the productive input.

ficient to show that any transfer of labor from the “maximal productivity” allocation reduces the value of output in the market. Transferring worker  $i$  (i.e., a small amount of labor) from his highest productivity firm to some other firm reduces the value of output in the market: the value of the sending firm’s output falls more than the value of the receiving firm’s output rises.<sup>2</sup> Therefore, the optimal match assigns each worker to the firm in which his productivity value is highest.

The importance of this result draws in part from the absence of such a property in prototypical matching models. Consider the model of Koopmans and Beckmann (1957). A key feature of the model is that assignments are one-to-one as in a monogamous marriage market (Becker 1973). Match values characterize the output or quality of each “marriage.” Although each worker has a highest match-value firm, efficient matching generally does not assign the worker to that firm (Koopmans and Beckmann 1957, p. 55; Becker 1973, pp. 824–25).

One way to structure the Koopmans-Beckmann model is to let the match values, denoted  $v_{ij}$ , be generated by a continuous function of indices of worker and firm quality:  $v_{ij} = f(l_i, k_j)$ , where  $f$  is an increasing concave function and  $f_{lk} > 0$  (Becker 1973; Sattinger 1980, pp. 98–101). This structure adds two features to the Koopmans-Beckmann model. First, worker productivity is well defined and given by  $f_l$ . Second, this is an ordered model. Matched with a firm of any quality level  $k$ , high  $l$  workers are more productive than low  $l$  workers. The optimal assignment matches the best worker with the best firm, down to the worst worker with the worst firm. In the ordered model, only the best worker matches with the firm in which his productivity value  $f_l$  is greatest.

<sup>2</sup> This can be established formally using two first-order Taylor series expansions. Let asterisks denote values in the maximal productivity assignment of workers to firms, and let firm 1 be  $i$ ’s maximal productivity match and firm 2 some other firm. Hence  $M_{i1}^* > M_{i2}^*$ . Let  $\Delta[P_1 F_1(L_1^*)]$  and  $\Delta[P_2 F_2(L_2^*)]$  denote the changes in the value of outputs at firms 1 and 2, respectively, that result from the transfer of worker  $i$  from firm 1 to firm 2:

$$\begin{aligned} \Delta[P_1 F_1(L_1^*)] + \Delta[P_2 F_2(L_2^*)] &= [P_1 F_1(L_1^* - x_{i1}) - P_1 F_1(L_1^*)] \\ &\quad + [P_2 F_2(L_2^* + x_{i2}) - P_2 F_2(L_2^*)] \\ &= P_1 \cdot \frac{\partial F_1}{\partial L_1} \cdot (L_1^* - x_{i1} - L_1^*) + P_2 \cdot \frac{\partial F_2}{\partial L_2} \cdot (L_2^* + x_{i2} - L_2^*) \\ &= -(M_{i1}^* - M_{i2}^*) < 0, \end{aligned}$$

where the derivatives are evaluated at  $L_1^*$  and  $L_2^*$ . The second step employs Taylor series expansions around  $L_1^*$  and  $L_2^*$ , and the final step follows from the definition of productivity value. Consequently, the total value of output falls from any such reassignment from the maximal productivity match.

By relaxing “monogamy” to allow for endogenous firm size, the matching model exhibits an intuitive property: in the optimal assignment, each worker matches with the firm in which his productivity value is highest. The next step is to determine whether a decentralized labor market supports the optimal assignment. It is here that rent sharing plays an important role.

### Rent Sharing and Wage Offers

Having characterized workers’ productivity values and the efficient allocation of workers to firms, I turn the analysis to the market’s pricing of labor. Must a worker be paid his productivity value in his optimal match? The answer is no. The shadow value of labor  $\omega_j$  determines the worker’s productivity value, not his wage payment. The marginal worker in firm  $j$  must be paid his productivity value, but firm  $j$  can *wage discriminate* against the inframarginal workers. Of course, each inframarginal worker has bargaining power, so one of a variety of mechanisms might determine wages. My approach is to analyze bargaining over wage contracts: the solution to a simple bargaining game determines the contract parameters. The analysis also allows for threats of monopsony wage setting by firms or monopoly wage setting by unionized workers to influence the contractual solution.

The wage contract is a simple rent-sharing agreement. Worker  $i$  and firm  $j$  sign a contract that pays the worker, if employed, a wage equal to his opportunity wage plus a share of the rents to the match. More generally, the wage contract generates wage offers to all workers. If worker  $i$ ’s opportunity wage exceeds his productivity value in firm  $j$ , firm  $j$  offers a wage equal to  $M_{ij}$ .

The wage contract governs how, in an auction, the  $J$  firms simultaneously bid for the  $I$  workers. In each round of bidding, a worker receives  $J$  bids. To each firm, he privately announces his best alternative wage offer. Firms follow the wage contract in revising their wage offers. The auction closes when no firm revises its wage offer to any worker—that is, when wage offers are mutually consistent. At the close of the auction, each worker accepts his highest-paying job offer.

