Temple University
Economics 1902
Spring 2011
Exam 3

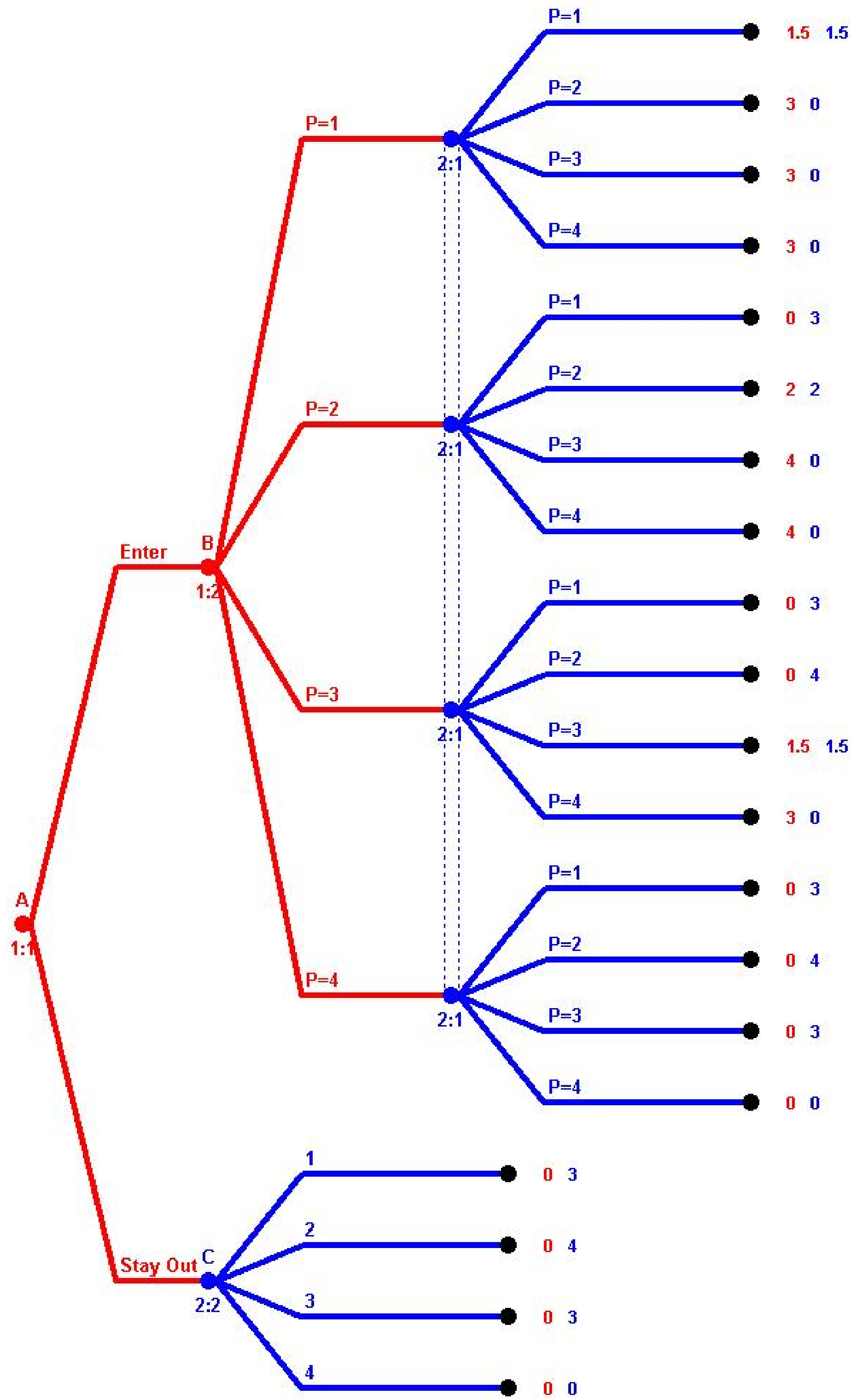
Name ____

Directions: You have two hours for the exam and must complete all questions. Your answers must be your own. You may neither give nor receive help. This is a closed book exam. You may use a calculator, but not a cell phone during the exam. It is advisable to show your work.

1. Subgame Perfection. There are two firms (Art's Advanced Autos, known henceforth as AAA, and Mike's Monster Motors, known henceforth as MMM) that set prices in a market with this demand curve: Q=4–P, where P is the lower of the two prices. (Note that the demand curve can also be written as P=4-Q.) If the two prices are different then the lower priced firm sells all of the output. If the two firms post the same price, then each gets half the market. Prices can only be posted in whole numbers and costs are zero.

   AAA is the incumbent firm. Prior to the price competition phase MMM has to make an entry decision. If MMM stays out then AAA retains its monopoly and makes a profit maximizing decision based on the given demand curve. If MMM enters then price competition ensues.

   a. Draw the extensive form of the game and identify the subgames. Be sure to label the players, nodes, and strategies. **The extensive form of the game tree is on the next page. There are two proper subgames at nodes B and C. The whole game is also a subgame.**



b. If MMM enters the market, what must be the price of the good?

