Evaluating Structural Vector Autoregression Models in Monetary Economies

(Job Market Paper)

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Abstract

This paper uses Monte Carlo simulations to evaluate alternative identification strategies in VAR estimation of monetary models, and to assess the accuracy of measuring money instability as a cause of output fluctuations. I construct theoretical monetary economies using general equilibrium models with cash-in-advance constraints, which also include technology shocks, labor supply shocks, and monetary shocks. Particularly, two economies are characterized: one is fully identified and satisfies the long-run restriction; another is not fully identified and the portion of temporary technology shocks is mixed with demand shocks when applying the long-run restriction. Based on each theoretical model, artificial economies are then generated through Monte Carlo simulations, which allow me to investigate the reliability of structural VAR estimation under various identifying restrictions. Applying short-run, medium-run, and long-run restrictions on the simulated data, I check for the bias between the average VAR estimates and the true theoretical claim. The findings show that short-run and medium-run restrictions tend to work more robustly under model uncertainty, particularly because the bias for measuring the effects of monetary shocks using long-run restriction could increase substantially when the underlying economy includes unidentified temporary shocks. This experiment supports the claim that monetary shocks contribute no more than one third of the cyclical variance of post-war U.S. output, and suggests that their contribution could in fact be substantially less.

Keywords: VAR Estimation, Monetary Shocks, Business Cycle Fluctuations

JEL Classifications: E3, E4, C1, C3

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1 Introduction

Structural vector autoregressions (SVARs) are widely used by economists to study sources of business cycle fluctuations. Ideally, if the actual economy follows a well-specified theoretical model that identifies monetary, preference, and technology shocks, and predicts the economy's response, we would study the economy simply based on that model. In reality, because we do not have completely successful models of this sort, most evidence for the effects of different shocks has to come via SVARs. However, the theoretical models, though far from perfect, can then serve as useful lab experiments to check the reliability of structural VARs.

In this paper, we use Monte Carlo simulations of reasonably calibrated dynamic general equilibrium models to evaluate SVAR models for estimating contributions of different shocks to the business cycle fluctuations. In particular, I evaluate alternative identification strategies in structural VAR estimation of monetary models, and to assess the accuracy of measuring money instability as a cause of output fluctuations.

1.1 Motivation

Since the seminal work of Kydland and Prescott (1982), followed by Hansen (1985) and many others, the real business cycle (RBC) economists have claimed a central role for technology shocks driving macroeconomic fluctuations in industrialized economies. In these studies, macroeconomic fluctuations are interpreted as equilibrium responses to exogenous shocks, in an environment with perfect competition and intertemporally optimizing agents, and the role of monetary policy is assumed to be, at most, secondary.

In contrast, the traditional Keynesian monetary models assume a more important role of monetary shocks by emphasizing sticky nominal prices and wage settings, and other market frictions. It is not until more recently that the new generation of small-scale monetary business cycle models with a more extensive range of shocks, (see Christiano, Eichenbaum, and Evans 2005, and Smets and Wouters 2007 among others) demonstrates that monetary policy shocks, as an exogenous source, may contribute only a small fraction of the forecast variance of output at all horizons.

Structural VARs have played a fundamental role in advancing research in this area. Many studies, based on SVARs, confirm that in the postwar United States the exogenous technology shocks measured by the conventional Solow residual account for more than half the fluctuations. This is consistent with the classic work of Shapiro and Watson (1988), who employed structural

VAR estimation and found that monetary shocks account for about 30 percent of short-run output variability (see Table 1)[1]. All these findings are important for economists and policymakers to understand the sources of business cycle fluctuations, in order to evaluate the effects of macroeconomic stabilization policies.

Table 1: Percentage of Variance due to Monetary Shocks (from Shapiro and Watson 1988)

Quarter	Output	Hours	Inflation	Interest rate
1	28	36	89	83
4	28	40	82	71
8	20	31	82	72
12	17	27	84	74
20	12	20	86	79
36	8	12	89	85
∞	0	0	94	94

However, ever since the early years that SVARs were introduced, there have been controversies. Cochrane (1994) did an extensive survey on empirical works in the literature that use SVAR models to estimate the impacts of technology and monetary shocks. He found large variation of the results, and plausible variations can generate numbers from 0 to 100% for both money and technology shocks. These variation, as he suggested, might depend on specification uncertainty, choice of statistics, and sampling variation.

Lately, the traditional approach using SVARs to discriminate theoretical macroeconomic models, which compares the impulse responses from SVARs run on the real data to the theoretical impulse responses from the model, has also been called into question (for example, Chari, Kehoe and McGrattan 2005, Kehoe 2006, and Christiano, Eichenbaum, and Vigfusson 2006 among others). Moreover, Chari, Kehoe and McGrattan (2005) show that the popular SVAR methodology may generate significant bias when estimating a RBC economy due to the short length of lags assumed and insufficient data points used. Christiano, Eichenbaum, and Vigfusson (2006), on the other hand, suggest that identified VAR can still be useful for discriminating among competing economic models even confidence intervals are wider in the case of long-run restrictions.

This paper focuses on the robustness of the most widely used identification strategies in SVAR models. In particular, in order to assess the accuracy of measuring money instability as a cause

[1] Shapiro and Watson (1988) use a five-variable VAR method, which include two nominal variables (the inflation rate and the nominal interest rate) and three real variables (output, hours, and the relative price of oil.) Their estimation strategy assumes that, among the five shocks, two shocks which are associated with nominal variables have no permanent effects on real variables in the long run.

of the output fluctuations, the robustness of long-run, short-run, and medium-run identifying restrictions is explored systematically via an economic experiment. The motivation goes in line with Faust (1998) and Uhlig (1998, 2005)[2].

The following issues about identified VAR models are the main focus of this paper:

(1) Long-run identifying assumptions vs. Short-run identifying assumptions.

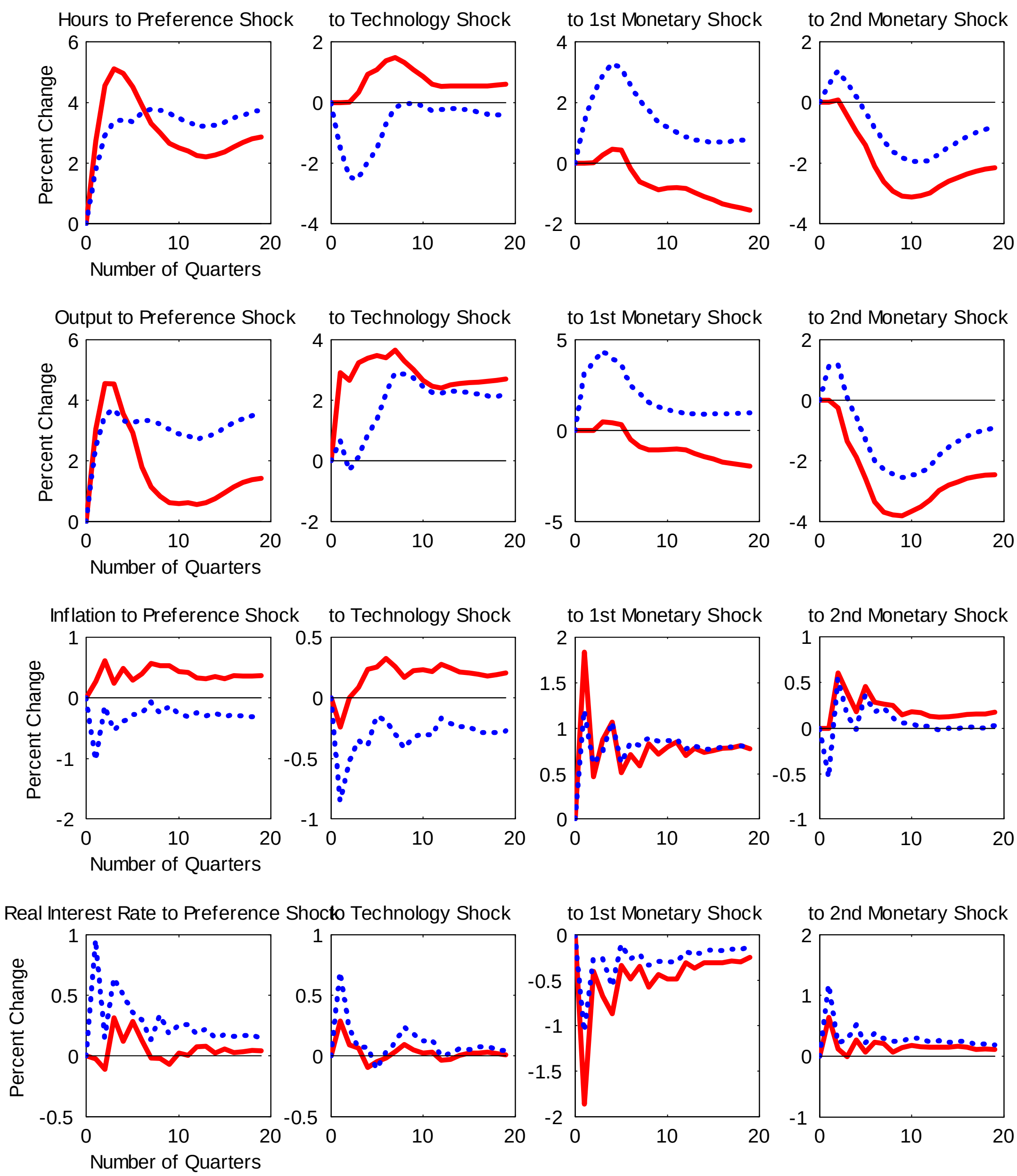
The simplicity of long-run identifying methodology, pioneered by Blanchard and Quah (1989), Gali (1999), King, Plosser, Stock, and Watson (1991) and Shapiro and Watson (1988), has contributed to its broad appeal. The principle behind the identifying restriction considers a kind of monetary growth model that does not have a long-run Phillips curve. In this essence, real variables, in the long run, are determined by real factors only. Nominal shocks can affect real variables and relative prices in the short run, but not in the long run. Accordingly, the long-run neutrality of money is imposed on statistical VAR models as an identifying restriction to disentangle shocks. However, the recent literature (see Chari, Kehoe, and McGrattan 2005, and Christiano, Eichenbaum, and Vigfusson 2006[3]) has debated whether it is likely to yield reliable inferences. Many concerns have been raised. For example, it is difficult to estimate precisely the long-run effects of shocks using a short data sample. Faust and Leeper (1997) emphasize that SVARs with long-run restrictions may perform poorly when estimated over the sample periods typically utilized. Also, as discussed by Cooley and Dwyer (1998) and Lippi and Reichlin (1993), a short-ordered VAR may provide a poor approximation of the dynamics of the variables in the VAR if the true data-generating process has a VARMA representation. McGrattan (2006) also shows that the identifying assumptions of VARMAs are too minimal, and the range of estimates are so large as to be uninformative for most statistics that business cycle researchers need to distinguish alternative theories.

Hence, we wonder if another popular identifying methodology, the short-run identifying scheme, would serve as a more robust estimation tool. Christiano, Eichenbaum, and Vigfusson (2006) test the performance of short-run identifying restrictions using a specific sticky-price model. They argue that with short-run restrictions, structural VARs perform well in their model. Nevertheless,

[2] Faust (1998) and Uhlig (1998, 2005) proposed that the choice of identifying restrictions is important when applying identified VAR. They examine the contribution of monetary policy shocks as a source of output fluctuations with sign restrictions. Sensitivity analyses are performed by comparing the impulse response functions with those that are believed to be "reasonable" by the conventional wisdom, i.e. "After a contractionary monetary shock, the interest rate goes up, while GDP and prices go down."

[3] Chari, Kehoe and McGrattan (2005) raise concerns on the long-run restriction when estimating the response of hours to technology shocks. Researchers have used the fall in hours in response to technology shocks to discriminate the RBC models. They claimed a poor approximation of VARs. On the contrary, Chistiano, Eichenbaum, and Vigfusson (2006) point out that, under some assumptions, VARs still work.

Figure 1: Impulse Responses to One-Standard-Deviation Shocks



Legend: —— short-run identification, - - - - long-run identification for short-run restrictions, the main critique, pointed out by Lucas and Stokey (1987), is that the particular class of short-run identifying assumptions in the literature does not apply to a broad class of theoretical models, and hence are difficult to guide the development of a broad class of research. As Christiano, Eichenbaum, and Evans (1998, p. 68) explain, the typical short-run restrictions imply that "*the time t variables in the Fed's information set do not respond to the time t realizations of the monetary policy shock.*" If the monetary authority at time t sets its policy as a function of time t variables, including output, consumption, and investment, as it does in Christiano, Eichenbaum, and Evans (2005), then the model must have specific timing assumptions. It means that, in a quarterly model, after a monetary shock is realized, private agents cannot adjust their output, consumption, and investment decisions during the remainder of the quarter. However, this timing assumption is not satisfied in the primary models in the monetary literature, for example, the neoclassic monetary models without the government recursive reaction rules.

Therefore, it is very important to understand the consequences of using different identification strategies. As a motivation, Figure 1 presents the impulse responses derived from SVARs using postwar real US data, which shows the big difference between using short-run identification and long-run identification.

(2) Permanent vs. Transitory Component in the Solow Residual.

Related with the above discussions, the key identifying assumption in the long-run restriction approach is that only technology innovations can affect labor productivity in the long run, e.g. Blanchard and Quah (1989) and Gali (1999) among others. Numerous researchers have used this approach to assess how technology shocks affect macroeconomic variables, and to quantify the importance of technology shocks in accounting for output and employment fluctuations.

This identification assumes all permanent shocks to labor productivity are solely from technology. While, it implicitly rules out the transitory component of technology shocks with this specification. If a positive technology shock is permanent, it should cause a large rise in consumption and output in the long run. However, if parts of the technology shocks are just transitory, it should also cause a rise in consumption and output in the short term, but no effect in the long run.

Does the long-run restriction rule out the transitory component as technology shocks? The conventional Solow residual is often treated as an appropriate measure of technology. King, Plosser, Stock and Watson (1991) show that the common-stochastic-trend / cointegration implication is consistent with postwar U.S. data. However, as shown in King and Rebelo (1999), when the actual sequence of technology shocks (proxied by the estimated disturbances of an autoregressive (AR) process for the Solow residual) is fed as an input into the model, the resulting equilibrium paths of output and labor input track surprisingly well the observed historical patterns of those variables. Obviously, the disturbance of an AR process decays eventually, and has no permanent effect.

Moreover, the wealth effects of transitory and permanent technology shocks are different. A permanent shift in productivity has a smaller effect on labor than a persistent but temporary shock. When the shock is temporary, there is small wealth effect that depresses labor supply but temporarily high wages and real interest rates induce individuals to work harder. When the shock is permanent, there are much larger wealth effects and the pattern of intertemporal substitution in response to wages is reversed since the future wages are high relative to current wages. The permanent shock assumption implies that the shocks are expected to have an equal effect on current and expected future productivity.

Meantime, Basu and Kimball (1997), Burnside, Eichenbaum and Rebelo (1995), and Rotemberg (2005) point out that the conventional Solow residual can explain only the part of technology shock, and some unobservable transitory capital utilization variation comove with it. We may then wonder whether our ignorance of the underlying transitory components of Solow residual affects our measures of fluctuations when applying structural VARs.

1.2 My Approach

All these questions cast serious doubts on the SVAR methodology as well as our understanding of business cycle fluctuations. This paper, therefore, undertakes a systematic investigation.

In this paper, we use Monte Carlo simulations of reasonably calibrated dynamic general equilibrium models to evaluate SVAR models for estimating contributions of different shocks to the business cycle fluctuations. In particular, we compare the estimated response of macroeconomic variables to monetary shocks derived from applying various identifying scheme with the “true” response implied by the theoretical models.

Two economies are constructed and both are cash-in-advance monetary models with endogenous capital accumulation that includes shocks to labor supply, total factor productivity, and money supply. However, one is fully identified and satisfies the long-run restriction, while another is not fully identified and the portion of temporary technology shocks is mixed with demand shocks when applying the long-run restriction.

We then generate Monte Carlo simulations from each model using an empirically reasonable sample length of 200 quarters. The structural VAR that we estimate using the simulated data includes the difference of hours worked, the GDP growth, the inflation rate, and the real interest rate. One appealing feature of this specification is that a low-ordered VAR(i.e., four lags) provides a close approximation to the true data-generating process in the benchmark parameterization of each of the models considered. This allows us to interpret the bias in the estimated impulse responses as arising almost exclusively from the small sample problems emphasized by Faust and Leeper (1997).

Applying short-run, medium-run, and long-run identifying restrictions on the simulated data, we check for the bias between the average VAR estimates and the true theoretical claim. The findings show that short-run and medium-run restrictions tend to work more robustly under model uncertainty; particularly because the bias for measuring the effects of monetary shocks using long-run restriction could increase substantially when the underlying economy includes unidentified temporary shocks. This experiment supports the claim that monetary shocks contribute no more than one third of the forecast error variance of post-war U.S. output, but even that could be an overstated upper bound.

1.3 Structure of the Paper

The structure of this paper is as follows. Section 2 lays out the baseline cash-in-advance model adapted from Cooley and Hansen (1989) and describes the calibration. We then set up an alternative model by introducing a transitory component in technology innovations. Section 3 specifies the structural VARs used on the Monte Carlo simulated data. Section 4 summarizes the structural VAR and estimation results based on the calibrated model. Section 5 concludes. Simulation details are in the Appendix.

2 A Neoclassical Monetary Economy

In this section, we start with a classic stochastic cash-in-advance model extended from Cooley and Hansen (1989) as the underlying monetary economy. This model is among a large class of monetary models, including those with various types of frictions and various sources of shocks. In this model, the only shock that affects labor supply in the long run is a shock to labor preference in the utility function; the only shock that affects labor productivity in the long run is a shock to technology growth trend. This property lies at the core of the long-run identifying restriction used by Blanchard and Quah (1989), King, et al (1991), Gali (1999), and many other papers. Demand shocks, which are defined as shocks to increase the price level and the output in the short run, are monetary shocks in this setup.

We then consider a variant of the model which includes a temporary supply shock. The temporary supply shock here is defined as a temporary shock to the Solow residual. As addressed by Basu and Kimball (1997), Burnside, Eichenbaum and Rebelo (1995), and Rotemberg (2005), the conventional Solow residual includes not only the permanent part of technology shock, but some unobservable transitory shocks, including capital utilization variation which comove with the permanent technology shocks. The reason that I consider this variant is to explore the robustness of different identifying restrictions, when econometricians face model uncertainty. Since the temporary shock to the Solow residual would not change the long-run growth trend of the labor productivity, we then analyze quantitatively how much our ignorance of the underlying transitory components may affect our VAR estimation results. As we will see, when applying the long-run identifying restriction to this variant of the model, temporary supply shocks to the Solow residual cannot be distinguished from other temporary demand shocks, e.g. the monetary shocks. Therefore, if the technology is regarded as only the permanent portion in the Solow residual, the impact of technology is underestimated. Accordingly, the portion of monetary shocks are exaggerated by the long-run restriction.

On the other hand, the short-run restrictions with specific timing assumptions are not satisfied in both theoretical models. However, although no assumptions are directly made on the recursiveness of variables, the contemporaneous effects of monetary policy are small in both calibrated examples. It allows the VAR estimation based on short-run identifying schemes to work statistically.

Considering the potential problems with long-run and short-run restrictions, the medium-run identifying restrictions pointed out by Uhlig (2004) provide a practically reasonable strategy to estimate VAR. We also discuss the properties of the estimation results with this identifying scheme.