Properties of the market equilibrium become clear by examining the market in three stages. First, given the shadow prices of labor, I solve for the set of mutually consistent wage offers. Second, these wage offers generate the supply of labor to each firm as a function of the shadow prices of labor. Armed with the marginal productivity schedule and the labor supply schedule facing each firm, I use standard results from general equilibrium theory to solve for a unique set of equilibrium shadow prices of labor. This produces a complete description of employment, productivity, and wages in a labor market with wage contracts. Third, given the equilibrium contractual payoffs to firms and workers, I investigate whether such wage contracts would be signed. Are there values of the rent-sharing

parameter for which the payoffs under wage contracts dominate payoffs under monopsonistic wage setting of firms or monopoly wage setting of union workers? These three stages are developed in turn.

Since a firm's wage offer to a worker depends on his opportunity wage, which also depends on outside wage offers, wage offers are interrelated. Let  $w_{ij}$  denote worker  $i$ 's wage offer from firm  $j$ , and let  $w'_{ij} \equiv \max_{k \neq j} w_{ik}$  define worker  $i$ 's opportunity wage, his best alternative wage offer. Formally, the wage contract is a piecewise linear function of worker  $i$ 's productivity value in firm  $j$  and  $i$ 's opportunity wage:

$$w_{ij} = \begin{cases} w'_{ij} + \beta_{ij} \cdot [M_{ij} - w'_{ij}] & \text{if } M_{ij} \geq w'_{ij}, \\ M_{ij} & \text{if } M_{ij} < w'_{ij}, \end{cases} \quad (5)$$

where  $0 < \beta_{ij} \leq 1$  is the rent-sharing parameter. The rent-sharing parameter reflects the determinants of bargaining strength other than threat points in a cooperative bargaining game. Aside from a refinement introduced below, one may think of  $\beta_{ij}$  as the generalized Nash solution to the bargaining game that is played if the firm employs the worker.

With all firm-worker pairs writing these wage contracts, the wage offers are interrelated. For given shadow prices of labor, is there a unique, mutually consistent set of wage offers  $\mathbf{w}_i = (w_{i1}, \dots, w_{ij})$  to each worker  $i$ ? For each worker  $i$ , (5) is a system of  $J$  equations in  $J$  unknowns. In the Appendix, I use the contraction-mapping theorem to establish that a unique solution exists. The solution,  $\mathbf{w}_i$ , is the vector of offers received by worker  $i$ . For convenience, index firms such that  $w_{i1} \leq \dots \leq w_{ij}$ . Since  $w_{ij} \leq w_{ij}$ , rejected wage offers equal productivity values. For  $j = 1, \dots, J-1$ ,

$$w_{ij} = M_{ij}. \quad (6a)$$

Therefore, the accepted wage is a convex combination of the worker's productivity values in his best two matches:

$$\begin{aligned} w_{ij} &= \beta_{ij}M_{ij} + (1 - \beta_{ij})w_{i,J-1} \\ &= \beta_{ij}M_{ij} + (1 - \beta_{ij})M_{i,J-1} \\ &= M_{i,J-1} + \beta_{ij}(M_{ij} - M_{i,J-1}). \end{aligned} \quad (6b)$$

Although a firm does not have direct knowledge of a worker's productivities on all other jobs, market wage offers reveal the only relevant information, the worker's productivity value on his next-best job.

### Labor Market Equilibrium

There exists a set of mutually consistent wage offers for any vector of shadow prices of labor  $\mathbf{\omega} = (\omega_1, \dots, \omega_j)$ . But mutual consistency is only

one property of equilibrium wage offers. Equilibrium wage offers must also clear the labor market. Thus, the second stage is to solve for the equilibrium vector of shadow prices  $\omega^* = (\omega_1^*, \dots, \omega_I^*)$ , which in turn generate equilibrium productivity values, wage offers, and matching. A key result is that labor is not perfectly elastically supplied to any firm. With downward-sloping demand and upward-sloping supply functions at the firm level, each firm appears as its own labor market.

The supply of labor to firm  $j$  is given as the solution to the  $I$  workers' matching choice problem. The labor supply function  $L_j^*(\cdot)$  is the sum of individual supplies:

$$L_j^*(\omega) = \sum_{i=1}^I X_j(s_i) \cdot D_{ij}(\omega), \quad (7)$$

where each indicator variable  $D_{ij}(\omega)$  equals one if the wage offers satisfy  $w_{ij} > w'_{ij}$ , and equals zero otherwise. (The previously imposed indexing convention is relaxed.) Equations (6a) and (6b) imply that the supply indicators  $D_{ij}$  do not depend on the rent-sharing parameters. Consequently, the supply of labor to firm  $j$  does not depend on bargaining strength in any match.

Firm  $j$ 's labor supply is an increasing function of its shadow price of labor and a decreasing function of the shadow prices in all other firms. In particular, firm  $j$ 's labor supply is an increasing step function of  $\omega_j$ . With shadow prices in other firms taken as given, firm  $j$ 's wage offers to all  $I$  workers increase with  $\omega_j$ . This results in a marginal worker switching from some other firm to firm  $j$ . A sufficiently higher  $\omega_j$  draws in another worker. Thus, varying  $\omega_j$  "sweeps out" the distribution of workers. For large  $I$ , it is innocuous to abstract from the discontinuity of supply and to treat each firm's labor supply as a continuous function of shadow prices. Thus, indivisibilities are ignored.