If MMM enters then the two firms are playing a subgame that is a game of imperfect information that starts at node B. This is a Bertrand competition game like the one we played some time ago. This subgame has a dominant strategy solution. That solution is <P=1, P=1> and results in total demand Q=3, which they share equally. The payoffs are then (1.5, 1.5).

c. If MMM does not enter the market, what will be the price of the good? **When MMM does not enter the market, AAA has a monopoly. The profit maximizing price and output are P=2 and Q=2, for profit equal to 4**

d. What are the strategies used by MMM and AAA in the subgame perfect equilibrium?

Each player has to have a plan that includes a strategy for each subgame in which they have a move. Since we have drawn the game with MMM making the first move we will put his plan first.

<(Enter, P=1); (P=1, P=2)>

The solution to the game is that MMM enters and both players charge P=1.

2. Static Games of Incomplete Information. There are two firms in the telecommunications market. One firm (Kate's Kalkulators, or KK for short) currently produces a tablet computer. The other firm (Sally's Satcom, or SS for short) can produce either a competing tablet computer or a performance enhancing add-in for KK's computer, but not both.
When SS produces a competing tablet computer then the payoff matrix for the two firms is given by

KK and SS are both producing **substitute** tablet computers			Sally's Satcom		
			Price		
			High	Medium	Low
Kate's Kalkulators	Price	High	5, 5	0, 8	0, 6
		Medium	8, 0	4, 4	0, 6
		Low	6, 0	6, 3	3, 3

When SS produces the performance enhancing add-in for KK's computer then the two goods are complements. The consequence is that if KK lowers its price then SS's sales will increase also. Furthermore, for its rival's price given, a firm's profit is increasing in its own price. The result is the following payoff table:

KK and SS are producing **complementary** products			Sally's Satcom		
			Price		
			High	Medium	Low
Kate's Kalkulators	Price	High	5, 5	6, 3	10, 1
		Medium	3, 6	4, 4	5, 2
		Low	1, 10	2, 5	3, 3

KK is the incumbent firm and they do not know what product SS is going to bring to the market. It is **KK's belief that it is equally likely** that SS will bring either a tablet computer or add-in to market. However, SS knows which product they intend to bring to market.

a. When SS produces a product that is a complement for KK's tablet does SS have a dominant strategy? **Yes. High is SS's dominant strategy.**

b. When producing a complementary product does SS have any dominated strategies? **Medium and Low are both dominated by High.**

c. When SS produces a substitute for KK's tablet does SS have a dominant strategy? **No, SS does not have a dominant strategy.**

d. When producing a substitute product does SS have any dominated strategies? **Yes, Medium and Low both dominate High.**

e. Use Harsanyi's method to convert this from a game of incomplete information to a game of imperfect information.

		Sally's Satcom								
		H_sH_c	H_sM_c	H_sL_c	M_sH_c	M_sM_c	M_sL_c	L_sH_c	L_sM_c	L_sL_c
Kate's Kalk-ulators	H	5, (5,5)	5.5, (5,3)	7.5, (5,1)	2.5, (8,5)	3, (8,3)	5, (8,1)	2.5, (6,5)	3, (6,3)	5, (6,1)
	M	5.5, (0,6)	6, (0,4)	6.5, (0,2)	3.5, (4,6)	4, (4,4)	4.5, (4,2)	1.5, (6,6)	2, (6,4)	2.5, (6,2)
	L	3.5, (0,10)	4, (0,5)	4.5, (0,3)	3.5, (3,10)	4, (3,5)	4.5, (3,3)	2, (3,10)	2.5, (3,5)	3, (3,3)

f. In your representation of the game of imperfect information does SS have a dominant strategy? **No.**

g. In your representation of the game of imperfect information does SS have any dominated strategies?

Yes. LsHc dominates LsMc and LsLc. MsHc dominates MsMc and MsLc. MsHc also dominates HsHc, HsMc and HsLc. I have highlighted the dominated strategies in yellow.

h. Does KK have a dominant strategy in your game of imperfect information? **Even after taking out SS's dominated strategies, KK does not have a dominant strategy.**

i. Does KK have any dominated strategies in your game of imperfect information? **Yes, L dominates M. I have highlighted the dominated strategy in green.**

j. What is/are the Nash solution(s) to the game? **Once we have eliminated all dominated strategies we can see that SS plays M_sH_c and KK plays L.**

3. Auctions. A rare fossil has been found by R. K. Aulogist in Montana. RK has decided to auction the fossil. He knows that there are two museums that each values his fossil at $5 million, while the next possible buyer values it at only $4 million. These valuations are known to all of the potential bidders.

   a. If RK holds an open outcry, or English, auction how much can he expect to receive if tie bids are resolved by the toss of a fair coin? Explain. **Those who value the fossil at $4 million will drop out as soon as the price reaches even 1 cent over their valuation. The remaining two museums will continue to push the price up until it reaches $5 million, at which point the fair coin will be tossed.**

   b. If RK holds a sealed bid, first price auction how much can he expect to receive? Explain. Again, tie bids are resolved by the toss of a fair coin. **The dominant strategy in a sealed bid auction is to bid one's reservation price or valuation. Again, the fossil will sell for $5 million.**

   c. If RK holds a sealed bid, second price auction how much can he expect to receive? Explain. Again, tie bids are resolved by the toss of a fair coin. **In a sealed bid second price auction the dominant strategy is to bid one's valuation. If you value the fossil at $5 million you will bid that amount in the hope that the second highest bid is less than yours. In the case there are two who value the fossil at $5 million. The second highest bid will equal the highest bid and the seller receives $5 million.**

The take away from this problem is that the seller gets $5 million from any of the three formats since there are two people who value the fossil at that price.