The parameterizations of both neoclassic monetary models are discussed in Section 2.3. Following that, I provide a full description of the conventional VAR-based strategies with different identifying restrictions for estimating the contributions of demand shocks (monetary shocks) to output in the short and medium horizons.

However, we understand that this particular configuration and experiments in the paper do not identify the whole class of macroeconomic or monetary models, rather it identifies some types of models. Thus, the results are not specific to any specific economy, but some economies with the discussed properties.

2.1 A Cash-in-Advance Model with Production

Aggregate output, Y_t, is produced according to the following constant returns-to-scale technology, where K_t and H_t are the aggregate capital stock and labor input respectively:

$$Y_t = K_t^{\theta}(Z_t V_t H_t)^{1-\theta}, \quad 0 < \theta < 1. \tag{1}$$

Given the assumption of constant returns, I assume, without loss of generality, that there is only one competitive firm. In addition, the firm will make zero profits in equilibrium. In the above, there are two types of technology shocks, Z_t and V_t.

The first part of the Solow residual is contributed by the technology shock, $\log Z_t$. It follows a random walk with drift, whose law of motion evolves according to an exogenous process of the form:

$$\log Z_{t+1} - \log Z_t = \bar{\mu}_z + \sigma_z \epsilon_{zt+1}, \tag{2}$$

where $\sigma_z > 0$, and the random variables $\{\epsilon_{zt}\}_{t=1}^{\infty}$ are time independent and normally distributed with zero mean and unit variance. The constant, $\bar{\mu}_z$, is a drift term in the random walk process. It drives the long-run growth trend of output. Technology shocks $\{\epsilon_{zt}\}_{t=1}^{\infty}$ have permanent effects on the level of total factor productivity in the economy.

We also consider a transitory component, $\log V_t$, for the technology, whose law of motion is governed by a mean-reverting $AR(1)$ process:

$$\log V_{t+1} = \rho_v \log V_t + \sigma_v \epsilon_{vt+1}, \tag{3}$$

where $0 < \rho_z < 1$, $\sigma_v > 0$, and the random variables $\{\epsilon_{vt}\}_{t=1}^{\infty}$ are time independent and normally distributed with zero mean and unit variance. Technology shocks $\{\epsilon_{vt}\}_{t=1}^{\infty}$ only have transitory effects on the level of total factor productivity. In the long run, the process of $\log V_t$ keeps stationary.

The reasons to consider the transitory component include two aspects. First, as addressed by Basu and Kimball (1997), Burnside, Eichenbaum and Rebelo (1995), and Rotemberg (2005), the conventional Solow residual includes not only the permanent part of technology shock, but some unobservable transitory shocks, for example, capital utilization variation. This inclusion can better fit the economy and explain both the permanent and transitory effects of technology shocks. Second, this paper tries to explore the robustness of different identifying restrictions, when econometricians face a model uncertainty. Since the temporary shock to the Solow residual would not change the long-run growth trend of the labor productivity, the long-run identifying restriction can only disentangle the permanent component of the technology. Taking this as a variant model enables us to analyze whether our ignorance of the underlying transitory components affects our measures of fluctuations when applying structural VARs.

The portion of output that is not consumed is invested in physical capital. Investment in period t produces productive capital in period $t+1$, so

$$K_{t+1} = (1-\delta)K_t + X_t, \tag{4}$$

where $\delta \in (0,1)$ is the constant depreciation rate.

The firm seeks to maximize profit, which is equal to $Y_t - w_t H_t - r_t K_t$. The first-order conditions for the firm's problem yield the following functions for the wage rate and rental rate of capital:

$$w(\log Z_t, \log V_t, K_t, H_t) = (1-\theta)\left(\frac{Y_t}{H_t}\right), \tag{5}$$

and

$$r(\log Z_t, \log V_t, K_t, H_t) = \theta\left(\frac{Y_t}{K_t}\right). \tag{6}$$

The economy is populated by a large number of identical households, which obtain utility from consumption and leisure. Their preferences are summarized by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, 1-h_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha \log c_{1t} + (1-\alpha)\log c_{2t} - \gamma\left(\frac{h_t}{\chi_t}\right)\right], \tag{7}$$

where $0 < \beta < 1$ and $0 < \alpha < 1$. Here β denotes the discount factor and E_0 is the expectation operator conditional on the information set available as of time $t = 0$.

There are two types of consumption goods; c_{1t} is a "cash good" and c_{2t} is a "credit good." The difference is that previously accumulated cash balances are required in order to purchase units of c_{1t}. Hours worked enters linearly in the utility function. This is a reduced-form utility function, which results from the "indivisible labor" assumption.

The labor supply h_t is subject to a permanent preference shock χ_t, that follows the stochastic process

$$\log \chi_{t+1} - \log \chi_t = \sigma_\chi \epsilon_{\chi t+1} \tag{8}$$

where $\sigma_\chi > 0$, and the random variables $\{\epsilon_{\chi t}\}_{t=1}^{\infty}$ are time independent and normally distributed with zero mean and unit variance. As noticed by Gali (2005), this shock can be an important source of fluctuations, as it accounts for permanent shifts in the marginal rate of substitution between goods and labor (see Hall 1997).

In the beginning of any period t, a representative household has currency holdings equal to $m_t + (1 + R_{t-1})b_t + T_t$, where m_t is currency carried over from the previous period and the second term is principal plus interest from government bond holdings, b_t. The third term is a nominal lump sum (or tax) paid at the beginning of period t. Households then acquire bonds, which they carry into the next period, b_{t+1}. This leaves the household with $m_t + (1+R_{t-1})b_t + T_t - b_{t+1}$ unit of currency for purchasing goods; the household has no access to additional currency after this point. Thus, purchases of cash goods must satisfy the cash-in-advance constraint,

$$P_t c_{1t} \leq m_t + (1 + R_{t-1})b_t + T_t - b_{t+1}, \tag{9}$$

where P_t is the price level in period t. It turns out that this constraint will hold with equality as long as the nominal interest rate is always positive. This requirement will be satisfied throughout the following analysis.

Household allocations must also satisfy the following sequence of budget constraint:

$$c_{1t} + c_{2t} + x_t + \frac{m_{t+1}}{P_t} + \frac{b_{t+1}}{P_t} \leq w_t h_t + r_t k_t + \frac{m_t}{P_t} + \frac{(1 + R_{t-1})b_t}{P_t} + \frac{T_t}{P_t}. \tag{10}$$

The household expenditures include purchases of the two consumption goods (c_{1t}, c_{2t}), investment (x_t), money to be carried into the next period (m_{t+1}), and government-issued bonds (b_t). The funds available for these purchases include income from capital and labor, currency carried over from the previous period, the principal and interest from bond holdings, and nominal transfers.

Real government consumption, G_t, and the per capita stock of money, M_t, are assumed to be realizations of exogenous stochastic processes. In addition, a government policy also includes sequences of nominal transfers net of taxes, T_t, and nominal government debt, B_t, that satisfies the following government budget constraint for each period t :

$$P_t G_t + T_t = M_{t+1} - M_t + B_{t+1} - (1 + R_{t-1})B_t, \tag{11}$$

where the initial stock of government debt, B_0, is given. In addition, the government policy must satisfy the condition that $(1 + R_{-1}B_0)$ plus the expected present value of government purchases and net transfer payments equals the expected present value of seignorage revenues.

To make the study focus on the impact of monetary shocks and not the impact of changes in government spending, we assume G_t equal to a constant for all $t \geq 0$. In particular, set $G_t = 0$. In this case, a money injection can be used to directly finance lump sum transfers or to retire existing government debt. The first of these is analogous to the "helicopter drop" described in Friedman (1969), and the second is a standard open market operation. An implication of Ricardian equivalence in this economy is that given B_0 and a particular realization of the money supply process, as long as the present-value government budget constraint is satisfied, the time path of B_t $(t \geq 0)$ and T_t $(t \geq 0)$ does not matter for the equilibrium allocations. Thus, these two methods for injecting new money are equivalent in this economy. Hence, with no loss in generality, we assume that $B_t = 0$ for all $t \geq 0$. In addition, we assume that B_0 is equal to zero. Together these assumptions imply that no bonds are held in this economy and that $T_t = M_{t+1} - M_t$ for each t.

The per capita money supply is assumed to grow at the rate $e^{\mu_t} - 1$ in period t. That is,

$$M_{t+1} = e^{\mu_t} M_t, \tag{12}$$

where μ_t is revealed at the beginning of period t. Therefore, $T_t = (e^{\mu_t} - 1)M_t$.

The total money growth rate μ_t is assumed to follow an $ARMA(1,1)$ process, equal to the sum of two underlying processes: an AR process μ_{1t} and a white noise process μ_{2t}. The evolution process is defined as follows:

$$\mu_t = \bar{\mu}_m + \mu_{1t} + \mu_{2t}. \tag{13}$$

The average growth rate of money is equal to a constant number $\bar{\mu}_m$. The random variable μ_{1t} is assumed to evolve according to the autoregressive process:

$$\mu_{1t+1} = \rho_m \mu_{1t} + \sigma_1 \epsilon_{1t+1}, \tag{14}$$

where $0 < \rho_m < 1$, and $\sigma_1 > 0$. The random variable $\{\epsilon_{1t}\}_{t=1}^{\infty}$ are are time independent and normally distributed with zero mean and unit variance. σ_1 is the conditional standard deviation. In other words, μ_{1t} follows a mean-reverting process, and it represents the persistent component of monetary policy shocks.

The random variable μ_{2t} is assumed to be a white noise at time t. It evolves according to the process

$$\mu_{2t} = \sigma_2 \epsilon_{2t} \tag{15}$$

where $\sigma_2 > 0$, and the random variables $\{\epsilon_{2t}\}_{t=1}^{\infty}$ are are time independent and normally distributed with zero mean and unit variance. σ_2 is the standard deviation of the random monetary shocks.

With the above specification, the persistent monetary shock ϵ_{1t} affects the money growth μ_{1t} and some real variables for more than one period, but does not change the equilibrium in the long run. The random monetary shock ϵ_{2t} only affects the money growth rate and inflation contemporaneously, but nothing beyond that.

2.2 Bellman Equation and Linearized Solution

This subsection lays out the Bellman equation of the monetary economy. The solutions are then characterized by applying the log-linearization method.

As described previously, in this model the labor supply shock χ_t has a permanent effect on hours H_t as well as on output Y_t, consumption C_t, investment I_t, and capital K_t. In addition, Z_t has a long-run effect on Y_t, C_t , I_t and K_t, but no long-run effect on H_t, which implies that Z_t alone can have a long-run effect on labor productivity. Hence, this model is perfectly compatible with the first difference specification for hours and the identification assumptions used in the SVAR previously estimated.

To analyze the behavior of the model, we first apply a stationary-inducing transformation to those real variables that share a common trend with the level of technology Z_t and labor supply shock χ_t. The variables are rescaled as follows:

$$\tilde{h}_t = \frac{h_t}{\chi_t},\ \tilde{Y}_t = \frac{Y_t}{Z_t\chi_t},\ \tilde{c}_{1t} = \frac{c_{1t}}{Z_t\chi_t},\ \tilde{c}_{2t} = \frac{c_{2t}}{Z_t\chi_t},\ \tilde{x}_t = \frac{x_t}{Z_t\chi_t},\ \tilde{K}_{t+1} = \frac{K_{t+1}}{Z_t\chi_t}.$$

With the transformations, a well-behaved deterministic steady state exists. We then compute the solution of the model from a log-linearization of the stationary equilibrium conditions around this deterministic steady state using the numerical algorithm of Anderson and Moore (1985) and Uhlig (1999), which provides an efficient implementation of the solution method proposed by Blanchard and Kahn (1980).

Following Shapiro and Watson (1988), we assume the hours follow a non-stationary process. This assumption receives empirical support. First, standard univariate unit root tests (Augmented Dickey Fuller Test) generally do not reject the unit-root hypothesis for hours worked (Gali and

Rabanal, 2004; Francis and Ramey, 2005; Christiano et al., 2004). Second, Gali (2005) regresses hours on the consumption-output ratio (in logs). When the data are unfiltered, he finds that this regression has little explanatory power. However, with data in first-differences, the fit is greatly improved, consistently with basic neoclassical theory. Christiano et al. (2005b) run a closely related regression and obtain a highly autocorrelated residual, suggesting the presence of permanent labor-supply wedges.

Of course, as many economists pointed out, modeling hours as a non-stationary process is controversial. The maximal number of hours that a person can work a day is bounded. Hence, no model that takes into account this physical constraint can yield a unit root in the logarithm of hours. However, the specification of the utility function ignores any physical bound: it is always possible to find a sequence of χ_t such that h_t will eventually exceed any positive limit $\bar{H}$, even if χ_t is a stationary shock. Meanwhile, the log-linear solutions ignore this upper bound. This problem is all the more reinforced as we assume that χ_t follows a random walk. Thus, our model as well as our solution procedure yield dynamics that should be viewed as a local approximation, a view that is widespread in the literature with similar utility functions.

At each period, the representative individual maximizes the future expected utility by choosing her current cash-good consumption, credit-good consumption, hours, future capital stock, and cash balance. In the normalized form, the representative individual decision can be stated recursively:

$$\begin{aligned} V(\tilde{K}, \tilde{k}, \tilde{m}, \log Z, \log V, \log \chi, \mu) &= \max_{\{\tilde{c}_1, \tilde{c}_2, \tilde{h}, \tilde{k}', \tilde{m}'\}} \{\alpha \log \tilde{c}_1 + (1-\alpha)\log \tilde{c}_2 - \gamma \tilde{h} \\ &\quad + \beta E V(\tilde{K}', \tilde{k}', \tilde{m}', \log Z', \log V', \log \chi', \mu')\} \end{aligned} \tag{16}$$

subject to

$$\tilde{c}_1 = \frac{\tilde{m} + e^{\mu} - 1}{e^{\mu}\tilde{P}} \tag{17}$$

$$\tilde{c}_2 + \tilde{k}' + \frac{\tilde{m}'}{\tilde{P}} = \tilde{w}(z, V, \tilde{K}, \tilde{H})\tilde{h} + \frac{Z}{Z'}\frac{\chi}{\chi'} r(z, V, \tilde{K}, \tilde{H})\tilde{k} + \frac{Z}{Z'}\frac{\chi}{\chi'}(1-\delta)\tilde{k} \tag{18}$$

$$
\begin{aligned}
\log Z' &= \log Z + \bar{\mu}_z + \sigma_z \epsilon'_z, && (19)\\
\log V' &= \rho_v \log V + \sigma_v \epsilon'_v, && (20)\\
\log \chi' &= \log \chi + \sigma_\chi \epsilon'_\chi && (21)\\
\tilde{K}' &= \tilde{K}'(\tilde{K}, \log Z, \log V, \log \chi, \mu), && (22)\\
\tilde{H} &= \tilde{H}(\tilde{K}, \log Z, \log V, \log \chi, \mu), && (23)\\
\tilde{P} &= \tilde{P}(\tilde{K}, \log Z, \log V, \log \chi, \mu) && (24)
\end{aligned}
$$

where we normalize the money demand m_t by the quantity of money supply M_t, and denote the normalized money holding by $\tilde{m}_t \equiv \frac{m_t}{M_t}$. Also the nominal price is normalized to be $\tilde{P}_t \equiv \frac{P_t Z_t \chi_t}{M_{t+1}}$.

The last three lines of the constraints give the functional relationship between the aggregate state ($\tilde{K}$, $\log Z$, $\log V$, $\log \chi$, μ), and normalized per capita investment, hours, and price level. In the equilibrium, the competitive equilibrium is defined as:

Definition 1 *A (Normalized) Recursive Competitive Equilibrium consists a set of normalized decision rules for the household,* $\tilde{c}_1(s)$, $\tilde{c}_2(s)$, $\tilde{k}'(s)$, $\tilde{m}(s)$ *and* $\tilde{h}(s)$, *where* $s = (\tilde{K}, \tilde{k}, \tilde{m}, \log Z, \log V, \log \chi, \mu)$; *a set of normalized per capita decision rules,* $\tilde{K}'(\tilde{K}, \log Z, \log V, \log \chi, \mu)$ *and* $\tilde{H}(\tilde{K}, \log Z, \log V, \log \chi, \mu)$; *pricing functions* $\tilde{P}(\tilde{K}, \log Z, \log V, \log \chi, \mu)$, $\tilde{w}(\tilde{K}, \tilde{H}, \log Z, \log V)$, *and* $r(\tilde{K}, \tilde{H}, \log Z, \log V)$; *and a value function* $V(s)$, *such that*

(1)Household optimization problem is solved: Given the normalized pricing functions and the per capital decision rules, $V(s)$ *solves the functional equation in (16), and* $\tilde{c}_1(s)$, $\tilde{c}_2(s)$, $\tilde{k}'(s)$, $\tilde{m}(s)$ *and* $\tilde{h}(s)$ *are the associated decision rules;*

(2)Firm optimization is satisfied: The functions $\tilde{w}$ *and* r *are given by the firm's maximization problem; and*

(3)Markets clear: Individual decisions are consistent with aggregate outcomes.

$$
\begin{aligned}
\tilde{k}'(\tilde{K}, \tilde{K}, 1, \log Z, \log V, \log \chi, \mu) &= \tilde{K}'(\tilde{K}, \log Z, \log V, \log \chi, \mu)\\
\tilde{h}(\tilde{K}, \tilde{K}, 1, \log Z, \log V, \log \chi, \mu) &= \tilde{H}(\tilde{K}, \log Z, \log V, \log \chi, \mu)\\
\textit{and} \;\; \tilde{m}'(\tilde{K}, \tilde{K}, 1, \log Z, \log V, \log \chi, \mu) &= 1 \quad \textit{for all } (\tilde{K}, \log Z, \log V, \log \chi, \mu).
\end{aligned}
$$

To solve for the decision rule, the linear-quadratic approximation of this economy is used. The decision from the linear-quadratic approximation is also linear for the economy. The log-linearing procedure of deriving the maximization is included in the appendix. In the following subsection, the detail of the parameterization is discussed.

2.3 Parameterization

In order to derive results from the artificial economies, we follow Kydland and Prescott (1982) by choosing parameter values based on growth observations and the results of studies using microeconomic data. Table 1 summarizes the calibrated values of most of the model's parameters. The model is calibrated at a quarterly frequency so that $\beta = 1.03^{-0.25}$ and $\delta = 0.019$. The capital share parameter θ is set to 0.40, the ratio of investment to output is 0.25, the quarterly ratio of capital to output is 13.28, and the fraction of time spent working is 0.31. Again, the resulting parameter values are derived from the indivisible labor assumption. Hence, the parameter γ, which determines the fraction of time spent working at the steady state, is set to 2.53.