In the absence of discontinuities, establishing the existence of a general equilibrium in the labor market is entirely conventional. Since the units of labor are gross substitutes across firms, a unique, globally stable, general equilibrium is guaranteed to exist. This equilibrium equates marginal productivity schedules, equation (3), to labor supply schedules, equation (7), simultaneously across all firms to yield solution vectors  $\omega^* = (\omega_1^*, \dots, \omega_I^*)$  and  $L^* = (L_1^*, \dots, L_I^*)$ . Since each firm is its own market, the equilibrium shadow prices of labor generally vary across firms.<sup>3</sup> And unlike

<sup>3</sup> For particular specifications of the model, there is a common shadow price of labor. If skills are mapped into units of labor by firm-specific *linear* functions of a *single* skill, then rents vanish, and the shadow prices converge. Firms that value the skill most employ more labor, which lowers the marginal product of labor. For this specification, the two effects offset exactly.

in hedonic pricing models (e.g., Tinbergen 1956; Rosen 1974), the underlying skills are not priced out in equilibrium.

The equilibrium shadow prices  $\omega^*$  are employed directly in determining equilibrium productivity values  $M_{ij}^*$ , equilibrium wage offers  $w_{ij}^*$ , and equilibrium values of the indicator variables,  $E_{ij}^*$  and  $D_{ij}^*$ , for each firm-worker pair.<sup>4</sup> Equilibrium productivity values are  $M_{ij}^* \equiv \omega_j^* \cdot x_{ij}$ . Again adopt the indexing convention that  $w_{i1} \leq \dots \leq w_{iJ}$ , then worker  $i$ 's vector of equilibrium wage offers  $w_i^*$  satisfies

$$w_{ij}^* = M_{ij}^*, \quad j = 1, \dots, J-1, \quad (8a)$$

and

$$w_{ij}^* = \beta_{ij} M_{ij}^* + (1 - \beta_{ij}) M_{i,J-1}^*. \quad (8b)$$

*Equilibrium wage offers are flexible.*—The  $w_{ij}^*$  vary with productivity values. The accepted wage offer is increasing in the rent-sharing parameter  $\beta_{ij}$  and the worker's productivity value within the consummated match  $M_{ij}^*$ . The accepted wage  $w_{ij}^*$  is also increasing in the worker's best alternative productivity  $M_{i,J-1}^*$ . Worker  $i$ 's bargaining power in any other firm does not affect his accepted wage  $w_{ij}^*$ .<sup>5</sup>

*Equilibrium wage offers induce efficient matching.*—The market's allocation of labor is efficient even though each worker is paid less than his productivity value; for  $j \neq J$ ,  $M_{ij}^* = w_{ij}^* \leq w_{ij}^* \leq M_{ij}^*$ .<sup>6</sup> Since the  $\beta_{ij}$  govern the sharing of rents, the rent-sharing parameters do not influence the market's

<sup>4</sup> With the assistance of Andre Litster, I have developed an interactive GAUSS program that computes the complete equilibrium. The program begins by drawing workers from distributions of skills, and firms from distributions of production technologies. Discontinuities, from which I abstract in the text, are treated explicitly in solving for the equilibrium shadow prices. For matching problems with 15 skills, 100 workers, and 10 firms, the algorithm iterates to the equilibrium within 15–20 seconds on a 33-MHz 386 microcomputer.

<sup>5</sup> One could compute general equilibrium comparative statics effects, but the results would be quite standard. For instance, a new worker added to the labor market would be matched with the firm in which he is most productive. The increased supply of labor to this firm drives down its shadow price of labor, which leads marginal workers to find jobs in other firms more attractive. The increased supply to other firms changes shadow prices there, and these feed back to the original firm. Partial equilibrium comparative statics would miss the equilibrium feedback.

<sup>6</sup> Equilibrium values of the indicator variables are  $E_{ij}^* = 1$  for demand because  $w_{ij}^* \leq M_{ij}^*$ , and  $D_{ij}^* = 1$  for supply because  $w_{ij}^* \leq w_{ij}^*$  for all  $j \neq J$ ; and  $E_{ij}^* = D_{ij}^* = 0$  for all  $j \neq J$ .

allocation of labor. In particular, the wage contract produces an ordering of wage offers that is coincident with the ordering of productivity values.<sup>7</sup>

Identifying the source of rents is important for understanding the equilibrium. Although heterogeneity of both firms and workers is required, the discrete heterogeneity of firms is crucial. For instance, there can be rents associated with employing each of a dozen identical workers. If no other firm values their skills so highly, one firm employs all 12, each at a wage less than the common productivity value. (In typical monogamous matching models, competition among the 12 would drive the wage down to the opportunity wage, and the firm would capture all the rents.) However, with even two identical firms, rents to any worker employed by either firm would vanish: the worker's productivity value is the same in both firms, so the worker's wage must equal his productivity value.<sup>8</sup> Thus, rents must be supported by differences across firms, such as differences in product markets, production technologies, capital stocks and vintages of capital, locations, or how skills are valued.