4. Bargaining. You are bargaining with a local dealer over the price of a Toyota Camry. The list price, the price on the sticker in the window of a new car, of the version that you want is $20,000. The invoice price, the cost to the dealer, is $18,000.
   a. Assume that your utility and the utility of the dealer are both proportional to dollars exchanged. What is the Nash solution to this bargaining game? **The Nash solution will be $19,000. This is similar to the simple examples we did in class.**
   b. Now suppose that a dealer located 60 miles away has agreed to sell the car at $18,500. With this outside option available to you, what is the Nash solution? **With the outside offer your greatest offer falls from $20,000 to $18,500. You and the dealer now split the difference and a deal is struck for $18 ,250.**

5. Evolutionarily Stable Games. The game in Bekoff(2003) is a two period game. In each period each player must decide to be fair(F) or not fair (NF). There is no per se immediate payoff to playing fairly in the first period. However, playing fairly in the first period imparts some knowledge of how to be fair and cooperate in the second period when there is a resource at stake. The total resource to be gained in the second period is z, but we'll make the exercise a little easier by normalizing this to z=1. By learning how to play fairly in the first period and implementing that knowledge in the second period the players can change the probability of obtaining the resource z, which is then shared between them. The share of the resource earned by Player 1 is $0<j<1$, and that earned by player 2 is the complement, (1-j). The player using NF in the second period against an opponent playing F in the second period obtains the share $j>0.50$. In the event that the two players implement the same second period strategy, they share the resource equally. The probability distribution of obtaining the resource in the second period, contingent on the strategic plans implemented, can be summarized in the following table:

Probabilities of obtaining resource					
		Player 2			
		F_1, F_2	F_1, NF_2	NF_1, F_2	NF_1, NF_2
Player 1	F_1, F_2	1.0	α	β	γ
	F_1, NF_2	α	η	δ	η
	NF_1, F_2	β	δ	ε	ϕ
	NF_1, NF_2	γ	η	ϕ	η

The ordering on the probabilities is $\alpha>\gamma>\eta$, $\delta>\phi>\eta$, and $\beta>\varepsilon>\eta$. Combining the probabilities with the shares yields the payoff table where we show only the payoffs for player 1.

Payoffs for Player 1					
		Player 2			
		F_1, F_2	F_1, NF_2	NF_1, F_2	NF_1, NF_2
Player 1	F_1, F_2	0.5	$(1-j)\alpha$	β	γ
	F_1, NF_2	$j\alpha$	0.5η	$j\delta$	0.5η
	NF_1, F_2	0.5β	$(1-j)\delta$	ε	$(1-j)\phi$
	NF_1, NF_2	$j\gamma$	0.5η	$j\phi$	0.5η

a. Does player 1 have any dominated strategies? **Yes** If so, what are they? **NF_1, NF_2 is a dominated strategy.**

b. Given the symmetry in the game, does player 2 have any dominated strategies? **Yes** If so, what are they? **NF_1, NF_2 is a dominated strategy.**

With respect to player 1, define an evolutionarily stable strategy (ESS) as one for which its corresponding diagonal payoff is larger than any other payoff in the column. For example, player 1's strategy of (F_1, NF_2) is not an ESS since the payoff at {(F_1, NF_2), (F_1, NF_2)}, which is 0.5η, is not the strictly greatest in the column.

c. With respect to player 1, is it possible for NF_1, NF_2 to be an ESS? **No** Explain. **There is another strategy available to player 1 that results in a payoff in the same column that is equal to what he gets for NF_1, NF_2**

d. With respect to player 1, is it possible for NF_1, F_2 to be an ESS? **No** Explain. **The explanation is the same as that for part c.**

e. With respect to player 1, is it possible for F_1, NF_2 to be an ESS? **No** Explain. **Same explanation as the previous two parts.**

f. With respect to player 1, is it possible for F_1, F_2 to be an ESS? **Yes** Explain. **Suppose $j\alpha<0.50$. If player 2 plays and sticks to F_1, F_2 then F_1, F_2 is a best response for player 1. The symmetry in the game tells us that F_1, F_2 is also an ESS for player 2.**

g. Bekoff identifies the circumstances under which there is no ESS in the game. He imposes those circumstances and does a simulation to determine the mixed strategies that would be used by the two players. In his tables the players mix over all four strategies. Given your earlier answers do you think he did the simulation correctly? Explain. **There must be something wrong with Bekoff's simulation since we know that neither player would ever play NF_1, NF_2.**