The parameter α, which determines the relative importance of the cash and credit good in the utility function, is calibrated by considering the evidence of the velocity of money. The approach similar to that of Lucas (1988) is taken. He considers a cash-in-advance model and shows how the parameters of conventional money demand functions are related to the parameters of preferences. To illustrate this, the first-order conditions for the household's problem can be used to obtain the following expression:

$$\frac{C_t}{c_{1t}} = \frac{1}{\alpha} + \frac{1-\alpha}{\alpha} R_t \tag{25}$$

where $C_t = c_{1t} + c_{2t}$. Per capita real money balances held during period t are equal to c_{1t}, given that the cash-in-advance constraint holds with equality. This implies that the velocity of money with respect to consumption ($Velocity$) is:

$$Velocity_t = \frac{1}{\alpha} + \frac{1-\alpha}{\alpha} R_t \tag{26}$$

To identify the appropriate measure of consumption and the appropriate measure of money from which to construct the velocity, we use the measures from National Income and Product Accounts (NIPA). First, for total consumption C_t, we use consumption of nondurables and services. Conventional monetary aggregates that one might use to capture quantities subject to the inflation tax–the monetary base, or the non-interest bearing portion of $M1$–have drawback that they are too large. They imply velocities of less than unity, which is inconsistent with the model. Instead, we use the portion of $M1$ that is held by households. To measure the value of α, we use the regression implied by (26) using these data. The estimated equation is

$$Velocity = \underset{(0.03)}{1.14} + \underset{(0.01)}{0.19} * R^{TB3M}, \quad R^2 = 0.55$$

where R^{TB3M} is the rate on the three-month Treasury bills stated on a quarterly basis. The intercept of this regression implies an estimate of $\alpha = 0.84$. The advantage of the cash-in-advance model is that there is a direct connection between the parameters of the model and empirical relationships (e.g. the demand for money) that have been widely explored.

The monetary growth rate is calibrated by estimating an $ARMA(1,1)$ process for $M1$ growth over the sample period, 1954:I to 2006:IV. That estimation produces the following equation:

$$\Delta \log M_t = \underset{(0.0012)}{0.007} + \underset{(0.072)}{0.491} \Delta \log M_{t-1}$$

This implied average growth rate of money is 1.3 percent per quarter.

The parameter choices are summarized in Table 2. The approximate equilibrium decision functions can be computed by log-linearing the first-order conditions (See, for example, Uhlig 1999). The details are included in the appendix.

Table 2. Parameters Values in the Calibrated Models

Common Parameters		
$\beta = 1.03^{(-0.25)}$		$\bar{\mu}_z = 0.0037$
$\theta = 0.40$		$\bar{\mu}_m = 0.015$
$\delta = 0.019$		$\rho_m = 0.49$
$\alpha = 0.84$		$\sigma_1 = 0.0087$
$\gamma = 2.53$		$\sigma_2 = 0.002$
		$\sigma_\chi = 0.0101$
Benchmark Model (Case 1)		
$\sigma_v = 0$		$\sigma_z = 0.0148$
Additional Shocks (Case 2)		
$\rho_v = 0.95$	$\sigma_v = 0.0103$	$\sigma_z = 0.0104$

To measure the technology shocks, we use the conventional definition of the Solow residual. Technology shocks are measured by the total Solow residual, defined as

$$SR_t \equiv \frac{Y_t}{K_t^{\theta} {H_t}^{1-\theta}} \tag{27}$$

We consider two cases with and without temporary technology shocks in accordance with this definition:

Case 1 (benchmark model) *Firstly, we consider the benchmark model without the transitory technology shocks, in which we shut down the process of* $\log V_t$*. The production function is simply as follows:*

$$Y_t = K_t^{\theta}(Z_t H_t)^{1-\theta}. \tag{28}$$

Accordingly, we obtain the explicit form of the Solow residual as

$$SR_t = {Z_t}^{1-\theta} \tag{29}$$

This implies that

$$\log SR_t = (1-\theta)\log Z_t$$

In this benchmark model (Case 1), we obtain the time series of Z_t *from equation (27) and (29). Then we estimate the stochastic trend of* Z_t *and standard deviation using (2). We derive that the estimate of* $\bar{\mu}_z$ *is* 0.0037 *and the estimate of* σ_z *is* 0.0148.

It is straightforward to confirm that along the balanced growth path the following variables are stationary: $\tilde{h}_t = \frac{h_t}{\chi_t}$, $\tilde{Y}_t = \frac{Y_t}{Z_t\chi_t}$, $\tilde{c}_{1t} = \frac{c_{1t}}{Z_t\chi_t}$, $\tilde{c}_{2t} = \frac{c_{2t}}{Z_t\chi_t}$, $\tilde{x}_t = \frac{x_t}{Z_t\chi_t}$, $\tilde{K}_{t+1} = \frac{K_{t+1}}{Z_t\chi_t}$*. The benchmark economy has the following properties:*

- *the equilibrium labor supply is only affected by the preference shock* $\epsilon_{\chi t}$ *in the long run. That is, at any date, t,*

$$\lim_{j\to\infty}\frac{\partial \ln H_{t+j}}{\partial \epsilon_{\chi t}} = \sigma_\chi > 0,\ \ \lim_{j\to\infty}\frac{\partial \ln H_{t+j}}{\partial \epsilon_{zt}} = \lim_{j\to\infty}\frac{\partial \ln H_{t+j}}{\partial \epsilon_{1t}} = \lim_{j\to\infty}\frac{\partial \ln H_{t+j}}{\partial \epsilon_{2t}} = 0 \tag{30}$$

- *the equilibrium output is affected by both the preference shock* $\epsilon_{\chi t}$ *and the technology shock* ϵ_{zt}*, but not by the monetary shocks in the long run. That is, at any date, t,*

$$\lim_{j\to\infty}\frac{\partial \ln Y_{t+j}}{\partial \epsilon_{\chi t}} = \sigma_\chi > 0,\ \ \lim_{j\to\infty}\frac{\partial \ln Y_{t+j}}{\partial \epsilon_{zt}} = \sigma_z > 0,\ \ \lim_{j\to\infty}\frac{\partial \ln Y_{t+j}}{\partial \epsilon_{1t}} = \lim_{j\to\infty}\frac{\partial \ln Y_{t+j}}{\partial \epsilon_{2t}} = 0 \tag{31}$$

- *the real interest rate is not affected by all four shocks in the long run. At any date, t,*

$$\lim_{j\to\infty}\frac{\partial r_{t+j}}{\partial \epsilon_{\chi t}} = \lim_{j\to\infty}\frac{\partial r_{t+j}}{\partial \epsilon_{zt}} = \lim_{j\to\infty}\frac{\partial r_{t+j}}{\partial \epsilon_{1t}} = \lim_{j\to\infty}\frac{\partial r_{t+j}}{\partial \epsilon_{2t}} = 0 \tag{32}$$

- *last, the inflation rate is not affected by all four shocks in the long run. While, the price level is affected by all four shocks in the long run.*

$$\lim_{j\to\infty}\frac{\partial\pi_{t+j}}{\partial\epsilon_{\chi t}}=\lim_{j\to\infty}\frac{\partial\pi_{t+j}}{\partial\epsilon_{zt}}=\lim_{j\to\infty}\frac{\partial\pi_{t+j}}{\partial\epsilon_{1t}}=\lim_{j\to\infty}\frac{\partial\pi_{t+j}}{\partial\epsilon_{2t}}=0 \tag{33}$$

and

$$\begin{aligned}\lim_{j\to\infty}\frac{\partial\ln P_{t+j}}{\partial\epsilon_{\chi t}} &= -\sigma_\chi<0,\ \lim_{j\to\infty}\frac{\partial\ln P_{t+j}}{\partial\epsilon_{zt}}=-\sigma_z<0,\\ \lim_{j\to\infty}\frac{\partial\ln P_{t+j}}{\partial\epsilon_{1t}} &= \sigma_1>0,\ \lim_{j\to\infty}\frac{\partial\ln P_{t+j}}{\partial\epsilon_{2t}}=\sigma_2>0.\end{aligned} \tag{34}$$

The short-run effects of shocks are different from each other. In particular, the white-noise monetary shocks ϵ_{2t} only affect the inflation rate for one period, but nothing beyond that. It only leads to a one-period adjustment of the price level, and keeps all other real variables to be neutral.[4] *Regarding the liquidity effect, the white-noise monetary shocks ϵ_{2t} does not play any role because of its one-period effect. Instead, the mean-reverting monetary shocks ϵ_{1t} cause the liquidity effects on output, and real interest rates.*

By implementing the identifying restrictions based on these features, we can identify each shock, and even disentangle the two types of monetary shocks.

These features of our benchmark theoretical model (Case 1), including the long-run and short-run responses of endogenous variables, are shown in Figure A1, which provides the impulse responses of different variables to one-standard-deviation preference, technology, and monetary innovations.

Implications (30) - (34) are quite general since they follow from the assumptions on preferences and technology shocks for balanced growth. So extended models with additional endogenous variables and propagation mechanisms, including models with market friction, price rigidity, and interest rate rules, are consistent with these implications.

In the following example, I show a variant of the benchmark model. It includes a transitory component in the Solow residual. As discussed before, many economists have pointed out that the conventional Solow residual explains only a part of technology shock, and there also exists some unobservable transitory capital utilization variation. However, if we consider the Solow residual to be

[4] Note: the white-noise monetary shocks ϵ_{2t} only affect the inflation rate for one period, but nothing beyond that. To include this shock, we can well-identify each shock, and even disentangle the two types of monetary shocks.

two parts: the permanent component and the transitory component of technology, the consequent effects on the economy's equilibrium are unlike. Particularly, the wealth effects of transitory and permanent technology shocks vary substantially. A permanent shift in productivity has a smaller effect on labor than a persistent but temporary shock. When the shock is temporary, there is small wealth effect that depresses labor supply but temporarily high wages and real interest rates induce individuals to work harder. When the shock is permanent, there are much larger wealth effects and the pattern of intertemporal substitution in response to wages is reversed since the future wages are high relative to current wages. The permanent shock assumption implies that the shocks are expected to have an equal effect on current and expected future productivity. The properties are summarized as follows:

Case 2 (with a temporary component in the Solow residual) *We consider a different theoretical economy by adding the transitory technology shocks into the benchmark model. The production function becomes as follows*

$$Y_t = K_t^{\theta}(Z_t V_t H_t)^{1-\theta}. \tag{35}$$

With the specification of technology as the Solow residual

$$SR_t = (Z_t V_t)^{1-\theta} \tag{36}$$

This implies that

$$\log SR_t = (1-\theta)[\log Z_t + \log V_t]$$

Given the definition of the process $\log Z_t$ *and* $\log V_t$, *technology* $\log SR_t$ *then follows an* $ARIMA(1,1,1)$ *process. Transitory technology shocks* $\log V_t$ *is captured by the* MA *part of the process.*

Proof. *See Appendix A.* ■

In this alternative model (Case 2), we obtain the time series of SR_t *from equation (27), (35) and (36). Assume that the temporary technology shock contributes 50% of the variation to the growth rate of the Solow residual. We have the estimates* $\rho_v = 0.95$, $\sigma_v = 0.0103$, *and* $\sigma_z = 0.0104$. *With this variant, we keep* $\bar{\mu}_z = 0.0037$, *in order to reach the same economic growth trend as the benchmark economy.*

Also, it is easy to confirm that along the balanced growth path the following variables are stationary: $\tilde{h}_t = \frac{h_t}{\chi_t}$, $\tilde{Y}_t = \frac{Y_t}{Z_t \chi_t}$, $\tilde{c}_{1t} = \frac{c_{1t}}{Z_t \chi_t}$, $\tilde{c}_{2t} = \frac{c_{2t}}{Z_t \chi_t}$, $\tilde{x}_t = \frac{x_t}{Z_t \chi_t}$, $\tilde{K}_{t+1} = \frac{K_{t+1}}{Z_t \chi_t}$. *Then this model with* *additional shocks has some similar properties as before:*

- *the equilibrium labor supply is only affected by the preference shock* $\epsilon_{\chi t}$ *in the long run;*
- *the equilibrium output is affected by both the preference shock* $\epsilon_{\chi t}$ *and the technology shock* ϵ_{zt}*, but not by the monetary shocks in the long run;*
- *the real interest rate is not affected by all four shocks in the long run;*
- *also, the inflation rate is not affected by all four shocks in the long run. While, the price level is affected by all four shocks in the long run.*

  However, the properties of the temporary technology shocks, which is distinctive from the benchmark model, include the following aspects:
- *the temporary technology shocks do not affect the long-run growth trend of hours, output and labor productivity, though it contributes to the business cycle fluctuations as one type of supply shocks*

$$\lim_{j\to\infty}\frac{\partial \ln H_{t+j}}{\partial \epsilon_{vt}} = \lim_{j\to\infty}\frac{\partial \ln Y_{t+j}}{\partial \epsilon_{vt}} = \lim_{j\to\infty}\frac{\partial \ln Y_{t+j}/H_{t+j}}{\partial \epsilon_{vt}} = 0.$$

- *while, compared with the permanent technology shocks, the temporary component has the similar impacts on the real variables in the short run. That is, at any date t, for j = 1,2,3,4*

$$\frac{\partial \ln H_{t+j}}{\partial \epsilon_{vt}} > 0,\ \frac{\partial \ln Y_{t+j}}{\partial \epsilon_{vt}} > 0,\ \frac{\partial r_{t+j}}{\partial \epsilon_{vt}} > 0,\ \frac{\partial \pi_{t+j}}{\partial \epsilon_{vt}} < 0.$$

  Consequently, the temporary component in the Solow residual can not be identified with the long-run restriction, although it contributes to the short-run fluctuations of real variables as the permanent technology shocks.

These features of our alternative theoretical model Case (2) are shown in Figure A2, which provides the impulse responses of different variables to one-standard-deviation preference, technology, and monetary shocks.

As it is one variant from the benchmark model, extended models with additional temporary shocks on taxation, and government spending may deliver some consistent implications as this example.

For each of the above cases, I solve the equilibrium decision functions by log-linearizing the first-order conditions. (See Appendix A for the detailed procedure.) The theoretical impulse responses and the forecast error variance decomposition are then derived. These results are model-implied, and will be regarded as the "true" impulse response functions and "true" variance decomposition contributed by different shocks in our analysis.

In order to evaluate the SVARs, we generate a sequence of time series data of variables of interest based on our theoretical economies through Monte Carlo simulations. These simulated data are taken as an artificial economy, for which we know what the underlying true process is. By applying the SVAR methods with different identifying restrictions discussed in the following section, we then compare the discrepancy between a "true" economy and the one which econometricians would derive from their SVAR estimation.

Specifically, I use the true model plus random shocks to generate artificial data for 2000 times with the length of 200 for each. Since all the data are generated from the same data-generating process – the linearized state-space representation model, the underlying "true" impulse response functions and "true" variance decomposition are identical. I run SVAR estimation on each simulated economy. Comparing the average SVAR results of the 2000 economies with the underlying true process, we are then able to evaluate the reliability of the SVAR methods with various identifying restrictions. These findings will shed light on how we use and interpret SVARs when applied to actual data.

3 Structural VAR Specification

In this section, I specify the SVARs with different identifying restrictions. In section 3.1, we begin with the discussion of the relationship between a theoretical model to VARs. Under some assumptions, we show that the state-space representation, which is derived from the theoretical model, can be arranged as a form of $VAR(\infty)$. As long as $VAR(p)$ with finite lags is a good approximation of $VAR(\infty)$, econometricians may just use $VAR(p)$ to estimate the model. Furthermore, I discuss the identification of the economic shocks from the VAR residuals when applying different restriction schemes. In the literature, short-run and long-run restrictions are proposed to serve as the most popular identification assumptions, as well as the recently proposed medium-run restrictions (Uhlig, 2004). In section 3.2, I describe the different identifying schemes in details, then I apply them on the artificial data simulated from our theoretical models from Section 2.

3.1 Relates Theoretical Models to VARs

In this subsection, we discuss the relationship between a theoretical model and VAR specifications. Specifically, there are some conditions under which the reduced form of the theoretical model is a VAR with disturbances that are linear combinations of the economic shocks. We begin by showing how to put the reduced form of the theoretical model into a form of a state-space representation. Throughout, we analyze the log-linear approximations to model solutions.

Suppose the variables of interest in the theoretical model are denoted by the series of vectors $\{X_t\}_{t=0}^{\infty}$ where t is the time index. Let S_t denote the vector of state variables including the endogenous state variables and exogenous economic shocks at time t. All variables are measured by the percent deviation from steady state, and after scaling by Z_t and χ_t if the variables have trend components.[5] From the theoretical model, the log-linearization of the approximate solution for X_t is given by:

$$X_t = HS_t \tag{37}$$

and the state variables follow the law of motion:

$$S_t = FS_{t-1} + D\epsilon_t \tag{38}$$

where F and D denote the matrices of the law of motion, and ϵ_t is the economic shocks we discussed in the theoretical model. It is consistent with the theoretical model that ϵ_t have zero-mean and unit standard deviation. Moreover, the dimension of the shocks ϵ_t equal to that of the theoretical model.

As shown in the appendix, we find that the above state-space representation can be rearranged into an infinite-order VAR:

$$X_t = B_1 X_{t-1} + B_2 X_{t-2} + ... + u_t \tag{39}$$

with

$$u_t = C\epsilon_t$$

where u_t denotes the residual in $VAR(\infty)$ and ϵ_t denotes the economic shocks defined from the state-space representation derived from the theory. As shown in the appendix, this VAR residual u_t is a linear combination of the economic shocks ϵ_t. They are of the same dimension. The details about the relations between the coefficients of $VAR(\infty)$ and the initial state-space representation

[5] Capital k_t is nomalized by the technology trend Z_{t-1} and the preference trend χ_{t-1}.

are discussed in the appendix.

The relationship indicates why economists can use finite-order VAR models to analyze a theoretical dynamic model. This equation (39) clarifies a reduced form of solutions from a standard theoretical model can be equivalently identified by a VAR model. While, the equivalence is from the assumptions used to derive the $VAR(\infty)$ form from the state representation form. In summary, the assumptions include:

- observable variables X_t are of the same dimension as the economic shocks ϵ_t;
- the eigenvalues of the state-space representation are less than unity in absolute values;
- the matrix C is invertible, and can be well identified;
- and the finite-order $VAR(p)$ is a close approximation to $VAR(\infty)$ which requires the time series data are long enough for estimation.

These assumptions pose the possible concerns and the potential pitfalls of VAR estimation methods. Chari, Kehoe, and McGrattan (2005) show that the finite-order VARs provide a poor proxy given the small sample of postwar aggregate data. In the following section, we will discuss the specific identification problem using our theoretical models. To be specific, we want to answer the following questions in our theoretical simulated artificial economies:

- given the data limitation, how well we have measured in the past work on measuring the business cycle fluctuations;
- in order to identify economic shocks, how robust the different identifying restrictions are;
- last, what might be the comparatively most robust identification assumptions even if we are facing an uncertain real economy (model uncertainty).

3.2 $VAR(p)$ with Different Identifying Restrictions

In this section, we specify the $VARs$ with different identifying restrictions. As discussed in the previous subsection, the state-space representation, which is derived from the log-linearization of the first-order conditions for a theoretical model as in (37)-(38), can be rearranged as $VAR(\infty)$ processes. Since Sims (1989), most econometricians believe the finite-order $VAR(p)$ can provide a good approximation for the $VAR(\infty)$ process when the number of lags p is 4 or 6. Chari, Kehoe and McGrattan (2005) provide detailed discussion on the limitation of data and the optimal lag p in a RBC model. In their proposed model, the optimal p should be very large to fully recover the underlying $VAR(\infty)$.

However, given the limited postwar US data, we have only up to 200 quarterly data points, and in reality we have no way to increase the sample size unless time goes longer. Hence, we definitely need to better understand how much forecasting or estimating errors we would have from this widely used estimation tool. In section 2, we propose two monetary economies, both with the cash-in-advance constraints. Now we run the structural $VAR(p)$ on the simulated data with length of 200, as most researchers generally use on the real data.