### To Contract, or Not to Contract

The analysis to this point establishes the properties of a labor market with wage contracts. The third stage of the analysis explores whether firms and workers would agree to the wage contracts. Participation depends on anticipated bargaining strength and alternative mechanisms for determining wages. In deciding whether to agree to the wage contract, each side anticipates its *ex post* bargaining strength to determine its payoff from the wage contract. Weak bargainers—workers with low values and firms with high values of the rent-sharing parameter—might prefer the payoffs from unilateral wage setting. Moreover, *ex ante* threats not to contract over wages might influence equilibrium contractual wages.

Each firm has the option to behave as a monopsonist in marginalizing its rising supply curve of labor. If the  $\beta_{ij}$  equal one, firm  $j$  captures none of the rents; each worker is paid his productivity value. By rejecting the wage contracts, such a firm could capture some of the rents by reducing

<sup>7</sup> The property of efficiency allocations in models with thin markets is not novel. In the most familiar setting, exchange between two players in an Edgeworth box generally admits a large number of individually rational, Pareto-optimal allocations, which leaves room for bargaining. In a two-sided exchange economy with indivisibilities, Kaneko (1982) establishes that each one of a generally large number of core allocations can be supported by a set of competitive prices. Thus, it is well understood that thinness, indivisibility, and bargaining do not imply inefficiency. Nevertheless, efficiency of the rent-sharing equilibrium is fairly novel: abstracting from indivisibilities, a unique set of bargained wages generates an efficient allocation of labor despite the thinness of the labor market.

<sup>8</sup> As a result, replicating the economy—introducing twins for every firm and worker—drives all rents to zero.

employment, driving up marginal products, and paying a single low wage to all its workers. For  $\beta_{ij}$  close to zero, the wage contract approaches perfect wage discrimination by an employer, which dominates monopsony wage setting. Thus, there exists a critical value such that for  $\beta_{ij}$  less than this value the firm prefers the wage contract to monopsony wage setting. This auxiliary threat narrows the range of admissible values for the rent-sharing parameter.

In contrast, there are strong incentives for workers to unionize to set wages unilaterally. Even if workers capture all the rents to their matches, this would raise their wages to only their productivity values. Monopoly wage setting, however, enables workers to capture some producer surplus associated with labor's average product exceeding its marginal product. For now, assume competition from displaced workers would undercut monopoly wage setting, supporting the wage contracts and the rent-sharing equilibrium. Analysis of the effect of union bargaining on the rent-sharing equilibrium is deferred to Section II below.

### Relaxing the Informational Assumption

The model allows firms and workers to contract in terms of wage offers and productivity values. This would be possible if knowledge of these two variables were common to the employment pair. It would also be possible if private information were verifiable. With verifiable information, one side's disclosures can be freely verified by the other side. In games of persuasion Milgrom and Roberts (1986) establish that full disclosure is incentive compatible if information is verifiable: one rationally assumes the worst about information that remains suppressed. (Also see Grossman 1981; Milgrom 1981.) In this context, verifiability precludes bluffing or fraud in revealing private information about wage offers and productivity values.<sup>9</sup>

Alternatively, firms and workers might be symmetrically uniformed about wage offers and productivity values. With symmetrically incomplete information, the firm and worker agree about the expected value of productivity in this match and about the expected value of the worker's best outside wage offer. With these expected values replacing actual values in

<sup>9</sup> As Milgrom and Roberts (1986, p. 19) point out, penalties for false reports can substitute for the verifiability assumption without damaging the principal implication of full disclosure. If information were not directly verifiable, sufficiently sure and heavy penalties for false or distorted reporting would make full disclosure incentive compatible. Indeed, Riordan and Sappington (1988) demonstrate that noisy ex post information that is public can be used to overcome the problems of ex ante private information. In the context of employment matches, adding a random bonus/penalty term—contingent on the ex post information—might be sufficient to support full disclosure of wage offers and productivity values.

the wage contract, the model's equilibrium would match each worker to his highest *expected* productivity match.

Search is another alternative that weakens the informational assumption. Workers typically do not receive wage offers from every firm, so suppose each worker samples wage offers from only a subset of firms. Wage offers from other firms are set to zero. With this modification, the model generates a unique rent-sharing equilibrium given the sampling rule, and this equilibrium retains all its properties including efficiency.<sup>10</sup> However, markets would be thinner, which would increase the quantitative importance of rents and rent sharing.

### Turnover

To prepare for the applications that follow, it is useful to explore the rent-sharing model's implications for labor turnover. What are the effects of rents, thinness, and bargaining power on mobility from firm to firm? I consider two sources of turnover: dynamics along the path to the rent-sharing equilibrium, and shifts in the rent-sharing equilibrium resulting from changes in exogenous variables.

If the bidding process were to take place in real time, with employment occurring in each round of bidding, workers would move from firm to firm as the labor market converged to its unique rent-sharing equilibrium. Along this path, workers would be temporarily mismatched and would reshuffle toward superior matches. The general equilibrium dynamics are familiar, and bargaining adds nothing.

Turnover also results from shifting the rent-sharing equilibrium. If in a sequence of spot markets the exogenous variables and functions—product prices, capital stocks, production and skill-valuation functions, and the supply of skills—varied stochastically from period to period, the equilibrium matching of workers to firms would change. Through general equilibrium interactions, shifting the product price, capital stock, or production function in *any* firm would lead to turnover of *marginal* workers: declining firms lose marginal workers to other firms; growing firms gain marginal workers. Shocks to *any* firm's skill-valuation function or to the skills of *any* worker would generate more idiosyncratic turnover: some inframarket

<sup>10</sup> If search were costly and search intensity were endogenous, the efficiency of the rent-sharing equilibrium would be tempered. First, Mortensen (1978) demonstrates that “counteroffer matching” generates excessive search intensity as a form of rent seeking. The wage contract includes a counteroffer-matching component, so the model includes this force toward excessive search. Second, Mortensen (1986) also establishes that with rent sharing the worker confers an external benefit to his employer in finding a superior match, which is a force toward insufficient search. The two forces work in opposite directions, and which force dominates is not pinned down. Nevertheless, search intensity is generally not efficient.

ginal workers could separate from growing firms to be hired by declining firms.

Changes in bargaining strength, however, do not generate turnover. Indeed, the separation rate is independent of the rent-sharing parameters. The wage contract generates efficient matching in each period, so worker  $i$ 's best match in any period is independent of the rent-sharing parameters. Therefore, worker  $i$ 's probability of changing employers (i.e., the separation rate) is independent of his or anyone else's bargaining strength. This does not imply that quits are less frequent from jobs where workers have more bargaining power. Indeed, applying McLaughlin's (1991) efficient turnover model to the rent-sharing environment implies that quits are a decreasing function, and layoffs an increasing function, of bargaining power even though all turnover is efficient.<sup>11</sup>

## II. Applications

The rent-sharing model abstracts from many rich features of labor markets. Despite its level of abstraction, the model can be fruitfully applied to sharpen our understanding of a variety of labor market issues. The starting point is to identify markets where thinness is most salient. Rents should be quantitatively more important where firms are differentiated in terms of their valuations of workers: firms might operate in different product markets, or have different clients, locations, or amounts and vintages of equipment. One example is a single firm or an employment cartel (e.g., professional sports leagues and collegiate athletic associations) operating in a specialty area, with competition limited to firms outside the specialty. More generally, with many dissimilar firms in a market, a particular worker could be more highly valued by one of the firms. But with many similar firms in an area, such as gasoline stations or fast-food restaurants in a city, quantitatively important rents should not exist.

The model is most applicable where workers have specialized skills or training and search is costly. Literacy commands a price, but it does not

<sup>11</sup> In McLaughlin (1991), the quit (layoff) rate is the probability that the following joint event obtains: the worker separates from his employer, and the wage offer from the new employer exceeds (falls short of) the worker's preseparation wage. The more successful a worker is in capturing rents from his period- $t$  employer, the higher is his wage in period  $t$ . Since separation rates are independent of bargaining strength, greater bargaining strength on his period- $t$  job lowers (raises) the probability the worker separates to a higher (lower) paying job in period  $t + 1$ . Therefore, the quit rate is a decreasing function, and the layoff rate is an increasing function, of the worker's share of the rents on the job he separates from. Similarly, the greater the worker's bargaining strength on his new job, the more (less) likely his separation is labeled a quit (layoff). A formal derivation of these results, including the result on the invariance of the separation rate to bargaining power, is available from the author.

generate rents. The ability to cap oil fires, as a specialized skill, is both valuable and capable of supporting rents. But skills need not be unique. With relatively scarce skills, a firm's supply curve would slope up, and rents would obtain. Furthermore, even if there were many workers with a particular skill, the analysis from Section I indicates that search costs thin out the market.

Unskilled workers are not excluded from the model, but rents for these workers are not predicted to be quantitatively important. The analysis points to managers, professionals, scientists, and skilled technicians and craftsmen as the workers on jobs with potentially important rents. As an empirical guide, low turnover rates are likely to signal jobs with important rents to be shared.

The rent-sharing model applies particularly well to markets for superstars. These markets match the best athletes to teams, actors and actresses to parts, directors to scripts, vocalists to songs, musicians to orchestras, and authors to publishing companies. Since my consumption of a performance does not exclude your consumption, the public good feature of superstar markets supports large differences in productivity values for small differences in skills (Rosen 1981). For instance, the talents of one actress can be ideally suited to a part in a particular film. Being a little better for the part could be worth millions to the movie company, so even with competition from parts in other films, rents of millions of dollars might remain.

From this background, I turn to more detailed applications. To highlight the model's more novel aspects, I investigate the effects of bargaining over rents in three applications: union bargaining, interindustry wages differentials, and the relationship between pay and profit. Although the topics are familiar, the rent-sharing model offers fresh perspectives and new insights.

### Union Bargaining

Since union status is observable and almost certainly related to bargaining power, the application to union bargaining is natural. In standard models of union bargaining, with monopoly or efficient contracting unions, workers bargain collectively to capture some of the difference between the value of labor's average and marginal products at the competitive level of employment. So collective bargaining raises wages above a competitively determined productivity value at the expense of producer surplus.

The role of unions in bargaining for higher wages is different in the rent-sharing model. Each worker is generally paid less than his productivity value, so increasing bargaining power through unions pushes workers' wages up toward their productivity values. Thus, unions in the rent-sharing model attenuate monopsonistic exploitation of workers. This feature is similar to union wage bargaining to prevent a labor market monopsonist

from driving wages and employment down along the supply curve. However, the rent-sharing model extends this from collective bargaining to individual bargaining.