Given a single realization of data, the estimation procedure that we would follow is outlined as follows. This estimation procedure is simplified from Shapiro and Watson (1988)[6]. X_t is the 4×1 vector, which contains the log of hours worked per worker, $\Delta \log h_t$, the log difference of real output $\Delta \log y_t$, the ex-post real interest rate, $i_t - \pi_t$, and the inflation rate, π_t[7]. All level variables are expressed as a deviation from the model's non-stochastic steady state. The inclusion of the variables is standard in the empirical literature using VARs to identify technology shocks and monetary shocks. That is,

$$X_t = \begin{bmatrix} \Delta \log h_t, & \Delta \log y_t, & i_t - \pi_t, & \pi_t \end{bmatrix}' \tag{40}$$

Firstly, we consider the unrestricted VAR regression with finite lags, say p. Hence, we estimate

$$X_t = B_1 X_{t-1} + B_2 X_{t-2} + ... + B_p X_{t-p} + u_t, \quad E u_t u_t' = V \tag{41}$$

where the disturbance u_t is the residual of the $VAR(p)$ regression. The lag length, p, is chosen by using the information criterion in Schwarz (1978), where $p \in \{1, 2, ..., 10\}$. And the disturbance u_t is related to the fundamental economic shocks ϵ_t as follows: .

$$u_t = C\epsilon_t, \quad V = CC' \tag{42}$$

[6] Shapiro and Waston (1988) used the instrumental variables strategy including the growth rate of oil prices. To be simple, the theoretical model and VAR here drop the oil prices. A revised model including oil shocks and consequent results are in another paper of the author.

[7] Shapiro and Waston (1988) used the difference of the inflation rate, because they showed the inflation rates have a unit root by Argumented Dickey-Fuller test. Here I only use the level of inflation rates, since π_t from my model is stationary.

The economic shocks satisfy the following conditions $E(\epsilon_t) = 0$, $E[\epsilon_t \epsilon_t'] = I$, and $E[\epsilon_t \epsilon_{t+j}'] = 0$ for $j > 0$.

Equivalently, the structural VAR (41) can be expressed as the form:

$$B(L)X_t = u_t = C\epsilon_t \tag{43}$$

where $B(L) = I - B_1L - B_2L^2 - ... - B_pL^P$, and B_i for $i = 1, 2, ..., p$ is a square matrix of reduced-form parameters; L is the lag operator, and X_t, u_t , and ϵ_t are 4×1 vectors of endogenous variables, reduced-form innovations from residuals, and economic innovations, as defined before.

A researcher can estimate $B_1, B_2, ..., B_p$, and $V = Eu_tu_t'$. However, in order to obtain the impulse response functions and the variance decompositions, we need the matrix C corresponding to the specific economic shock in ϵ_t that is of interest. In order to identify the matrix C, additional identifying restrictions are required.

Here we discuss the long-run, short-run, and medium-run restrictions as follows. From equation (43), we derive the impulse response functions to the structural shocks from the following equation

$$X_t = B(L)^{-1}C\epsilon_t \tag{44}$$

Thus, letting $R(L) = B(L)^{-1}C = \sum_{i=0}^{\infty} R_iL^i$, it follows that the $(j \times k)$ element (for $j, k = 1, 2, 3, 4$) of R_i is the impulse response of the variable j to the shock k. The orders of the variable of X_t and the shocks ϵ_t are defined as previous.

The $k - step$ ahead forecast revision is defined as $E_t[X_{t+k}] - E_{t-1}[X_{t+k}]$. It tells the forecasting updates on variable X at time $t + k$ by receiving the extra information from time $t - 1$ to t. From equation (44), we find that

$$E_t[X_{t+k}] - E_{t-1}[X_{t+k}] = R_k\epsilon_t \tag{45}$$

The variance-covariance matrix of the $k - step$ ahead forecast revision Σ_k is

$$\Sigma_k = R_kR_k' \tag{46}$$

because the economic shocks satisfy $E[\epsilon_t\epsilon_t'] = I$. We can decompose it into the contributions of each shock $j = 1, 2, 3, 4$, then we have

$$\Sigma_k = \sum_{j=1}^{4} \Sigma_{k,j}, \quad \text{where } \Sigma_{k,j} = R_k E_{jj} R_k' \tag{47}$$

where $\Sigma_{k,j}$ is the variance-covariance contributed by shock j, and E_{jj} is a (4×4) zero matrix, with only the $j-th$ element on the diagonal replaced by 1. The fraction of the $k-step$ ahead forecast revision variance for variable i, explained by shock j is $\frac{(\Sigma_{k,j})_{(i\times i)}}{(\Sigma_k)_{(i\times i)}}$ where subscript $(i \times i)$ denotes the element $(i \times i)$ of the corresponding matrix.

(i) *Long-run Identifying Restriction*

The long-run identification of Blanchard and Quah (1989) and Gali (1999) consider that the demand shocks (i.e. monetary shocks in our theoretical models) have no effects to the real variables in the long run. In the model, the economic shocks include one permanent preference shock $\epsilon_{\chi t}$, one permanent technology shocks ϵ_{zt} , and two monetary shocks ϵ_{1t} and ϵ_{2t} for each period, i.e. $\epsilon_t = \begin{pmatrix} \epsilon_{\chi t} & \epsilon_{zt} & \epsilon_{1t} & \epsilon_{2t} \end{pmatrix}'$ at time t.

For example, if we consider the long-run restriction that monetary shock ϵ_{1t} has no effect on $\log y_t$ in the long run. Since we have $\Delta \log y_t$ as an element of X_t, the restriction means the $k-step$ ahead forecast revision when $k \to \infty$ must satisfy that

$$\lim_{k\to\infty} \sum_{s=1}^{k} \left(E_t[\Delta \log y_{t+s}] - E_{t-1}[\Delta \log y_{t+s}]\right) \text{ is uncorrelated with } \epsilon_{1t}.$$

In general, the long-run effects of the shocks are $R(1) = B(1)^{-1}C$, where $R(1) = \sum_{i=0}^{\infty} R_i$ and R_i is defined before as the impulse response of the VAR. The long-run restriction is:

$$\text{long-run restriction:} \quad B(1)^{-1}C = \begin{bmatrix} number & 0 & 0 & 0 \\ number & number & 0 & 0 \\ number & number & number & 0 \\ number & number & number & number \end{bmatrix} \tag{48}$$

and a sign restriction to rule out the case of $V = (-C)(-C)'$:

$$\text{sign restriction: diagonal elements of } B(1)^{-1}C \text{ are all positive} \tag{49}$$

The long-run restrictions associated with equation (48) are imposed through a Cholesky decomposition after estimating $B(L)$. $B(1)^{-1}C$ is estimated as the lower triangular Cholesky decomposition of $B(1)^{-1}VB(1)^{-1\prime}$ using least squares. This decomposition is used to solve for the matrix C.

(ii) *Short-run Identifying Restriction*

The standard short-run identification, or the recursiveness assumption assumes that "... *the time t variables in the Fed's information set do not respond to the time t realizations of the monetary policy shock.*" (Christiano, Eichenbaum, and Evans, 1998). Hence, assuming that the real variable at time t, output $\log y_t$, does not response to the time$-t$ monetary shock ϵ_{1t} contemporaneously, it means the one-step ahead forecast revision of output growth is not explained by ϵ_{1t}. In other words, this recursiveness assumption means that

$$E_t[\Delta \log y_t] - E_{t-1}[\Delta \log y_t] \text{ is uncorrelated with } \epsilon_{1t}.$$

To order the variables of X_t by the recursive sequence, we have that the time$-t$ monetary shocks do not affect the $time - t$ labor supply and $time - t$ output. Then we can do a Cholesky decomposition of the variance-covariance matrix of the $0 - step$ ahead forecast revision Σ_0, by letting

$$CC' = \Sigma_0 = V$$

where C is estimated as the lower triangular Cholesky decomposition of V.

The short-run restriction is:

$$\text{short-run restriction:} \quad C = \begin{bmatrix} number & 0 & 0 & 0 \\ number & number & 0 & 0 \\ number & number & number & 0 \\ number & number & number & number \end{bmatrix} \tag{50}$$

and a sign restriction to rule out the case of $V = (-C)(-C)'$:

$$\text{sign restriction: diagonal elements of } C \text{ are all positive} \tag{51}$$

(iii) *Medium-run Identifying Restriction*

Uhlig (2004) proposed a medium-run identifying scheme in order to estimate the labor productivity contributed by technology shock more robustly. In the short-run and long-run identifying schemes, the Cholesky decomposition of $\Sigma(k) = \sum_{i=0}^{k} \Sigma_i$ are used for $k = 0$ and $k = \infty$ respectively. Since Σ_i is the variance-covariance matrix of the $i - step$ ahead forecast revision, $\Sigma(k)$ is the cumulative variance-covariance matrix of the forecast revision for k steps. To do the Cholesky decomposition of $\Sigma(k) = C_k C_k'$, by letting C_k be the lower triangular matrix, it actually assumes that nominal shocks have no cumulative effects on some particular real variables after k periods. For some suitable $0 < k < \infty$, Uhlig (2004) shows that this identifying scheme work more robustly in some RBC models.

The medium-run restriction with parameter k is:

$$\text{medium-run restriction:} \quad \Sigma(k) = \sum_{i=0}^{k} \Sigma_i = C_k C_k' \tag{52}$$

$$\text{where } C_k = \begin{bmatrix} number & 0 & 0 & 0 \\ number & number & 0 & 0 \\ number & number & number & 0 \\ number & number & number & number \end{bmatrix}$$

and a sign restriction is that

$$\text{sign restriction: diagonal elements of } C_k \text{ are all positive} \tag{53}$$

In our Monte Carlo simulations, we generate 2,000 groups of time series data from each theoretical model we describe in Section 2. We keep every data sample with the length of 200 quarterly observations. Then we apply the estimation strategy and identifying schemes discussed above to each sample. For each theoretical model (Case 1 or Case 2), we apply the different identifying schemes on the same 2,000 groups of data. By comparing the average estimates using a specific identifying assumption to the "true" model-implied counterparts, some important findings stand out as shown in the following section.

4 Estimation Results

In this section, we analyze the properties of conventional VAR-based strategies for identifying the effects of technology shocks, preference shocks, and monetary shocks. We compare the estimated impulse response and variance decomposition of each shock with the "true" ones in the theoretical models.

We use the monetary model parameterizations discussed in the previous sections as the data generating processes. For each parameterization, we simulate 2,000 data sets of 200 observations each. The shocks $\epsilon_{\chi t}$, ϵ_{zt}, ϵ_{vt}, ϵ_{1t}, ϵ_{2t} are drawn from *i.i.d.* standard normal distributions. For each artificial data set, we estimate a six-lag VAR. The average, across the 2,000 data sets, of the estimated impulse response functions, and the variance decompositions, allows us to assess bias.

For each theoretical model, we estimate percentile-based confidence intervals across the 2,000 estimates from the 2,000 simulated artificial economies. The percentile-based confidence interval is defined as the top 2.5 percent and bottom 2.5 percent of the estimated coefficients in the dynamic response functions or the variance decompositions across the 2,000 data sets.[8]

We assess the accuracy of the impulse response estimators in the following way. First, we implement the SVARs with alternative identifying restrictions on one sequence of simulated data. The estimates of impulse responses, and associated variance decompositions are recorded to be the estimated results on one specific Monte Carlo simulation. We repeat the Monte Carlo simulation for 2,000 times. For each simulation, we conduct the same SVARs with the alternative identifying restrictions.

Because the underlying data generating process is from the same theoretical model, there exists only one "true" theoretical value for each estimate. Hence, we compute the average impulse response estimates, across 2,000 data sets simulated from the same theoretical model, and compare them with the "true" theoretical impulse response functions. If the VAR-based estimates were perfectly accurate, the average estimates would be the same as the theoretical impulse responses. Then we use the confidence interval that we obtained across 2,000 data sets to see whether the relevant true coefficients are contained in the 95% percentile-based confidence intervals.

We conduct the above experiments for both monetary models (Case 1 and Case 2). Monte Carlo simulations are firstly applied on the log-linearized solution to the theoretical model. For

[8] Another possible measure of the confidence interval is the bootstrap-based confidence interval, which is based on one series of data. Because we want to focus on the average performance of SVARs, here we use the percentile-based confidence intervals. Percentile-based confidence intervals report the average performance of VAR estimates among different simulated time-series variables.

each simulation, we set $X_t = [\Delta \log h_t,\ \Delta \log y_t,\ i_t - \pi_t,\ \pi_t]'$ in the VARs.[9] When we consider the first monetary model (Case 1), the data are generated from the four-shock's specifications. When we consider the second monetary model (Case 2), the data are generated from the five-shock's specifications.

4.1 Results for the four-shock specifications (Case 1)

Figure A1 displays the "true" impulse responses of hours worked, output, real interest rate, and price level to four shocks, including preference shock, technology shock, mean-reverting monetary, and random monetary shocks for the benchmark calibration of the cash-in-advance model as in Case 1. In each panel, the solid lines show the true responses from the model. The innovation occurs at date 1, and has been scaled to be the one-standard-deviation shocks. Consequently, the preference shock at date 1 would increase hours worked for 1% in the long run; the technology shock would increase output for 1% in the long run.

Figures A2–A5 report results generated from the VAR estimation when applying long-run restriction, short-run restriction, and medium-run identifying restrictions with $k = 4$ and 20, respectively. In each figure, the dashed line in each panel shows the mean of the impulse responses derived from applying our benchmark four-variable SVAR to the 2,000 artificial data samples (the median responses are nearly identical). The dotted lines show the 95% pointwise confidence interval of the SVAR impulse responses.

Tables A1–A8 report the variance decomposition results generated from the VAR estimation when applying alternative identifying restrictions. In each table, the "true" variance decompositions are shown in the left panel, and the right panel shows the mean of the variance decompositions derived from applying our four-variable SVAR (Case 1) to the 2,000 artificial data samples. The percentile-based 95% confidence intervals of the SVAR's variance decompositions are reported in the square brackets.

Long-run identifying restriction

As shown in Figure A2, the mean responses of hours worked, output, real interest rate, and price level have the same sign and qualitative pattern as the true impulse responses. As indicated by the pointwise confidence intervals, the SVARs give the correct sign of the responses for these variables. The mean estimates are always qualitatively in line with the true responses. Although the

[9] The choice of variables is similar to Shapiro and Watson (1988). They add one more variable about the oil price.

confidence interval is wide, it is unlikely for our experiments to get the impulse response functions with opposite signs to the theoretical ones.

Quantitatively, the SVAR with long-run restriction does not perform perfectly; however, as indicated in Table A1 and A2, the estimated variance decompositions of the SVAR provide a reasonable range for the true model. Although it seems that SVAR systematically underestimate fluctuations of hours and output explained by labor supply shocks, other estimates in both the short term and the long term perform pretty well. The average estimates are close to the true coefficients, with the bias less than 5% in Case 1.

Short-run identifying restriction

Our theoretical model does not actually impose any recursiveness assumptions on the monetary variables and other variables. Therefore, it does not strictly follow what Christiano, Eichenbaum, and Evans (1998, p. 68) assumed "*The economic content of the recursiveness assumption is that the time t variables in the Fed's information set do not respond to the time t realizations of the monetary policy shock.*" In spite of the facts, we impose the short-run restriction on the simulated data as a *pseudo* assumption. The results are shown in Figure A3.

It seems that, in our specific model, the mean responses of hours worked, output, real interest rate, and price level have the same sign and qualitative pattern as the true impulse responses. The short-run identification restriction looks robust. As indicated by the pointwise confidence intervals, the SVARs are likely to give the correct sign of the response for these variables. The mean estimates are qualitatively in line with the true responses. However, the fact that the confidence interval covers both the negative and positive areas of some variables (for example, the response of hours worked due to a technology shock), indicates the probability of getting an estimate with a wrong sign for the impacts of technology shocks on hours is still very nonnegligible according to the experiments.

Quantitatively, the SVAR with short-run restriction provides a reasonable estimation range for the true model, given the comparison of impulse responses (in Figure A3) and the variance decompositions (in Tables A3-A4). Although SVARs with short-run restriction do not impose the impacts on hours and output explained by monetary shocks (demand shocks) to be zero in the long run, which is contradictory to the neutrality of money, the estimates using the short-run restrictions in both the short term and the medium term perform well. The average estimates are close to the true coefficients.

Some explanations can help understand the findings. Based on the calibrated parameters derived from the postwar US money growth data on M2, the monetary shocks are assumed to contribute little in the theoretical model. Although the theoretical model does not have the timing properties, as Christiano, Eichenbaum, and Evans (1998) explained about the standard recursiveness of monetary policy shocks, the theoretical short-run impacts of monetary is negligible. Hence, in practice, the short-run restrictions are reasonable to be considered as a *pseudo* assumption due to the small fluctuations of monetary policy shocks in postwar US economy.[10]

Medium-run identifying restriction

As for the medium-run restrictions, we take two possible parameters for k, as $k = 4$ and 20. As discussed by Uhlig (2004), the medium-run restriction with the parameter to be k, means that we assume that the impacts of demand shocks decay after k periods. Here when we impose $k = 4$ (or 20), it assumes that the lower ordered shocks have no impacts after 4 (or 20) quarters. In extreme, the long-run restriction is equivalent to let $k = +\infty$. Whereas, the short-run restriction is equivalent to let $k = 0$.

We find that the results are similar to those applying the short-run restriction when $k = 4$, and similar to those of using the long-run restriction when $k = 20$. As discussed in the short-run and long-run restrictions, the impulse responses shown in Figures A4–A5 and the variance decompositions shown in Tables A5–A8 are close to the true coefficients.

Contrasting the three identifying restrictions

To gauge the size of the bias, Table 3 summarizes the average absolute difference between the mean estimates and the true variance decomposition due to monetary shocks in the horizon 1, 4, 8, 12, 20, 36 and 40 quarters. The most robust identifying restriction is expected to have the closest distance between the mean estimates and the true coefficients. Table 3 shows that all the three restrictions provide robust estimates. To be specific, the short-run restrictions provide more robust estimates for the short horizons; however, the long-run restrictions provide more reliable estimates for the long horizons.

[10] It has not been tested whether the short-run identifying restrictions will still work well in an economy with high monetary fluctuations, e.g. some high inflation countries. A separate paper of mine, which takes economies with relatively high inflation into consideration, is in progress. In that paper, I discuss different calibrated economies using Argentina and India data.

Table 3: (Case 1: 4-Shock Model) Distance between Mean Estimates and True Variance Decomposition due to Monetary Shocks (%)

Horizon	Identifying Assumption	Hours	Output	Real Interest Rate	Price
1	short-run	0.13	0.07	0.07	3.88
	medium-run with k=4	4.06	3.83	3.83	16.83
	with k=20	5.10	5.02	5.05	11.59
	long-run	4.56	4.60	4.62	10.56
4	short-run	1.55	1.56	0.71	5.34
	medium-run with k=4	2.49	2.24	1.96	17.87
	with k=20	3.59	3.43	3.34	13.05
	long-run	3.28	3.24	3.11	10.76
8	short-run	2.62	2.55	1.53	6.25
	medium-run with k=4	1.66	1.41	0.75	18.69
	with k=20	2.21	2.00	1.79	14.14
	long-run	2.02	1.88	1.66	11.36
12	short-run	3.02	2.96	2.05	6.26
	medium-run with k=4	1.57	1.33	0.54	18.6
	with k=20	1.63	1.40	1.02	14.15
	long-run	1.51	1.33	0.94	11.22
20	short-run	3.31	3.26	2.62	5.55
	medium-run with k=4	1.49	1.27	0.61	17.66
	with k=20	1.15	0.92	0.47	13.35
	long-run	1.08	0.88	0.44	10.37
36	short-run	3.46	3.44	3.07	3.81
	medium-run with k=4	1.39	1.21	0.79	15.52
	with k=20	0.78	0.59	0.22	11.44
	long-run	0.73	0.56	0.21	8.53
40	short-run	3.47	3.46	3.11	3.51
	medium-run with k=4	1.38	1.21	0.81	15.16
	with k=20	0.74	0.55	0.20	11.12
	long-run	0.69	0.52	0.20	8.22

Notes: Absolute value of difference between mean estimated and true model variance decomposition due to monetary shocks are reported in the table. Variance decomposition is in percent. The true model is Case-1 model as discussed in the paper.

4.2 Results for the five-shock specifications (Case 2)

Similar to the studies of Case 1, Figure A5 displays the "true" impulse responses of hours worked, output, real interest rate, and price level to five shocks, including one preference shock, two technology shocks, one mean-reverting monetary shocks, and one random monetary shocks for the cash-in-advance model as explained in Case 2. Here technology shocks include two parts, the permanent and transitory parts. Only the permanent technology shocks have effects on real variables. The transitory technology shocks play the similar effects on real variables in the short run, but the contributions decay eventually.

In each panel of Figure A5, the solid lines show the true responses from the theoretical model. The innovation occurs at date 1 and has been scaled to be the one-standard-deviation shocks. Similarly, the preference shock at date 1 would increase hours worked for 1% in the long run; the technology shock would increase output for 1% in the long run.

Figures A6–A10 report results generated from the VAR estimation when applying long-run identifying restriction, short-run identifying restriction, medium-run identifying restriction with $k = 4$ and 20. In each figure, the dashed line in each panel shows the mean of the impulse responses derived from applying our benchmark four-variable SVAR to the 2,000 artificial data samples (the median of the estimates is nearly identical to the mean). The dotted lines show the 95% pointwise confidence intervals of the SVAR's impulse responses.

Tables A9–A16 report variance decomposition results generated from the VAR estimation when applying long-run identifying restriction, short-run identifying restriction, medium-run identifying restriction with $k = 4$ and 20, respectively. In each table, the "true" variance decompositions are shown in the left panel, and the right panel shows the mean of the variance decompositions derived from applying our four-variable SVAR (Case 1) to the 2,000 artificial data samples. The percentile-based 95% confidence intervals of the SVAR's variance decompositions are reported.

Long-run identifying restriction

As shown in Figure A6, the mean responses of hours, output, real interest rate, and price level have the same sign and qualitative pattern as the true impulse responses. As indicated by the pointwise confidence intervals, the SVARs are giving the appropriate signs of the responses for these variables. The mean estimate is qualitatively in line with the true response. Similar to the common problems with long-run identifying restrictions (as we have seen in Case 1), the confidence intervals tend to be wide, although the probability of getting an estimate with a wrong sign is still very negligible from our findings.

Quantitatively, the SVARs with long-run restriction do not perform perfectly; as shown in Table A9 and A10, the estimated variance decompositions from the SVARs substantially overestimate the true ones from the model. In particular, SVARs systematically overestimate fluctuations of hours and output explained by monetary shocks in both the short term and the long term. The bias between the average estimates to the true coefficients is around 30%.

Comparing these findings with Shapiro and Watson (1988) (indicated in Table 1), we find some interesting results. The VAR estimates with long-run identifying restrictions from Shapiro and Watson (1988) is implemented on the actual postwar US data. In particular, they claimed that monetary shocks (demand shocks) contribute to about 30% of the cyclical variance of real output in the postwar United States. While, we again calculate these variance decompositions via VARs using Monte Carlo simulated data. Surprisingly, the contributions of monetary shocks to real variables, which we find via Monte Carlo simulations, are consistent with the ones shown by Shapiro and Watson. However, the actual contributions of monetary shocks are very different. In our underlying theoretical model, the contributions of monetary shocks to real variables are minimal. In fact, the model-implied variance decomposition from the model of Case 2 is actually substantially less than the estimated ones, and less than 5%.

Short-run identifying restriction

Again, in our specific model, the mean responses of hours, output, real interest rate, and price level have the same sign and qualitative pattern as the true impulse responses. The short-run identification restrictions still perform very robustly. As indicated by the pointwise confidence intervals, despite of the extra transitory shocks, the SVARs with short-run restrictions still give the right signs of the responses for these variables. The mean estimates are qualitatively in line with the true responses. However, the fact that the confidence interval covers both the negative and positive areas of some variables (for example, the response of hours worked due to the permanent technology shock), indicates the probability of getting an estimate with a wrong sign is nonnegligible from our experiments.

Quantitatively, the SVARs with short-run restrictions provide a reasonable range for the true model, given the comparison of impulse responses (in Figure A8) and the variance decompositions (in Table A12-A13). Although it might systematically estimate fluctuations of hours and output explained by monetary shocks (demand shocks) in the long run, most estimates in both the short term and the medium term perform pretty well. The average estimates are close to the true coefficients, and the confidence interval bands always include the true ones.

The explanations are similar to what we discussed before. In the model, the monetary shocks are assumed to contribute little to the real economy. This feature is based on the choice of the theoretical model, and the calibrated parameters from the postwar US data. Although the theoretical model does not have the timing properties, which required by the short-run restrictions, SVARs perform more robustly by imposing some *pseudo* short-run restrictions than the long-run restrictions. The reasons that the *pseudo* short-run timing assumption is robust is largely due to the small fluctuations of monetary policy shocks in postwar US data.

Medium-run identifying restriction

As for the medium-run restrictions, we take two possible parameters for k, as $k = 4$ and 20. The results are similar to those applying the short-run restriction when $k = 4$, and similar to those of using the long-run restriction when $k = 20$. As discussed in the short-run and long-run restrictions, the impulse responses shown in Figures A9–A10 and the variance decompositions shown in Tables A13–A16 have 5%–30% bias from the true ones. In general, the medium-run identifying restrictions are most robust than the long-run restrictions.

Contrasting the three identifying restrictions

Table 4 reports the average absolute value difference between the mean estimates and the true variance decomposition due to monetary shocks in the horizon 1, 4, 8, 12, 20, 36 and 40 quarters. The performances of the three identifying restrictions vary significantly in this case. The short-run restrictions provide closer estimates in the short horizons, while the long-run restrictions provide more reliable estimates for the long horizons. However, when we use the long-run restrictions to identify the contributions of various shocks, it tends to overestimate the contributions of monetary shocks to real variables. This estimation could be substantially larger than the actual ones. In the above example, we find that the contributions to the cyclical fluctuations of real output due to monetary shocks can indeed be substantially less. In all, the short-run and medium-run identifying restrictions are most robust. It is mainly because of the very small fluctuations of monetary policy shocks in the underlying calibrated model.

Table 4: (Case 2: 5-Shock Model) Distance between Mean Estimates and True Variance Decomposition due to Monetary Shocks (%)

Horizon	Identifying Assumption	Hours	Output	Real Interest Rate	Price
1	short-run	0.10	0.04	0.03	0.11
	medium-run with k=4	5.83	5.86	5.73	16.45
	with k=20	25.34	27.29	26.95	32.82
	long-run	27.38	36.82	36.50	6.42
4	short-run	1.82	1.89	0.93	1.47
	medium-run with k=4	3.63	3.48	2.97	17.44
	with k=20	22.79	24.59	23.70	33.12
	long-run	24.45	33.93	33.76	8.49
8	short-run	3.86	4.16	2.61	2.60
	medium-run with k=4	2.44	2.31	1.07	17.71
	with k=20	18.47	20.02	18.88	33.45
	long-run	20.56	29.67	29.58	8.87
12	short-run	5.30	6.09	4.43	2.84
	medium-run with k=4	2.76	2.94	1.11	17.03
	with k=20	15.01	16.27	14.42	32.68
	long-run	17.51	25.85	25.41	8.62
20	short-run	7.37	9.62	8.35	2.96
	medium-run with k=4	4.19	5.65	4.00	15.08
	with k=20	10.76	11.48	7.95	30.32
	long-run	13.73	20.03	18.19	8.42
36	short-run	9.47	14.81	15.95	3.07
	medium-run with k=4	6.3	10.49	11.89	11.43
	with k=20	7.37	7.65	3.61	25.89
	long-run	10.12	13.39	9.63	9.03
40	short-run	9.7	15.52	17.16	3.08
	medium-run with k=4	6.57	11.17	13.17	10.86
	with k=20	7.09	7.34	3.57	25.20
	long-run	9.68	12.58	8.72	9.19

Notes: Absolute value of difference between mean estimated and true model variance decomposition due to monetary shocks are reported in the table. Variance decomposition is in percent. The true model is Case-2 model as discussed in the paper.

5 Concluding Remarks

In conclusion, this paper uses Monte Carlo simulations to evaluate alternative identification strategies in VAR estimation of monetary models, and assess the accuracy of measuring price-level instability as a cause of the output fluctuations. Two economies are characterized: one is fully identified and satisfies the long-run restriction; another is not fully identified and the portion of temporary technology shocks is mixed with demand shocks when applying the long-run restriction. Based on each theoretical model, artificial economies are generated through Monte Carlo simulations. We investigate the reliability of structural VAR estimation under various identifying restrictions. Applying short-run, medium-run, and long-run restrictions on the simulated data, the bias between the average VAR estimates and the true theoretical claim has been studied. The findings show that short-run and medium-run restrictions tend to work more robustly under model uncertainty, particularly because the bias for measuring the effects of monetary shocks using long-run restriction could increase substantially when the underlying economy includes unidentified temporary shocks.

In light of our findings, the classic estimates in the literature that monetary shocks contribute about 30% of cyclical variance of post-war U.S. output could be an overstatement. Particularly, long-run restrictions may overlook transitory technology shocks, so they provide a lower bound for identifying the contribution of technology shocks. Consequently, the structural VARs tend to overestimate the contributions of monetary shocks to the business cycle fluctuations. We show that the bias could be substantial in our example. By comparing the various identifying schemes, we find the short-run, and the medium-run identifying restrictions work more robustly than the long-run identifying restriction.

Overall, VAR models with conventional long-run, short-run, and medium-run restrictions may provide different estimates. As we discussed, we provide an experiment using artificial economies, and the objective is to better understand statistics of interest in VAR models for economists and policymakers with practically reasonable sample sizes. The future research may include further studies on VARs with sign restrictions introduced by Faust (1995) and Uhlig (2005), and the state-space analysis pointed out by McGrattan (2007).

A TECHNICAL APPENDIX (Models and Solutions)

A.1 Nomenclature

Below there is the notation description for the variables in the model.

c_1 : per-capita consumption of "cash good"

c_2 : per-capita consumption of "credit good"

x : per-capita investment

k : per-capita net capital stock by households (beginning of period t stock has subscript t)

h : per-capita labor input

m : per-capita money demand (nominal)

T : per-capita government transfers (real)

K : total stock of capital demand in production

H : total labor input in production

r : real rental rate on capital

w : real wage rate

p : price level on the consumption goods

π : rate of inflation ($1 + \pi_t = \frac{p_t}{p_{t-1}}$)

M : nominal money supply

μ : growth rate of the money supply ($M_t = e^{\mu_t} M_{t-1}$)

μ_1 : part of growth rate of the money supply due to a mean-reverting monetary shock

μ_2 : part of growth rate of the money supply due to a random monetary shock ($\mu = \bar{\mu}_m + \mu_1 + \mu_2$)

χ : the permanent preference shock

Z : the permanent technology shock

V : the mean-reverting technology shock

A.2 Cash-in-advance Model

A.2.1 Maximization problems

Consider a monetary economy with households, firms, and the government. The representative household chooses consumption, investment, cash holding, and labor to solve the following maximization problem:

$$\max_{\{c_{1t},c_{2t},x_t,k_{t+1},m_t,h_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, h_t) \tag{54}$$

subject to

$$c_{1t} + c_{2t} + x_t + \frac{m_t}{p_t} \leq r_t k_t + w_t h_t + \delta k_t + \frac{m_{t-1}}{p_t} + \frac{T_t}{p_t} \tag{55}$$

$$c_{1t} \leq \frac{m_{t-1}}{p_t} + \frac{T_t}{p_t} \tag{56}$$

$$k_{t+1} = (1-\delta)k_t + x_t \tag{57}$$

$$c_{1t}, c_{2t}, x_t, h_t, m_t \geq 0 \text{ in all states}$$

taking processes for the goods price, rental rate, wage rate, the tax rates, and transfers as given. Also $T_t = M_{t+1} - M_t$.

Suppose the period utility function is of the form

$$U(c_{1t}, c_{2t}, 1-h_t) = \alpha \log c_{1t} + (1-\alpha)\log c_{2t} - \gamma \left(\frac{h_t}{\chi_t}\right)$$

The labor supply h_t is subject to a permanent preference shock χ_t, that follows the stochastic process

$$\log \chi_t - \log \chi_{t-1} = \sigma_\chi \epsilon_{\chi t+1}$$

where $\sigma_\chi > 0$ and the random variables $\{\epsilon_{\chi t}\}_{t=1}^{\infty}$ are time independent and normally distributed with zero mean and unit variance.

The representative firm solves a simple static problem at t :

$$\max_{\{K_t,L_t\}} F(K_t, H_t) - r_t K_t - w_t H_t \tag{58}$$

We consider a case with two technology innovations Z_t and V_t. One has a unit root in its law of motion, and another innovation process is stationary.

Suppose the firm now has a constant returns-to-scale Cobb Douglas production function of the form

$$Y_t = K_t^{\theta}(Z_t V_t H_t)^{1-\theta} \tag{59}$$

In the above function, Z_t is a unit-root process for technology whose law of motion is governed by:

$$\log Z_{t+1} - \log Z_t = g_z + \epsilon_{zt+1}$$

where the random variable ϵ_z is normally distributed with mean zero and standard deviation σ_z.

The government sets rates of taxes and transfers in such a way that their budget constraint at t, namely,

$$P_t G_t + T_t = M_{t+1} - M_t \tag{60}$$

$$G_t + \frac{T_t}{p_t} = \frac{(e^{\mu_{t+1}} - 1)M_t}{p_t} \tag{61}$$

is satisfied. In equilibrium, the following market clearing conditions must hold:

$$(c_t + x_t) + G_t = \exp(z_t)F(K_t, H_t) \tag{62}$$

$$k_t = K_t$$

$$h_t = H_t$$

$$m_t = M_t$$

The money stock follows a law of motion

$$M_t = e^{\mu_t} M_{t-1} \tag{63}$$

where the log of the gross growth rate of money supply evolves according to an autoregression of the following form. The random variable μ_t equals a constant term $\bar{\mu}_m$ plus two random components, μ_{1t} and μ_{2t}. The total μ_t is revealed to all agents at the beginning of period t.

$$\mu_t = \bar{\mu}_m + \mu_{1t} + \mu_{2t}.$$

The random variable μ_{1t} is assumed to evolve according to the autoregressive process,

$$\mu_{1t+1} = \rho_m \mu_{1t} + \sigma_1 \epsilon_{1t+1}, \quad 0 < \rho_m < 1. \tag{64}$$

The random variable $\{\epsilon_{1t}\}_{t=1}^{\infty}$ are time independent and normally distributed with mean zero and standard deviation 1. σ_2 represents the standard deviation. μ_{1t} follows a mean-reverting process.

The random variable μ_{2t} is assumed to be a random shock, which is a white noise at time t. It evolves according to the process

$$\mu_{2t} = \sigma_2 \epsilon_{2t}$$

The random variables $\{\epsilon_{2t}\}_{t=1}^{\infty}$ are time independent and normally distributed with mean zero and standard deviation 1. σ_2 represents the standard deviation of the random monetary shocks.

A.2.2 First-order conditions

I now derive the first-order conditions in this monetary economy. The Lagrangian for the household optimization problem is given by

$$\begin{aligned}
\mathfrak{L} = E_0 \sum_t \beta^t \{ & U(c_{1t}, c_{2t}, h_t) \\
& + \eta_t \{(r_t k_t + w_t h_t + \frac{m_t}{p_t} + \frac{T_t}{p_t} - c_{1t} - c_{2t} - x_t - \frac{m_{t+1}}{p_t}\} \\
& + \lambda_t \{\frac{m_t + T_t}{p_t} - c_{1t}\} \\
& + \zeta_t \{(1-\delta)k_t + x_t - k_{t+1}\}\}
\end{aligned}$$

The control variables are $c_{1t}, c_{2t}, x_t, h_t, m_{t+1}, k_{t+1}$. And the state variables are z_t, μ_t, and k_t. Here, I am assuming that the investment decision will be interior.

The relevant first-order conditions are found by taking derivatives of $\mathfrak{L}$ with respect to c_t, x_t, h_t, k_{t+1},and m_t :

$$\begin{aligned}
[c_{1t}] &: \frac{\alpha}{c_{1t}} = \lambda_t + \eta_t \\
[c_{2t}] &: \frac{1-\alpha}{c_{2t}} = \eta_t \\
[x_t] &: \eta_t = \zeta_t \\
[h_t] &: \frac{\gamma}{\chi_t} = \eta_t w_t \\
[k_{t+1}] &: \zeta_t = \beta E_t \{\eta_{t+1} r_{t+1} + \zeta_{t+1}(1-\delta)\} \\
[m_{t+1}] &: \frac{\eta_t}{p_t} = \beta E_t \{\frac{\lambda_{t+1} + \eta_{t+1}}{p_{t+1}}\}
\end{aligned}$$

Normalize the price level p_t by the expected money supply at time $t+1, M_{t+1}$ and the money demand m_t by the current money supply M_t, then we have

$$\hat{p}_t = \frac{p_t}{M_{t+1}} \tag{65}$$

$$\hat{m}_t = \frac{m_t}{M_t} \tag{66}$$

Eliminating multiplier ζ_t yields:

$$\frac{\alpha}{c_{1t}} = \lambda_t + \eta_t \tag{67}$$

$$\frac{1-\alpha}{c_{2t}} = \eta_t \tag{68}$$

$$\frac{\gamma}{\chi_t} = \eta_t w_t \tag{69}$$

$$\eta_t = \beta E_t\{\eta_{t+1}(r_{t+1} + 1 - \delta)\} \tag{70}$$

$$\eta_t = \beta E_t\{(\eta_{t+1} + \lambda_{t+1})\frac{\hat{p}_t}{\hat{p}_{t+1}}\frac{M_{t+1}}{M_{t+2}}\} \tag{71}$$

In addition, there are the first-order conditions for the firm's static problem. There are

$$w_t = F_2(K_t, H_t) \tag{72}$$

$$r_t = F_1(K_t, H_t) \tag{73}$$

Finally, we have a resource constraint given by (62).

In the equilibrium, we note that $\hat{m}_t = 1$.

A.2.3 Normalization

We detrend real GDP and the GDP expenditure components by $Z_t\chi_t$ and the capital stock, K_t, by $Z_{t-1}\chi_{t-1}$. Normalize the variable, labor supply, H_t by χ_t. Denote the normalized variable X_t as $\hat{X}_t = \frac{X_t}{Z_t\chi_t}$. As follows, we list all the normalized variables

$$\begin{array}{lllll} \hat{k}_t = \frac{k_t}{Z_t\chi_t}, & \hat{c}_{1t} = \frac{c_{1t}}{Z_t\chi_t}, & \hat{c}_{2t} = \frac{c_{2t}}{Z_t\chi_t}, & \hat{w}_t = \frac{w_t}{Z_t}, & \hat{x}_t = \frac{x_t}{Z_t\chi_t} \\ \hat{H}_t = \frac{H_t}{\chi_t}, & \hat{Y}_t = \frac{Y_t}{Z_t\chi_t}, & \hat{\lambda}_t = \lambda_t(Z_t\chi_t), & \hat{\eta}_t = \eta_t(Z_t\chi_t), & \bar{\hat{p}}_t = \hat{p}_t(Z_t\chi_t) \end{array}$$

Note: the interest rate r_t and the inflation rate π_t do not need the normalization, because they are stationary and not affected by the unit-root process of Z_t and χ_t.