Allowing workers to bargain separately limits the role of the rent-sharing union. In a first stage, the rent-sharing union bargains collectively over the rules that govern subsequent bargaining between individuals workers and employers. In setting the environment for individual bargaining, the union improves its workers' bargaining position in their individual negotiations. This structure broadly characterizes the role of unions in sports, where collective bargaining focuses on free agency and arbitration rather than unionwide wages. Thus, modeling unions in negotiating the rules for individual bargaining fills a gap in the literature.

In superstar markets with concentrated employers, collusion among employers to suppress salaries is common (e.g., Quirk and Fort 1992). Forming an employment cartel—via the reserve clause in a professional sports league, or amateur status in collegiate athletic associations, or the studio system in Hollywood—turns a fairly thick market with small rents into a thin market with large rents. Competition among employers, which would push wages up toward productivity values, is limited by reserve or other “no tampering” clauses, which are enforced by cartel sanctions (e.g., blackballing). Without competition from within the profession, a star performer’s best outside opportunity lies outside his specialty. This drives a large wedge between a star’s productivity value and his best alternative. Since bargaining in these artificially created thin markets is typically done at the individual level, the rent-sharing model applies.

In sports, the response to such employment cartels has been collective bargaining. Counter to the approach of standard models of union bargaining, the sports union does not bargain a wage for its players. The difference between the productivity value of a star athlete and a journeyman player is too great to absorb the wage compression that characterizes most union wage structures. Heterogeneity precludes the standard solution, so collective bargaining in professional sports is over the bargaining environment, such as the reserve clause, salary arbitration, free agency, compensation for lost free agents, and amateur drafts. Each player is left to bargain his own wage.

From the perspective of the rent-sharing model, forming the union improves the bargaining position of players and drives up pay.<sup>12</sup> Free agency

<sup>12</sup> Hollywood’s studio system, which tied actors, actresses, writers, and directors to particular studios in the 1930s and 1940s, was not reformed by collective bargaining. (At the time, Hollywood was completely unionized by guilds, such as the Screen Actors Guild.) In 1944 the California Supreme Court ruled that Warner Brothers’ long-term contract with Olivia De Haviland violated the state’s anti-peonage laws. As a substitute for collective bargaining, this ruling granted free

promotes competition, which thickens the market and raises each player's threat point; salary arbitration improves a player's bargaining power given his threat point. The biggest gains are to star performers, as marginal performers are paid close to their expected productivity values even under the reserve clause (or studio system). Competition among players prevents any player's pay from exceeding his expected productivity. This property of rent-sharing unions sharply contrasts with the implications of traditional models of union bargaining, in which the union's goal is to raise wages above the competitively determined productivity value. In addition, rent-sharing unions do not distort employment or turnover decisions. Again, this efficiency property applies to neither the monopoly union model nor the efficient contracting model in general equilibrium (Layard and Nickell 1990).

The rent-sharing model captures individual salary negotiations within collective bargaining agreements in professional sports as a response to monopsonistic employment cartels. It also helps to explain the existence of salary floors in these agreements. Marginal players have the least to gain from collective bargaining in a rent-sharing union. To entice these players to support the players union, they are paid off with salary floors as part of the collective bargaining agreement. Since salary floors overpay marginal players, an otherwise efficient rent-sharing equilibrium can be distorted.

### Interindustry Wage Differentials

As many have recognized, differences in wages across industries for workers with similar skills are inconsistent with the textbook competitive model. Aside from temporary deviations from long-run equilibrium, or unobserved differences in ability or attributes of jobs, industry wage effects should not exist. The persistence of estimated interindustry wage differentials leads Krueger and Summers (1987, 1988) to reject the framework of competitive labor markets in favor of rent sharing in a segmented labor market. But rent sharing and segmented labor markets are not intrinsically linked. How much of the interindustry wage structure is attributable to efficient rent sharing? And how much reflects labor market segmentation?

The equilibrium model of rent sharing might account for interindustry wage differentials if rents or bargaining power vary across industries. Suppose the importance of matching—supported by relative diversity of firms, scarcity of skills, and costliness of search—varies across industries. Rents are small in industries where firms and workers are fairly homogeneous. Alternatively, in industries where firms are diverse, workers have specialized

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agency, which thickened the market and drove up the pay of actors, actresses, writers, and directors. By 1945, only about 25% of screen actors and actresses, 18% of screen writers, and 33% of motion picture directors remained under long-term studio contracts (Gomery 1986, p. 10).

skills and training, and search is costly, there would be a bargaining-determined premium to specialized skills.

These implications are supported empirically.<sup>13</sup> First, high-wage industries tend to have large firms: the correlation between industry wage effects and establishment size is positive. Second, industries with low turnover rates tend to pay higher wages: the correlation between industry wage effects and average job tenure in the industry is positive. These are two examples of a broader empirical theme: industry wage effects are largely attributable to observable characteristics of industries (e.g., capital intensity, establishment size, accounting returns, and unionization) and to characteristics of their workforces (e.g., average education and turnover), many of which tie closely to thinness. And since thinness-related characteristics of industries persist, the model captures the strong regularity that industry wage effects persist.