A.2.4 The steady states

From here on, I make the following functional form assumptions and auxiliary choices:

$$U(c_{1t}, c_{2t}, 1-h_t) = \alpha \log c_{1t} + (1-\alpha)\log c_{2t} - \gamma\left(\frac{h_t}{\chi_t}\right)$$

I turned off the taxation distortion at first to solely consider a two-shock version of the model.

In this economy, there are four shocks $\log\frac{Z_t}{Z_{t-1}}$, $\log\chi_t$, μ_{1t} and μ_{2t}, one endogenous state variable $\hat{k}_t$. Besides, the control variables are $\hat{c}_{1t}, \hat{c}_{2t}, \hat{x}_t, \hat{h}_t, \hat{m}_t, \hat{k}_{t+1}$. The first-order conditions can be simplified as:

$$\frac{\alpha}{\hat{c}_{1t}} = \hat{\lambda}_t + \hat{\eta}_t \tag{74}$$

$$\frac{1-\alpha}{\hat{c}_{2t}} = \hat{\eta}_t \tag{75}$$

$$\hat{w}_t = \frac{\gamma}{\hat{\eta}_t} \tag{76}$$

$$\hat{c}_{1t} = \frac{1}{\bar{\hat{p}}_t} \tag{77}$$

$$\hat{y}_t = \hat{c}_{1t} + \hat{c}_{2t} + \hat{x}_t \tag{78}$$

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t\left(\frac{Z_{t-1}\chi_{t-1}}{Z_t\chi_t}\right) + \hat{x}_t \tag{79}$$

$$\hat{w}_t = (1-\theta)\frac{\hat{y}_t}{h_t} \tag{80}$$

$$r_t = \theta\frac{\hat{y}_t}{\hat{k}_t}\left(\frac{Z_t\chi_t}{Z_{t-1}\chi_{t-1}}\right) \tag{81}$$

$$\hat{y}_t = \hat{k}_t^{\theta}(V_t H_t)^{1-\theta}\left(\frac{Z_t\chi_t}{Z_{t-1}\chi_{t-1}}\right)^{-\theta} \tag{82}$$

$$\pi_{t+1} = \frac{\bar{\hat{p}}_{t+1}}{\bar{\hat{p}}_t} e^{\bar{\mu}_m + \mu_{1t+1} + \mu_{2t+2}}\left(\frac{Z_t\chi_t}{Z_{t-1}\chi_{t-1}}\right) \tag{83}$$

$$\hat{\eta}_t = \beta E_t\{\hat{\eta}_{t+1}(r_{t+1} + 1 - \delta)\left(\frac{Z_t\chi_t}{Z_{t+1}\chi_{t+1}}\right)\} \tag{84}$$

$$\hat{\eta}_t = \beta E_t\{\frac{\hat{\eta}_{t+1} + \hat{\lambda}_{t+1}}{\pi_{t+1}}\left(\frac{Z_t\chi_t}{Z_{t+1}\chi_{t+1}}\right)\} \tag{85}$$

Hence, from (84), we have in the steady state that

$$r^{ss} + (1-\delta) = \frac{e^{\bar{\mu}_z}}{\beta}$$

From (74-85), we have

$$\frac{\hat{y}^{ss}}{\hat{k}^{ss}} = \frac{r^{ss}}{\theta} e^{-\bar{\mu}_z}$$

The inflation rate in the steady state is

$$\pi^{ss} = e^{\bar{\mu}_m - \bar{\mu}_z}$$

Accordingly, we can get the steady state value for $\hat{c}_1^{ss}, \hat{c}_2^{ss}, \hat{k}^{ss}, \hat{h}^{ss}, \hat{y}^{ss}, \bar{\hat{p}}^{ss}, r^{ss}, \hat{w}^{ss}$.

A.2.5 Log-linearization

Approximate equilibrium decision functions can be computed by linearization the first-order conditions and applying the standard methods (for example, Uhlig (1999) uses log-linearization methods).

In this simple example, I derive the log-linearization around the steady states as follows:

$$0 = \frac{\alpha}{\hat{c}_1^{ss}}(\log \hat{c}_{1t} - \log \hat{c}_1^{ss}) + \hat{\lambda}^{ss}(\log \hat{\lambda}_t - \log \hat{\lambda}^{ss}) + \hat{\eta}^{ss}(\log \hat{\eta}_t - \log \hat{\eta}^{ss}) \tag{86}$$

$$0 = \frac{1-\alpha}{\hat{c}_2^{ss}}(\log \hat{c}_{2t} - \log \hat{c}_2^{ss}) + \hat{\eta}^{ss}(\log \hat{\eta}_t - \log \hat{\eta}^{ss}) \tag{87}$$

$$0 = (\log \hat{w}_t - \log \hat{w}^{ss}) + (\log \hat{\eta}_t - \log \hat{\eta}^{ss}) \tag{88}$$

$$0 = (\log \hat{c}_{1t} - \log \hat{c}_1^{ss}) + (\log \bar{\hat{p}}_t - \log \bar{\hat{p}}^{ss}) \tag{89}$$

$$0 = \hat{c}_1^{ss}(\log \hat{c}_{1t} - \log \hat{c}_1^{ss}) + \hat{c}_2^{ss}(\log \hat{c}_{2t} - \log \hat{c}_2^{ss}) + \hat{x}^{ss}(\log \hat{x}_t - \log \hat{x}^{ss}) - \hat{y}^{ss}(\log \hat{y}_t - \log \hat{y}^{ss}) \tag{90}$$

$$\begin{aligned}0 = {} & k^{ss}(\log \hat{k}_{t+1} - \log k^{ss}) - (1-\delta)\hat{k}^{ss} e^{-(\bar{\mu}_z + \bar{\mu}_m)}(\log \hat{k}_t - \log \hat{k}^{ss}) + (1-\delta)\hat{k}^{ss} e^{-(\bar{\mu}_z + \bar{\mu}_m)}[(\log \frac{Z_t}{Z_{t-1}} - \bar{\mu}_z) \\ & + \log \frac{\chi_t}{\chi_{t-1}}] - \hat{x}^{ss}(\log \hat{x}_t - \log \hat{x}^{ss})\end{aligned} \tag{91}$$

$$0 = (\log \hat{w}_t - \log \hat{w}^{ss}) + (\log \hat{h}_t - \log \hat{h}^{ss}) - (\log \hat{y}_t - \log \hat{y}^{ss}) \tag{92}$$

$$0 = (\log \hat{r}_t - \log \hat{r}^{ss}) + (\log \hat{k}_t - \log \hat{k}^{ss}) - (\log \hat{y}_t - \log \hat{y}^{ss}) - (\log \frac{Z_t}{Z_{t-1}} - \bar{\mu}_z) - \log \frac{\chi_t}{\chi_{t-1}} \tag{93}$$

$$0 = (\log \hat{y}_t - \log \hat{y}^{ss}) - \theta(\log \hat{k}_t - \log \hat{k}^{ss}) - (1-\theta)(\log \hat{h}_t - \log \hat{h}^{ss}) + \theta(\log \frac{Z_t}{Z_{t-1}} - \bar{\mu}_z) + \theta \log \frac{\chi_t}{\chi_{t-1}} \tag{94}$$

$$0 = (\log \pi_t - \log \pi^{ss}) - (\log \bar{\hat{p}}_t - \log \bar{\hat{p}}^{ss}) + (\log \bar{\hat{p}}_{t-1} - \log \bar{\hat{p}}^{ss}) - (\log \frac{Z_t}{Z_{t-1}} - g_z) - \log \frac{\chi_t}{\chi_{t-1}} - \mu_{1t} - \mu_{2t} \tag{95}$$

$$0 = E_t[(\log \hat{\eta}_{t+1} - \log \hat{\eta}^{ss})] + \beta E_t[r^{ss} e^{-(\bar{\mu}_z + \bar{\mu}_m)}(\log r_{t+1} - \log r^{ss})] - (\log \hat{\eta}_t - \log \hat{\eta}^{ss}) \tag{96}$$

$$0 = E_t[\log \pi_{t+1} - \log \pi^{ss}] - \beta E_t[\frac{1}{\pi^{ss}} e^{-(\bar{\mu}_z + \bar{\mu}_m)} (\log \hat{\eta}_{t+1} - \log \hat{\eta}^{ss})] - E_t[\frac{\hat{\lambda}^{ss}}{\hat{\eta}^{ss} + \hat{\lambda}^{ss}} (\log \lambda_{t+1} - \log \lambda^{ss})] \tag{97}$$

$$+ (\log \hat{\eta}_t - \log \hat{\eta}^{ss})$$

The equilibrium decision function for control and state variables X_t has the form

$$\log X_{t+1} = d_{x0} + d'_{xs} s_t$$

where our state vector is obvious to be $s_t = (1, \log \frac{Z_t}{Z_{t-1}}_t, \log \frac{\chi_t}{\chi_{t-1}}, \mu_{1t}, \mu_{2t}, \log \hat{k}_t)'$.

B APPENDIX (SVARs and State Representation)

B.1 Relation of the Theoretical Model to SVAR

In this section, we now discuss the relation between a theoretical model and VAR specifications. Specifically, we establish conditions under which the reduced form of the theoretical model is a VAR with disturbances that are linear combinations of the economic shocks. We begin by showing how to put the reduced form of the theoretical model into a form of a state-space representation. Throughout, we analyze the log-linear approximations to model solutions.

Suppose the variables of interest in the theoretical model are denoted by the series of vectors $\{X_t\}_{t=0}^{\infty}$ where t is the time index. Let S_t denote the vector of state variables including the endogenous state variables and exogenous economic shocks at time t. All variables are measured by the percent deviation from steady state, and after scaling by Z_t[11] if the variables have a technology trend. From the theoretical model, the log-linearization of the approximate solution for X_t is given by:

$$X_t = HS_t$$

and the state variables follow the law of motion:

$$S_t = FS_{t-1} + D\epsilon_t$$

[11] capital k_t is nomalized by Z_{t-1}

where ϵ_t denotes the independent identically-distributed economic shocks in the reduced form of the theoretical model with $E(\epsilon_t) = 0$, $E(\epsilon_t \epsilon_t') = I$. Matrices H, F and D are constructed from the log-linear approximation to the model. S_t include the endogenous state variables s_t^1 and exogenous state variables s_t^2 from economic shocks.[12]

The empirical researchers observe the vector of variables Y_t, which is equal to X_t plus measurement error, v_t. We assume

$$Y_t = X_t + v_t$$

where v_t is the independent and identically distributed measurement error at time t, with a diagonal variance-covariance, $E[v_t v_t'] \equiv R$.

The standard state-space representation follows the form of process:

$$S_t = FS_{t-1} + D\epsilon_t \tag{98}$$

$$Y_t = HS_t + v_t \tag{99}$$

We now focus on the case with no measurement errors. In this discussion, we set $v_t = 0$, so that $X_t = Y_t$ are observable variables. In addition, we assume that the number of elements in ϵ_t coincides with the number of elements in Y_t. It assumes the dimension of the exogenous shocks are the same as the dimension of the series of observable variables. In fact, it may be true that we observe more variables in Y_t than the exogenous shocks.

We reduce the log-linearization of the model solution as the form:

$$S_t = FS_{t-1} + D\epsilon_t \tag{100}$$

$$X_t = HS_t \tag{101}$$

where ϵ_t is the $[k \times 1]$ vector of economic shock, $E[\epsilon_t \epsilon_{t+j}'] = 0$ if $j \neq 0$ and $E[\epsilon_t \epsilon_t'] = I_{k \times k}$. Given the assumption that the number of elements in ϵ_t coincides with the number of elements in X_t, the observable variables X_t are also $[k \times 1]$ vectors. The state variables S_t are $[n \times 1]$ vectors. Accordingly, coefficient matrices have the dimensions that F is $[n \times n]$, D is $[n \times k]$, and H is

[12] We partition the coefficient matrices F and D. It is easy to show that

$$\begin{aligned} s_t^1 &= F_{11} s_{t-1}^1 + F_{12} s_{t-1}^2 + D_1 \epsilon_t \\ s_t^2 &= \qquad\qquad\quad F_{22} s_{t-1}^2 + D_2 \epsilon_t \end{aligned}$$

where the exogenous state variables s_t^2 are independent of the endogenous state variables s_{t-1}^1 at time $t-1$.

$[k \times n]$. n is the number of total state variables, and k is the number of economic shocks. Since the dimensions of economic shocks and the observables are equal, it is easy to find $n > k$.

From equation (100) and (101), we obtain

$$X_t = HFS_{t-1} + HD\epsilon_t \tag{102}$$

Define $C \equiv HD$. Assume that C is invertible, then we derive

$$X_t = HFS_{t-1} + C\epsilon_t \tag{103}$$

and

$$\epsilon_t = C^{-1}X_t - C^{-1}HFS_{t-1} \tag{104}$$

Substituting (104) into (100), we obtain:

$$S_t = MS_{t-1} + DC^{-1}X_t \tag{105}$$

where $M \equiv (I - DC^{-1}H)F$. As long as all the eigenvalues of M are less than unity in absolute value, the above equation with the lag operator expression as

$$(I - ML)S_t = DC^{-1}X_t$$

allows us to express the state variables as linear combination of the lagged observable variables:

$$\begin{aligned} S_t &= (I - ML)^{-1}DC^{-1}X_t \\ &= DC^{-1}X_t + MDC^{-1}X_{t-1} + M^2DC^{-1}X_{t-2} + M^3DC^{-1}X_{t-3} + ... \end{aligned} \tag{106}$$

Substituting (106) into (104), we find the economic shocks is a function of the lagged variables of X_t :

$$\begin{aligned} \epsilon_t = C^{-1}X_t - C^{-1}HFDC^{-1}X_{t-1} - C^{-1}HFMDC^{-1}X_{t-2} - C^{-1}HFM^2DC^{-1}X_{t-3} \\ ... - C^{-1}HFM^pDC^{-1}X_{t-p-1} - ... \end{aligned}$$

After rearranging, we derive an infinite order VAR form from the reduced state representation (100) and (101), which shows

$$X_t = HFDC^{-1}X_{t-1} + HFMDC^{-1}X_{t-2} + HFM^2DC^{-1}X_{t-3} + ... + C\epsilon_t \tag{107}$$

This expression is a form of a standard infinite order $VAR(\infty)$, which is usually seen as

$$X_t = B_1X_{t-1} + B_2X_{t-2} + B_2X_{t-3} + ... + u_t \tag{108}$$

where

$$u_t = C\epsilon_t \tag{109}$$

$$B_i = HFM^{i-1}DC^{-1}, \quad i = 1, 2, 3, ... \tag{110}$$

Here u_t is the regression errors, which is a linear combination of the economic shocks ϵ_t. We observe that error terms u_t is independent of X_{t-i} ,$i \geq 1$. It provides the relation of the theoretical model to the VAR estimation in empirical analysis.

Lemma 1 *(Fernandez-Villaverdez, Rubio-Ramirez, and Sargent) If C $(C \equiv HD)$ is invertible and the eigenvalues of M $(M \equiv (I - DC^{-1}H)F)$ are less than unity in absolute values, then the state representation reduced from the theoretical model*

$$S_t = FS_{t-1} + D\epsilon_t \tag{111}$$

$$X_t = HS_t \tag{112}$$

implies X_t has the infinite-order VAR representation form as

$$X_t = B_1X_{t-1} + B_2X_{t-2} + B_3X_{t-3} + ... + u_t \tag{113}$$

where $E(\epsilon_t\epsilon_t') = I$ and $E(\epsilon_t\epsilon_{t+j}') = 0$ if $j \neq 0$. The disturbances u_t is related to the economic shocks ϵ_t by (109), and coefficient matrices in $VAR(\infty)$ are related to the state-representation coefficient matrices by (110).

Under these assumptions, the state representation implies a $VAR(\infty)$ representation. In practice, we suppose that $VARs$ with finite lags in regressions could provide a close estimation as a

$VAR(\infty)$ with infinite lags, then it shows the practical importance of $VARs$.

B.2 Relation of the Long-run Restriction to its Economic Meaning

Among the economic shocks $\{\epsilon_t\}_{t=0}^{\infty}$, we assort them into two types: real shocks $\{\epsilon_t^{\text{real}}\}_{t=0}^{\infty}$ and nominal shocks $\{\epsilon_t^{\text{nominal}}\}_{t=0}^{\infty}$. The criterion used for the classification is to determine if the economic shocks have permanent impacts on the steady states of the economy. For example, in our monetary economy, monetary shocks $\{\epsilon_{Mt}\}_{t=0}^{\infty}$ are modeled as "neutral", which means the disturbances of the monetary policy have no impact on the steady state of the economy (e.g. the levels of output, total factor productivity, and hours worked, etc.) On the other hand, the technology shocks $\{\epsilon_{zt}\}_{t=0}^{\infty}$ and $\{\epsilon_{vt}\}_{t=0}^{\infty}$ have permanent effects to change the steady state of the economy of corresponding variables, as asserted and applied in $VARs$ by Blanchard and Quah (1989), Gali (1999), Shapiro and Watson (1988).

To formalize the economic meaning of "neutrality" and "non-neutrality", we assume that the information set at time t, Ω_t, includes the state variables (both endogenous state variables and exogenous shocks) and observables before time t

$$\Omega_t = (S_{t-s}, X_{t-s};\ \text{where } s \geq 0)$$

From the state representation, (101), it shows that X_t is a function of S_t for each $t \geq 0$. It means that

$$\Omega_t = (S_{t-s};\ \text{where } s \geq 0)$$

The state variables $\{S_t\}$ include endogenous state variables (e.g. $\widehat{\log k_t}$ in our monetary model) and exogenous state variables (e.g. technology and monetary shocks). To differ these two types of state variables, we denote the endogenous state variables as s_t^1, and the exogenous ones as s_t^2. Hence, the state variables are $S_t = (s_t^1, s_t^2)'$. Given Eq. (100) $S_t = FS_{t-1} + D\epsilon_t$, we derive the state transitional paths

$$\begin{bmatrix} s_t^1 \\ s_t^2 \end{bmatrix}_{[n\times 1]} = \begin{bmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{bmatrix} \begin{bmatrix} s_{t-1}^1 \\ s_{t-1}^2 \end{bmatrix}_{[n\times 1]} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \epsilon_t \tag{114}$$

To be specific, the coefficient matrices F and D have the partitioned forms as in the above equation. The law of motions (114) shows that the state variable S_t is a function of the history of the past disturbances $\{\epsilon_{t-j}\}_{j=0}^{\infty}$. Therefore, the information set at time t has the property

$$\Omega_t = (\epsilon_{t-s};\ \text{where } s \geq 0) \tag{115}$$

Classifying the shocks into real and nominal, we obtain $\Omega_t = \left(\epsilon_{t-s}^{\text{real}}, \epsilon_{t-s}^{\text{nominal}};\ \text{where } s \geq 0\right)$. The following proposition states the property of the information set Ω_t:

Proposition 1 *The information set at time t,* $\Omega_t = (S_{t-s}, X_{t-s};\ \textit{where } s \geq 0)$, *which include both the state variables and control variables till time t, can be reduced to the form:*

$$\Omega_t = (\epsilon_{t-s};\ \textit{where } s \geq 0) = \left(\epsilon_{t-s}^{real}, \epsilon_{t-s}^{nominal};\ \textit{where } s \geq 0\right) \tag{116}$$

where ϵ_t *is the economic shocks happened at time t. It shows that the information set is the history of all the economic shocks.*

Blanchard and Quah (1989), Gali (1999), Shapiro and Watson (1988) argue that only the real economic shocks $\{\epsilon_t^{\text{real}}\}_{t=0}^{\infty}$, but not the nominal shocks $\{\epsilon_t^{\text{nominal}}\}_{t=0}^{\infty}$, have the permanent impacts on the steady states of the economy. Regarding the variable of interest is a_t, their statement implies that

$$\lim_{j\to\infty} [E_t a_{t+j} - E_{t-1} a_{t+j}] = f(\epsilon_t^{\text{real}}) \tag{117}$$

where $E_t[.] = E[.|\Omega_t]$ denotes the expectation, conditional on the information set $\Omega_t = (\epsilon_{t-s};\ \text{where } s \geq 0)$. The prediction of the future fluctuation a_{t+j} before and after economic shocks at time t is only determined by the real economic shocks. Those shocks, which are neutral, will not affect the prediction of steady states in the economy. Given the log-linearization approximation of model solutions, $f(.)$ is a linear function.