From the perspective of efficient rent sharing, would industry wage effects for second-shift floor sweepers make sense? Not if the market for sweepers were thick. Thus, the rent-sharing model predicts that industry wage effects vary by occupation. This is consistent with the evidence of smaller industry wage effects for blue-collar workers. But a substantial component of industry wage effects is common across occupations. If industry characteristics determine bargaining strength, industry wage effects would have a component that is common across occupations.

That application rates tend to be higher in high-wage industries is also consistent with efficient rent sharing. In the model's simplest form, workers apply to *all* jobs to discover their most productive matches. So the search extension is important for capturing interindustry patterns in applications data. The option value from search suggests that workers would sample high-paying industries (and occupations) to check whether they have the skills that command the wage premia. (A worker without such skills would receive a low wage offer and would escape to a higher-paying job in a "low wage" industry.) Thus, the search extension predicts a positive correlation between application rates and industry wage effects, if the latter are the product of efficient rent sharing.

Since movers tend to be marginal workers, the rent-sharing model predicts zero or small industry wage effects for interindustry movers. The evidence points to smaller industry wage effects for movers, but estimated industry wage effects for movers are not zero. Self-selection is an issue. In

<sup>13</sup> For the empirical regularities of interindustry wage differentials, I draw evidence from Krueger and Summers (1987, 1988), Dickens and Katz (1987a, 1987b), Murphy and Topel (1987), Holzer, Katz, and Krueger (1991), and Bils and McLaughlin (1993). Groshen (1991) documents similar regularities for *establishment* wage differentials. Applying the rent-sharing model to establishment wage differentials is similar.

particular, even if industry wage effects vanished for interindustry movers, cross-sectional industry wage effects could reflect firms sharing rents with their inframarginal workers. However, jumps in pay from moving into or out of an industry would not be consistent with efficient rent sharing. To the extent these jumps in pay are empirically important and are not attributable to interindustry differences in the attributes of jobs, the evidence would point to labor market segmentation rather than efficient rent sharing.<sup>14</sup>

Nevertheless, since the efficient rent-sharing model is broadly consistent with the empirical regularities of interindustry wage differentials, two conclusions of Krueger and Summers (1987, pp. 43–44; 1988, p. 281) seem premature. First, without segmented labor markets, important interindustry wage differentials do not “create a *prima facie* case for the existence of involuntary unemployment.” Second, their policy proposal—subsidizing high-wage industries—would distort an efficient equilibrium if the interindustry wage structure arises from efficient rent sharing.

### Wages and Profits

In the short run, interindustry wage differentials can reflect the influences of product demand and technology shocks. But the standard competitive model implies that wages in a firm are not related to the determinants of its revenue: even short-run wage effects must be zero at the firm level. Since the firm is a price taker in the labor market, fluctuations in its product price or marginal productivity schedule generate only employment fluctuations. Although firm level co-movements of wages and profits (or other measures of firm performance) are not consistent with a competitive labor market, the rent-sharing model predicts such co-movements and retains the main themes of the competitive model.

The efficient rent-sharing model implies that wages in a firm do depend on the firm’s performance. Since the firm’s labor supply schedule slopes up, variables that affect the value of marginal product schedule also affect wages. Measures of firm performance, such as profits, are predicted to covary positively with wages. And the strength of the relationship between wages and profits should depend on the thinness of the market: where firms are relatively diverse, skills relatively scarce, and search relatively costly, the model predicts a stronger relationship between wages and profits. Thus, the link between pay and profit is predicted to be stronger where turnover rates are low.

<sup>14</sup> Not all the evidence is consistent with equilibrium rent sharing. Why are wages so high in construction? The high turnover rates in construction signal a thick market, so industry wages should reflect either the usual competitive forces or unionism. Since estimated industry wage effects are an increasing function of union density, the relative success of construction unions—rather than equilibrium rent sharing—is the answer.

These results apply to the literature on executive compensation, which relates an executive's pay to the performance of his firm. Team incentives—linking individual pay to firm performance to motivate workers—are the standard economic interpretation of executive compensation. But the rent-sharing model also casts some light on the evidence. Since the market for top executives is thin, the rent-sharing model predicts a positive relationship between executive compensation and firm performance. And it helps to explain why the relationship is stronger for chief executive officers (CEOs) than for line workers and clerical staff: the labor market for CEOs is thinner. The rent-sharing model also predicts a stronger link where workers are more heterogeneous. Models of team incentives predict the opposite: to deter adverse selection, the link between individual pay and firm performance must be weaker if workers are heterogeneous (McLaughlin 1993).

### III. Conclusions

Imagine a social planner confronted with the seemingly simple job of allocating 100 heterogeneous workers among 10 heterogeneous firms. To determine where any one worker is valued most, the planner must know where all the other workers are allocated. Add to this complication that the planner is unlikely to know how the workers' diverse skills are valued by each firm. Would a market help or hinder matters? Each firm knows how to value its potential employees, but pricing in a thin market complicates the problem.

In this article, two great economic insights of this century are merged to generate an efficient market solution to such a problem. Hayek (1945, p. 521) articulated the important informational function of the pricing system when individuals possess dispersed bits of incomplete knowledge of the "particular circumstances of time and place." Coase (1937) recognized that pricing is costly, so contractual relations can substitute for prices in markets with frictions. My model of rent sharing reflects a mix of market exchanges and contracts.