Before we start to discuss the relation between this statement with the long-run restriction in VAR, we assume that the assumptions in Lemma 1 hold. According to Lemma 1, the state representation form (100) and (101) has the infinite-order VAR representation form as $X_t = B_1 X_{t-1} + B_2 X_{t-2} + B_2 X_{t-3} + ... + u_t$. Or equivalently, we denote the $VAR(\infty)$ as

$$(I - B(L))X_t = u_t$$

u_t is related to the fundamental economic shocks ϵ_t as follows: $u_t = C\epsilon_t$, where $E(\epsilon_t) = 0$, $E[\epsilon_t \epsilon_t'] = I$, and $E[\epsilon_t \epsilon_{t+j}'] = 0$ for $j > 0$. L is the lag operator, and $B(L) = B_1 L + B_2 L^2 + B_3 L^3 + ...$.

We have the transformation

$$
\begin{aligned}
X_t &= (I - B(L))^{-1} u_t \\
&= (I - B(L))^{-1} C \epsilon_t
\end{aligned}
$$

From this transformation, we find the variables of interest are linear functions of the history of fundamental economic shocks. An explicit expression of the VAR representation shows

$$
X_t = (I - B(L))^{-1} C \epsilon_t \tag{118}
$$

$$
= \sum_{i=0}^{\infty} \mathfrak{D}_i L^i \epsilon_t = \sum_{i=0}^{\infty} \mathfrak{D}_i \epsilon_{t-i} \tag{119}
$$

econometricians call $\{\mathfrak{D}_i\}$ impulse response function matrices, which interpret the impact on variable vector X_t from the economic shocks ϵ_{t-i}. Or, alternatively, we have the impulse response function matrices as

$$
\mathfrak{D}_i = \frac{\bigtriangledown X_i}{\bigtriangledown \epsilon_{t-i}} \tag{120}
$$

This can be concluded in the following proposition:

Proposition 2 *Suppose that the variable, a_t, is non-stationary, and its first-order difference $\Delta a_t \equiv a_t - a_{t-1}$ is stationary. Hence, if the assumptions in Lemma 1 hold, we obtain*

$$
\Delta a_t = \sum_{i=0}^{\infty} \mathfrak{D}_i \epsilon_{t-i} \tag{121}
$$

where $\mathfrak{D}_i$ is the impulse response function matrix of the economic shock happened i period ago.

Proof. See the argument above. ∎

Recall the statement of “long-run restriction” by Blanchard and Quah (1989), Gali (1999), Shapiro and Watson (1988), which is obtained in the formal expression (117). Now we discuss the relation of this statement with the long-run restriction in $VARs$.

Proposition 3 *Assume that the assumptions in Lemma 1, and the assumptions about $\{a_t\}$ in Proposition 2 hold. The statement that only real shocks have permanent impacts on the steady state of the economy, or*

$$
\lim_{j \to \infty} [E_t a_{t+j} - E_{t-1} a_{t+j}] = f(\epsilon_t^{real}) \tag{122}
$$

is equivalent to the following equation

$$\sum_{i=0}^{\infty} \mathfrak{D}_i^{nominal} = 0 \tag{123}$$

where $\mathfrak{D}_i^{nominal}$ is the impulse response function from the nominal economic shocks in the stationary process $\Delta a_t = \sum_{i=0}^{\infty} \mathfrak{D}_i \epsilon_{t-i}$.

Proof. Applying the properties of projection (e.g. Hamilton 1994), we obtain

$$E_t a_{t+j} - E_{t-1} a_{t+j} = \{E(a_{t+j} - E(a_{t+j}|\Omega_{t-1})(\epsilon_t - E(\epsilon_t|\Omega_{t-1}))'\}\{E(\epsilon_t - E(\epsilon_t|\Omega_{t-1})(\epsilon_t - E(\epsilon_t|\Omega_{t-1}))'\}^{-1} \times (\epsilon_t - E(\epsilon_t|\Omega_{t-1})) \tag{124}$$

Applying that $E(\epsilon_t|\Omega_{t-1}) = 0$, $E(\epsilon_t \epsilon_t') = I$, we reduce the equation to

$$E_t a_{t+j} - E_{t-1} a_{t+j} = \{E(a_{t+j}\epsilon_t')\}\epsilon_t \tag{125}$$

As explained, the left hand side of the equation implies the impact of the extra information obtained at time t. It shows that it equals the projection of the variable a_{t+j} on the economic shocks at time t.

Because $a_{t+j} = \sum_{i=0}^{j}(\Delta a_{t+j-i}) + a_{t-1}$, we reduce the term in the right-hand side of (125) as

$$\begin{aligned}\{E(a_{t+j}\epsilon_t')\} &= E[\sum_{i=0}^{j}(\Delta a_{t+j-i}) + a_{t-1}]\epsilon_t' \\ &= \sum_{i=0}^{j} E[(\Delta a_{t+j-i}) \cdot \epsilon_t']\end{aligned}$$

From Proposition 2, we derive that $\Delta a_t = \sum_{i=0}^{\infty} \mathfrak{D}_i \epsilon_{t-i}$. Hence, $E[(\Delta a_{t+j-i}) \cdot \epsilon_t'] = E\mathfrak{D}_{j-i}\epsilon_t\epsilon_t' = \mathfrak{D}_{j-i}$, where we use $E\epsilon_t\epsilon_t' = I$ and $E\epsilon_{t+j}\epsilon_t' = 0$ otherwise.

The statement (122) is now reduced to

$$\lim_{j\to\infty} [E_t a_{t+j} - E_{t-1} a_{t+j}] = \left[\lim_{j\to\infty} \sum_{i=0}^{j} \mathfrak{D}_i\right] \cdot \epsilon_t = \left[\sum_{i=0}^{\infty} \mathfrak{D}_i\right] \cdot \epsilon_t = f(\epsilon_t^{\text{real}})$$

Suppose $\mathfrak{D}_i^{\text{real}}$ and $\mathfrak{D}_i^{\text{nominal}}$ denote impulse responses of the real and nominal economic shocks.

Hence, it shows that

$$\left[\sum_{i=0}^{\infty} \mathfrak{D}_i^{\text{nominal}}\right] = 0$$

■

From (118) and (119), we have

$$\left[\sum_{i=0}^{\infty} \mathfrak{D}_i\right] = (I - B(1))^{-1}C$$

where $B(1) = B_1 + B_2 + B_3 + ...$ in equation (118). Hence, the long-run restriction means

$$(I - B(1))^{-1}C = \begin{bmatrix} number & \underline{0} \\ number & number \end{bmatrix} \tag{126}$$

where $\underline{0}$ is a row vector of zeros for the columns matching the nominal economic shocks, and whose row matches the variable of interest Δa_t in regression.

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Table A1: (LONG-RUN RESTRICTION) VARIANCE DECOMPOSITION OF HOURS, AND OUTPUT (%)
(Means and 95% Bounds over 1000 Estimates, 4-Shock Model)

	Fraction of hours explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	90.54	9.33	0.13	70.35	24.95	4.70
				[12.43, 99.05]	[0.06, 83.04]	[0.01, 22.12]
4	91.4	8.54	0.06	72.6	24.06	3.34
				[14.11, 98.50]	[0.32, 82.45]	[0.13, 14.34]
8	92.34	7.62	0.04	75.44	22.51	2.06
				[18.04, 98.78]	[0.34, 79.70]	[0.12, 8.41]
12	93.13	6.84	0.03	77.9	20.56	1.54
				[21.96, 99.10]	[0.31, 75.51]	[0.10, 6.16]
20	94.37	5.61	0.02	81.65	17.26	1.10
				[30.95, 99.41]	[0.22, 68.13]	[0.07, 4.37]
36	95.95	4.04	0.01	86.3	12.96	0.74
				[45.90, 99.65]	[0.14, 53.28]	[0.04, 3.03]
∞	100	0	0			
	Fraction of output explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	48.5	51.43	0.07	35.78	59.55	4.67
				[0.20, 85.65]	[10.45, 98.26]	[0.00, 22.59]
4	48.53	51.45	0.02	36.56	60.18	3.26
				[0.59, 84.85]	[11.63, 97.71]	[0.12, 13.90]
8	48.53	51.46	0.01	37.3	60.8	1.89
				[0.83, 85.47]	[12.56, 97.94]	[0.09, 8.22]
12	48.53	51.46	0.01	37.58	61.09	1.33
				[0.83, 86.01]	[12.64, 98.13]	[0.07, 5.58]
20	48.53	51.46	0	37.79	61.33	0.88
				[0.72, 86.92]	[12.59, 98.44]	[0.04, 3.91]
36	48.54	51.46	0	37.92	61.52	0.56
				[0.65, 87.46]	[12.38, 98.74]	[0.02, 2.66]
∞	48.54	51.46	0			

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A2: (LONG-RUN RESTRICTION) VARIANCE DECOMPOSITION OF PRICE LEVEL, AND INTEREST RATE (%)
(Means and 95% Bounds over 1000 Estimates, 4-Shock Model)

	Fraction of prices explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	8.81	9.34	81.85	11.75	16.96	71.29
				[0.03, 47.87]	[0.04, 57.78]	[27.16, 97.90]
4	7.88	8.35	83.77	11.36	15.62	73.02
				[0.19, 44.30]	[0.28, 53.01]	[33.76, 97.57]
8	8.50	9.01	82.49	12.42	16.45	71.13
				[0.27, 46.48]	[0.35, 53.27]	[34.10, 96.76]
12	9.42	9.99	80.59	13.31	17.32	69.36
				[0.25, 48.55]	[0.34, 54.49]	[34.03, 96.99]
20	11.19	11.86	76.95	14.69	18.73	66.58
				[0.23, 53.53]	[0.37, 58.43]	[30.08, 96.89]
36	13.79	14.62	71.58	16.44	20.51	63.05
				[0.18, 59.77]	[0.41, 61.86]	[22.85, 96.70]
∞	17.7	18.76	63.54			
	Fraction of real interest rates explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	48.5	51.43	0.07	35.77	59.54	4.69
				[0.19, 85.66]	[10.43, 98.26]	[0.00, 22.68]
4	48.5	51.43	0.07	36.5	60.02	3.47
				[0.63, 85.11]	[11.59, 97.48]	[0.14, 14.62]
8	48.5	51.43	0.07	37.16	60.41	2.42
				[0.93, 85.21]	[12.66, 97.44]	[0.16, 9.84]
12	48.5	51.43	0.07	37.37	60.52	2.11
				[0.99, 85.59]	[12.65, 97.43]	[0.15, 8.69]
20	48.5	51.43	0.07	37.5	60.59	1.91
				[1.00, 86.44]	[12.52, 97.50]	[0.13, 8.01]
36	48.5	51.43	0.07	37.55	60.62	1.83
				[0.97, 86.97]	[12.43, 97.53]	[0.12, 7.59]
∞	48.5	51.43	0.07			

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A3: (SHORT-RUN RESTRICTION) VARIANCE DECOMPOSITION OF HOURS, AND OUTPUT (%)
(Means and 95% Bounds over 1000 Estimates, 4-Shock Model)

	Fraction of hours explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	90.54	9.33	0.13	100	0	0
				[100.00, 100.00]	[0.00, 0.00]	[0.00, 0.00]
4	91.4	8.54	0.06	97.49	0.90	1.61
				[91.95, 99.69]	[0.04, 4.06]	[0.07, 7.11]
8	92.34	7.62	0.04	95.49	1.84	2.66
				[85.29, 99.46]	[0.08, 8.63]	[0.15, 12.15]
12	93.13	6.84	0.03	94.21	2.74	3.05
				[80.22, 99.41]	[0.10, 12.99]	[0.15, 14.17]
20	94.37	5.61	0.02	91.83	4.84	3.33
				[70.84, 99.33]	[0.12, 21.39]	[0.13, 15.58]
36	95.95	4.04	0.01	87.81	8.72	3.47
				[58.40, 99.28]	[0.17, 36.78]	[0.10, 16.26]
∞	100	0	0			
	Fraction of output explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	48.50	51.43	0.07	77.75	22.25	0
				[71.10, 83.04]	[16.96, 28.90]	[0.00, 0.00]
4	48.53	51.45	0.02	76.01	22.41	1.58
				[61.57, 87.41]	[11.54, 36.32]	[0.05, 6.93]
8	48.53	51.46	0.01	74.84	22.59	2.56
				[55.69, 90.01]	[8.08, 41.80]	[0.12, 11.56]
12	48.53	51.46	0.01	74.39	22.65	2.96
				[53.20, 91.54]	[6.71, 44.18]	[0.11, 13.24]
20	48.53	51.46	0	74.04	22.70	3.26
				[50.82, 92.49]	[5.51, 46.07]	[0.09, 14.41]
36	48.54	51.46	0	73.82	22.73	3.44
				[49.31, 93.28]	[4.78, 47.16]	[0.06, 15.46]
∞	48.54	51.46	0			

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A4: (SHORT-RUN RESTRICTION) VARIANCE DECOMPOSITION OF PRICE LEVEL, AND INTEREST RATE (%)
(Means and 95% Bounds over 1000 Estimates, 4-Shock Model)

	Fraction of prices explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	8.81	9.34	81.85	17.71	4.32	77.97
				[7.67, 30.47]	[0.34, 10.91]	[64.43, 88.92]
4	7.88	8.35	83.77	16.84	4.72	78.44
				[3.99, 35.49]	[0.29, 15.45]	[58.62, 92.82]
8	8.50	9.01	82.49	18.08	5.68	76.23
				[2.96, 41.93]	[0.25, 20.52]	[50.32, 93.89]
12	9.42	9.99	80.59	19.41	6.27	74.32
				[2.40, 46.30]	[0.23, 23.00]	[44.59, 94.08]
20	11.19	11.86	76.95	21.55	7.05	71.4
				[1.95, 53.81]	[0.22, 25.45]	[35.97, 95.11]
36	13.79	14.62	71.58	24.27	7.96	67.78
				[1.54, 62.73]	[0.19, 29.00]	[25.79, 96.28]
∞	17.7	18.76	63.54			
	Fraction of real interest rates explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	48.5	51.43	0.07	77.75	22.25	0
				[71.10, 83.02]	[16.98, 28.90]	[0.00, 0.00]
4	48.5	51.43	0.07	75.93	22.39	1.68
				[61.65, 87.07]	[11.70, 36.27]	[0.06, 7.14]
8	48.5	51.43	0.07	74.62	22.59	2.79
				[55.76, 89.42]	[8.66, 41.74]	[0.15, 11.77]
12	48.5	51.43	0.07	74.13	22.64	3.23
				[53.60, 90.49]	[7.52, 43.55]	[0.15, 13.49]
20	48.5	51.43	0.07	73.81	22.67	3.52
				[51.84, 91.23]	[7.10, 44.83]	[0.15, 14.87]
36	48.5	51.43	0.07	73.7	22.69	3.62
				[51.10, 91.67]	[6.69, 45.07]	[0.14, 15.17]
∞	48.5	51.43	0.07			

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A5: (MEDIUM-RUN RESTRICTION, k=4) VARIANCE DECOMPOSITION OF HOURS, AND OUTPUT (%)
(Means and 95% Bounds over 1000 Estimates, 4-Shock Model)

	Fraction of hours explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	90.54	9.33	0.13	93.26	2.55	4.19
				[77.79, 99.76]	[0.00, 12.41]	[0.01, 17.46]
4	91.4	8.54	0.06	95.79	1.66	2.55
				[86.64, 99.43]	[0.07, 7.73]	[0.13, 9.42]
8	92.34	7.62	0.04	97.24	1.07	1.69
				[91.09, 99.60]	[0.08, 4.73]	[0.10, 6.19]
12	93.13	6.84	0.03	97.05	1.35	1.6
				[90.27, 99.58]	[0.08, 5.76]	[0.10, 6.18]
20	94.37	5.61	0.02	95.62	2.87	1.51
				[83.50, 99.57]	[0.10, 13.69]	[0.08, 6.33]
36	95.95	4.04	0.01	92.27	6.32	1.40
				[70.25, 99.59]	[0.10, 27.32]	[0.07, 6.60]
∞	100	0	0			
	Fraction of output explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	48.5	51.43	0.07	69.79	26.31	3.90
				[39.23, 93.11]	[3.95, 55.17]	[0.01, 15.60]
4	48.53	51.45	0.02	71.24	26.49	2.26
				[42.77, 90.73]	[7.75, 54.08]	[0.19, 8.06]
8	48.53	51.46	0.01	72.12	26.46	1.42
				[46.28, 88.62]	[10.40, 51.30]	[0.13, 4.80]
12	48.53	51.46	0.01	72.22	26.44	1.34
				[46.79, 88.60]	[10.34, 51.14]	[0.10, 4.91]
20	48.53	51.46	0	72.29	26.43	1.28
				[46.77, 88.77]	[10.49, 51.49]	[0.08, 5.87]
36	48.54	51.46	0	72.36	26.43	1.22
				[46.31, 88.95]	[10.29, 52.09]	[0.05, 6.29]
∞	48.54	51.46	0			

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A6: (MEDIUM-RUN RESTRICTION, k=4) VARIANCE DECOMPOSITION OF PRICE LEVEL, AND INTEREST RATE (%)
(Means and 95% Bounds over 1000 Estimates, 4-Shock Model)