I introduce a simple self-enforcing contract that governs how wage offers respond to productivity within the employment pair and outside wage offers. Placed in the thin-market setting of heterogeneous firms and workers, these interrelated wage contracts generate an efficient allocation of labor across firms. Thus, the model reminds us that market inefficiencies are not the inevitable consequence of thinness. In addition, since the allocation of workers to firms is invariant to the distribution of bargaining strength, the results support thin-market models that assume each worker's wage equals his (expected) marginal product (e.g., Jovanovic 1979).

The distinctly neoclassical flavor of the model stems from allowing diverse skills to be transformed into units of labor that can be aggregated across workers. With endogenous firm size, each firm appears as a distinct labor market with a downward-sloping demand for, and an upward-sloping

supply of, labor. Consequently, these results clarify and extend Buchanan and Tollison's (1981) analysis of a firm facing a rising supply price of a heterogeneous factor of production.

It will surprise no one that the model abstracts from many of the rich features of the labor market. Despite the level of abstraction, the applications to union bargaining, interindustry wage differentials, and the relationship between pay and profit suggest that the model accounts for many salient features of labor markets.

## Appendix

### Mutually Consistent Wage Offers

The purpose of this Appendix is to establish the existence of a unique solution to (5), a system of  $J$  equations for each worker  $i$ . To do so, I employ the contraction-mapping theorem, which also guarantees convergence.

The system of  $J$  equations (5) can be written as a single functional equation for worker  $i$ . (The  $i$  subscript is suppressed throughout.) For  $j \in D = \{1, \dots, J\}$ , the wage-offer function  $w(j)$  maps from  $D$  into the nonnegative subset of the real line:

$$w: D \subseteq R \rightarrow R^+. \quad (\text{A1})$$

Consequently, the single functional equation is

$$\begin{aligned} w(j) &= \beta(j)M(j) + [1 - \beta(j)] \cdot \min\{M(j), f[v(j)]\} \\ &= (Tv)(j) \quad \text{for all } j \in D, \end{aligned} \quad (\text{A2})$$

where  $\beta(j) \in (0, 1]$ ,  $M(j) \in [0, \bar{M}]$ ,  $f[v(j)] = \max_{k \neq j} v(k)$ , and  $T$  is a functional operator.

Let  $S = \{w: D \rightarrow [0, \bar{M}]\}$  be the space of bounded functions  $w$  with the sup norm as its metric. Note that, in equation (A2),  $T$  maps  $S$  into  $S$ .

**PROPOSITION.** In (A2),  $T: S \rightarrow S$  is a contraction mapping.  
*Proof.* By Blackwell (1965), it is sufficient to establish the following two conditions.

1) *Monotonicity*.— $w, v \in S$  and  $w(j) \leq v(j)$  for all  $j \in D$  implies that  $(Tw)(j) \leq (Tv)(j)$  for all  $j \in D$ .

2) *Discounting*.—For  $w \in S$ ,  $\alpha \in R^+$ , and some  $\gamma \in [0, 1)$ ,  $[T(w + \alpha)](j) \leq (Tw)(j) + \gamma\alpha$  for all  $j \in D$ .

Since  $w(j) \leq v(j)$  implies  $\max_{k \neq j} w(k) \leq \max_{k \neq j} v(k)$  for all  $j \in D$ , monotonicity is immediate:

$$\begin{aligned} (Tw)(j) - (Tv)(j) &\leq 0 \\ \text{as } \min\{M(j), f[w(j)]\} &\leq \min\{M(j), f[v(j)]\}. \end{aligned} \quad (\text{A3})$$

For discounting,

$$\begin{aligned}
 [T(w + \alpha)](j) &= \beta(j)M(j) + [1 - \beta(j)] \cdot \min\{M(j), f[w(j)] + \alpha\} \\
 &= \beta(j)M(j) + [1 - \beta(j)] \cdot \min\{M(j), f[w(j)] + \alpha\} \\
 &\leq \beta(j)M(j) + [1 - \beta(j)] \cdot \min\{M(j) + \alpha, f[w(j)] + \alpha\} \quad (\text{A4}) \\
 &\leq \beta(j)M(j) + [1 - \beta(j)] \cdot \min\{M(j), f[w(j)]\} \\
 &\quad + [1 - \beta(j)] \cdot \alpha \\
 &\leq (Tw)(j) + [1 - \beta(j)] \cdot \alpha.
 \end{aligned}$$

With  $\beta(j) \in (0, 1]$ , define  $\gamma = 1 - \max_j \beta(j)$ ; thus,  $\gamma \in [0, 1)$ . This establishes the discounting condition. Therefore,  $T$  is a contraction mapping. Q.E.D.

By the contraction-mapping theorem, there exists a unique function  $w^*: D \rightarrow R^+$  that solves the functional equation  $w(j) = (Tw)(j)$ . In addition, from any initial function  $w_0 \in S$ , the sequence  $w_{n+1}(j) = (Tw_n)(j)$  converges to  $w^*(j)$ . For this application, convergence is generally quick. One can show that convergence obtains in no more than four rounds.

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