	Fraction of prices explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	8.81	9.34	81.85	24.18	10.81	65.02
				[0.31, 85.85]	[0.01, 56.77]	[2.62, 96.14]
4	7.88	8.35	83.77	23.1	10.99	65.91
				[0.66, 82.59]	[0.15, 56.41]	[3.76, 96.02]
8	8.50	9.01	82.49	24.42	11.79	63.79
				[0.87, 84.81]	[0.23, 59.47]	[2.86, 95.68]
12	9.42	9.99	80.59	25.79	12.22	61.99
				[0.92, 86.61]	[0.21, 60.03]	[2.06, 95.46]
20	11.19	11.86	76.95	27.88	12.83	59.29
				[1.12, 88.08]	[0.21, 59.05]	[1.38, 95.25]
36	13.79	14.62	71.58	30.36	13.58	56.06
				[1.13, 88.56]	[0.22, 59.90]	[0.95, 95.72]
∞	17.7	18.76	63.54			
	Fraction of real interest rates explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	48.5	51.43	0.07	69.79	26.31	3.90
				[39.23, 93.14]	[3.96, 55.16]	[0.01, 15.53]
4	48.5	51.43	0.07	71.16	26.48	2.37
				[42.27, 90.59]	[7.81, 53.70]	[0.20, 8.37]
8	48.5	51.43	0.07	71.72	26.4	1.89
				[45.70, 88.31]	[10.49, 51.36]	[0.19, 6.26]
12	48.5	51.43	0.07	71.66	26.35	1.99
				[45.58, 88.25]	[10.46, 51.05]	[0.17, 7.20]
20	48.5	51.43	0.07	71.61	26.32	2.07
				[45.40, 88.25]	[10.45, 51.59]	[0.16, 8.37]
36	48.5	51.43	0.07	71.6	26.31	2.09
				[45.42, 88.38]	[10.39, 51.70]	[0.15, 8.68]
∞	48.5	51.43	0.07			

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A7: (MEDIUM-RUN RESTRICTION, k=20) VARIANCE DECOMPOSITION OF HOURS, AND OUTPUT (%)
(Means and 95% Bounds over 1000 Estimates, 4-Shock Model)

	Fraction of hours explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	90.54	9.33	0.13	80.46	14.31	5.23
				[29.40, 99.50]	[0.02, 65.56]	[0.00, 23.48]
4	91.4	8.54	0.06	83.08	13.27	3.65
				[34.37, 99.07]	[0.19, 60.78]	[0.14, 15.85]
8	92.34	7.62	0.04	86.08	11.67	2.25
				[42.88, 99.29]	[0.21, 54.36]	[0.12, 9.39]
12	93.13	6.84	0.03	88.29	10.05	1.66
				[49.39, 99.51]	[0.15, 48.01]	[0.09, 7.00]
20	94.37	5.61	0.02	91.1	7.74	1.17
				[60.06, 99.69]	[0.10, 37.91]	[0.06, 4.99]
36	95.95	4.04	0.01	93.55	5.66	0.79
				[69.99, 99.81]	[0.06, 28.22]	[0.04, 3.41]
∞	100	0	0			
	Fraction of output explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	48.5	51.43	0.07	46.59	48.32	5.09
				[1.18, 85.80]	[10.16, 95.59]	[0.00, 22.40]
4	48.53	51.45	0.02	47.53	49.01	3.45
				[1.69, 84.36]	[12.81, 95.34]	[0.11, 15.04]
8	48.53	51.46	0.01	48.45	49.54	2.01
				[1.71, 83.56]	[14.79, 96.13]	[0.08, 8.69]
12	48.53	51.46	0.01	48.82	49.77	1.41
				[1.70, 83.59]	[15.32, 96.69]	[0.06, 6.15]
20	48.53	51.46	0	49.11	49.96	0.93
				[1.65, 84.07]	[15.05, 97.11]	[0.04, 4.14]
36	48.54	51.46	0	49.29	50.12	0.59
				[1.48, 84.76]	[14.99, 97.42]	[0.02, 2.88]
∞	48.54	51.46	0			

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A8: (MEDIUM-RUN RESTRICTION, k=20) VARIANCE DECOMPOSITION OF PRICE LEVEL, & INTEREST RATE (%)
(Means and 95% Bounds over 1000 Estimates, 4-Shock Model)

	Fraction of prices explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	8.81	9.34	81.85	15.15	14.6	70.25
				[0.05, 62.74]	[0.03, 54.43]	[12.47, 98.08]
4	7.88	8.35	83.77	15.07	14.2	70.73
				[0.18, 64.29]	[0.22, 52.53]	[13.01, 97.61]
8	8.5	9.01	82.49	16.53	15.12	68.35
				[0.33, 65.58]	[0.32, 54.38]	[8.89, 96.95]
12	9.42	9.99	80.59	17.73	15.83	66.44
				[0.36, 66.93]	[0.33, 56.13]	[7.66, 96.96]
20	11.19	11.86	76.95	19.54	16.86	63.6
				[0.35, 68.61]	[0.40, 56.97]	[7.40, 96.87]
36	13.79	14.62	71.58	21.77	18.09	60.14
				[0.33, 71.44]	[0.40, 58.24]	[6.49, 96.54]
∞	17.7	18.76	63.54			
	Fraction of real interest rates explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	48.5	51.43	0.07	46.57	48.3	5.12
				[1.17, 85.83]	[10.16, 95.58]	[0.00, 22.62]
4	48.5	51.43	0.07	47.43	48.87	3.7
				[1.84, 83.91]	[12.74, 95.07]	[0.14, 15.35]
8	48.5	51.43	0.07	48.22	49.23	2.55
				[2.06, 83.15]	[14.73, 95.30]	[0.15, 10.41]
12	48.5	51.43	0.07	48.49	49.32	2.19
				[2.09, 83.13]	[15.05, 95.34]	[0.14, 8.88]
20	48.5	51.43	0.07	48.65	49.38	1.97
				[2.09, 83.68]	[15.15, 95.20]	[0.13, 8.29]
36	48.5	51.43	0.07	48.72	49.4	1.88
				[2.07, 84.11]	[14.72, 95.20]	[0.12, 8.06]
∞	48.5	51.43	0.07			

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A9: (LONG-RUN RESTRICTION) VARIANCE DECOMPOSITION OF HOURS, AND OUTPUT (%)
(Means and 95% Bounds over 1000 Estimates, 5-Shock Model)

	Fraction of hours explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	67.76	32.14	0.10	51.89	20.63	27.48
		6.98 / 25.16		[4.28, 95.70]	[0.09, 74.06]	[0.05, 83.73]
4	71.90	28.05	0.05	55.38	20.11	24.50
		6.72 / 21.34		[7.01, 95.01]	[0.28, 72.73]	[0.44, 78.63]
8	76.44	23.53	0.03	60.22	19.19	20.59
		6.31 / 17.22		[12.08, 95.26]	[0.37, 66.56]	[0.49, 71.21]
12	79.99	19.99	0.02	64.62	17.85	17.53
		5.88 / 14.11		[16.93, 95.92]	[0.35, 63.12]	[0.53, 62.77]
20	84.8	15.19	0.02	70.99	15.26	13.75
		5.04 / 10.15		[28.17, 96.99]	[0.32, 54.81]	[0.47, 48.56]
36	89.44	10.55	0.01	78.23	11.64	10.14
		3.76 / 6.79		[42.25, 97.96]	[0.24, 42.65]	[0.43, 33.02]
∞	94.75	5.24	0.01	79.17	11.14	9.69
	Fraction of output explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	26.45	73.52	0.04	23.46	39.68	36.86
		28.04 / 45.47		[0.10, 72.53]	[0.40, 93.46]	[0.13, 92.51]
4	28.14	71.84	0.01	24.77	41.29	33.94
		29.84 / 42.00		[0.44, 69.90]	[0.92, 93.04]	[0.52, 90.35]
8	30.22	69.78	0.01	26.46	43.86	29.68
		32.04 / 37.74		[0.63, 69.54]	[2.07, 93.14]	[0.60, 84.80]
12	32.07	67.93	0	28.09	46.05	25.86
		34.00 / 33.92		[0.86, 70.36]	[3.30, 93.39]	[0.51, 78.41]
20	35.16	64.84	0	30.74	49.23	20.04
		37.28 / 27.57		[1.18, 72.34]	[6.68, 93.93]	[0.40, 64.97]
36	39.31	60.69	0	33.96	52.65	13.39
		41.68 / 19.01		[1.27, 75.72]	[10.65, 94.61]	[0.29, 44.53]
	44.85	55.15	0	34.36	53.05	12.59

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A10: (LONG-RUN RESTRICTION) VARIANCE DECOMPOSITION OF PRICE LEVEL, AND INTEREST RATE (%)
(Means and 95% Bounds over 1000 Estimates, 5-Shock Model)

	Fraction of prices explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	8.53	12.27	79.21	11.18	16.04	72.78
		9.04 / 3.23		[0.02, 51.10]	[0.03, 61.97]	[16.35, 98.59]
4	7.61	11.45	80.94	11.37	16.17	72.46
		8.07 / 3.38		[0.19, 50.46]	[0.20, 61.63]	[17.07, 98.15]
8	8.16	12.62	79.22	12.28	17.37	70.34
		8.65 / 3.97		[0.22, 52.53]	[0.29, 61.61]	[14.93, 97.67]
12	9.01	13.97	77.02	13.11	18.49	68.41
		9.55 / 4.42		[0.23, 53.83]	[0.33, 62.79]	[13.67, 97.62]
20	10.65	16.06	73.28	14.71	20.42	64.87
		11.30 / 4.77		[0.23, 56.96]	[0.42, 67.92]	[13.43, 97.08]
36	13.21	18.21	68.58	17.32	23.13	59.55
		14.01 / 4.20		[0.26, 63.07]	[0.50, 74.08]	[8.69, 96.84]
∞	17.39	20.18	62.43	17.71	23.50	58.79
	Fraction of real interest rates explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	26.45	73.52	0.04	23.47	39.99	36.54
		28.04 / 45.47		[0.11, 72.21]	[0.40, 93.50]	[0.14, 92.49]
4	28.66	71.29	0.04	24.84	41.71	33.46
		30.39 / 40.90		[0.48, 69.59]	[1.00, 92.80]	[0.62, 89.52]
8	31.22	68.73	0.05	26.62	44.20	29.18
		33.11 / 35.63		[0.93, 69.58]	[2.82, 92.53]	[0.81, 83.08]
12	33.19	66.76	0.05	28.11	45.81	26.08
		35.19 / 31.57		[1.28, 69.67]	[4.75, 91.97]	[0.85, 75.30]
20	35.29	64.66	0.05	29.85	46.77	23.38
		37.41 / 27.25		[1.63, 70.60]	[8.39, 91.07]	[1.12, 63.33]
36	35.15	64.80	0.05	30.71	46.13	23.16
		37.27 / 27.53		[1.97, 70.54]	[10.49, 90.26]	[1.48, 58.08]
∞	33.12	66.84	0.05	30.73	46.01	23.26

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A11: (SHORT-RUN RESTRICTION) VARIANCE DECOMPOSITION OF HOURS, AND OUTPUT (%)
(Means and 95% Bounds over 1000 Estimates, 5-Shock Model)

	Fraction of hours explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	67.76	32.14	0.10	100.00	0.00	0.00
		6.98 / 25.16		[100.00, 100.00]	[0.00, 0.00]	[0.00, 0.00]
4	71.9	28.05	0.05	97.00	1.14	1.86
		6.72 / 21.34		[89.47, 99.72]	[0.04, 5.17]	[0.05, 8.72]
8	76.44	23.53	0.03	93.24	2.87	3.89
		6.31 / 17.22		[78.49, 99.33]	[0.14, 12.36]	[0.13, 16.85]
12	79.99	19.99	0.02	89.62	5.06	5.32
		5.88 / 14.11		[68.75, 99.06]	[0.21, 19.33]	[0.18, 23.06]
20	84.80	15.19	0.02	82.92	9.70	7.38
		5.04 / 10.15		[54.10, 98.52]	[0.37, 31.80]	[0.20, 29.31]
36	89.44	10.55	0.01	74.23	16.29	9.48
		3.76 / 6.79		[37.51, 97.78]	[0.56, 45.71]	[0.26, 37.67]
∞	94.75	5.24	0.01	73.15	17.14	9.71
	Fraction of output explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	26.45	73.52	0.04	81.30	18.70	0.00
		28.04 / 45.47		[75.72, 85.82]	[14.18, 24.28]	[0.00, 0.00]
4	28.14	71.84	0.01	79.95	18.14	1.90
		29.84 / 42.00		[66.21, 90.22]	[8.61, 31.31]	[0.05, 8.74]
8	30.22	69.78	0.01	78.64	17.19	4.17
		32.04 / 37.74		[59.68, 92.28]	[5.45, 34.64]	[0.14, 18.88]
12	32.07	67.93	0	77.83	16.07	6.10
		34.00 / 33.92		[55.86, 93.38]	[4.15, 35.73]	[0.20, 26.13]
20	35.16	64.84	0	75.99	14.39	9.62
		37.28 / 27.57		[49.31, 93.81]	[2.99, 35.29]	[0.25, 35.91]
36	39.31	60.69	0	72.58	12.62	14.81
		41.68 / 19.01		[39.10, 93.96]	[2.18, 35.57]	[0.28, 51.26]
∞	44.85	55.15	0	72.06	12.42	15.52

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A12: (SHORT-RUN RESTRICTION) VARIANCE DECOMPOSITION OF PRICE LEVEL, AND INTEREST RATE (%)
(Means and 95% Bounds over 1000 Estimates, 5-Shock Model)

	Fraction of prices explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	8.53	12.27	79.21	18.85	1.83	79.32
		9.04 / 3.23		[9.06, 30.64]	[0.02, 6.06]	[67.17, 89.45]
4	7.61	11.45	80.94	18.01	2.51	79.48
		8.07 / 3.38		[5.37, 34.52]	[0.12, 9.62]	[61.95, 92.74]
8	8.16	12.62	79.22	20.03	3.35	76.62
		8.65 / 3.97		[4.24, 44.04]	[0.12, 14.13]	[52.40, 94.07]
12	9.01	13.97	77.02	22.01	3.81	74.18
		9.55 / 4.42		[3.47, 50.30]	[0.13, 15.83]	[44.20, 94.38]
20	10.65	16.06	73.28	25.25	4.43	70.32
		11.30 / 4.77		[2.87, 60.71]	[0.13, 18.25]	[33.89, 95.25]
36	13.21	18.21	68.58	29.23	5.27	65.50
		14.01 / 4.20		[2.10, 72.68]	[0.13, 21.19]	[20.41, 95.84]
∞	17.39	20.18	62.43	29.71	5.40	64.89
	Fraction of real interest rates explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	26.45	73.52	0.04	81.30	18.70	0
		28.04 / 45.47		[75.72, 85.82]	[14.18, 24.28]	[0.00, 0.00]
4	28.66	71.29	0.04	80.04	17.88	2.08
		30.39 / 40.90		[66.48, 90.00]	[8.86, 30.79]	[0.06, 9.38]
8	31.22	68.73	0.05	78.40	16.77	4.83
		33.11 / 35.63		[59.36, 91.60]	[6.12, 33.37]	[0.22, 20.34]
12	33.19	66.76	0.05	76.92	15.79	7.29
		35.19 / 31.57		[55.16, 91.72]	[5.38, 33.59]	[0.33, 28.58]
20	35.29	64.66	0.05	73.53	14.99	11.48
		37.41 / 27.25		[47.10, 91.05]	[4.98, 32.63]	[0.50, 38.51]
36	35.15	64.80	0.05	69.40	15.12	15.48
		37.27 / 27.53		[39.90, 90.30]	[4.82, 32.48]	[0.66, 48.56]
∞	33.12	66.84	0.05	69.03	15.18	15.80

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A13: (MEDIUM-RUN RESTRICTION, k=4) VARIANCE DECOMPOSITION OF HOURS, AND OUTPUT (%)
(Means and 95% Bounds over 1000 Estimates, 5-Shock Model)

	Fraction of hours explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	67.76	32.14	0.10	90.62	3.45	5.93
		6.98 / 25.16		[68.63, 99.64]	[0.00, 16.78]	[0.01, 23.82]
4	71.9	28.05	0.05	94.12	2.20	3.68
		6.72 / 21.34		[80.77, 99.28]	[0.09, 9.99]	[0.18, 14.12]
8	76.44	23.53	0.03	95.93	1.59	2.47
		6.31 / 17.22		[87.33, 99.38]	[0.13, 6.28]	[0.15, 9.06]
12	79.99	19.99	0.02	94.74	2.48	2.78
		5.88 / 14.11		[84.32, 99.18]	[0.20, 9.11]	[0.17, 9.83]
20	84.8	15.19	0.02	90.11	5.68	4.21
		5.04 / 10.15		[69.18, 98.85]	[0.29, 19.93]	[0.20, 17.82]
36	89.44	10.55	0.01	82.58	11.10	6.31
		3.76 / 6.79		[50.79, 98.36]	[0.38, 36.05]	[0.24, 27.70]
∞	94.75	5.24	0.01	81.58	11.84	6.58
	Fraction of output explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	26.45	73.52	0.04	67.00	27.10	5.89
		28.04 / 45.47		[35.27, 92.60]	[4.37, 57.24]	[0.02, 23.67]
4	28.14	71.84	0.01	69.89	26.61	3.49
		29.84 / 42.00		[39.81, 90.81]	[7.74, 52.97]	[0.20, 12.66]
8	30.22	69.78	0.01	72.34	25.35	2.31
		32.04 / 37.74		[46.50, 89.22]	[9.73, 49.57]	[0.17, 8.07]
12	32.07	67.93	0	73.24	23.81	2.95
		34.00 / 33.92		[48.10, 88.99]	[8.96, 48.58]	[0.21, 10.69]
20	35.16	64.84	0	73.06	21.29	5.65
		37.28 / 27.57		[47.83, 89.20]	[7.14, 44.24]	[0.21, 24.67]
36	39.31	60.69	0	71.10	18.41	10.49
		41.68 / 19.01		[41.62, 90.05]	[4.33, 42.50]	[0.20, 45.30]
∞	44.85	55.15	0	70.76	18.07	11.17

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A14: (MEDIUM-RUN RESTRICTION, k=4) VARIANCE DECOMPOSITION OF PRICE LEVEL, & INTEREST RATE (%)
(Means and 95% Bounds over 1000 Estimates, 5-Shock Model)

	Fraction of prices explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	8.53	12.27	79.21	27.90	9.35	62.75
		9.04 / 3.23		[1.09, 88.62]	[0.01, 53.56]	[1.56, 95.06]
4	7.61	11.45	80.94	26.60	9.89	63.50
		8.07 / 3.38		[1.55, 86.94]	[0.15, 55.42]	[3.20, 94.41]
8	8.16	12.62	79.22	28.10	10.40	61.50
		8.65 / 3.97		[1.82, 87.63]	[0.21, 55.67]	[2.50, 93.93]
12	9.01	13.97	77.02	29.43	10.57	59.99
		9.55 / 4.42		[1.94, 86.77]	[0.17, 54.39]	[2.76, 93.83]
20	10.65	16.06	73.28	31.04	10.76	58.21
		11.30 / 4.77		[1.82, 87.04]	[0.18, 54.53]	[3.33, 94.57]
36	13.21	18.21	68.58	32.01	10.84	57.15
		14.01 / 4.20		[1.49, 86.33]	[0.18, 54.53]	[4.15, 95.11]
∞	17.39	20.18	62.43	32.05	10.84	57.11
	Fraction of real interest rates explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	26.45	73.52	0.04	67.09	27.14	5.77
		28.04 / 45.47		[35.34, 92.67]	[4.40, 57.21]	[0.02, 23.25]
4	28.66	71.29	0.04	70.20	26.33	3.47
		30.39 / 40.90		[40.06, 90.82]	[7.62, 52.66]	[0.28, 12.99]
8	31.22	68.73	0.05	71.88	24.53	3.59
		33.11 / 35.63		[45.99, 88.56]	[9.36, 48.11]	[0.37, 11.86]
12	33.19	66.76	0.05	71.39	22.79	5.83
		35.19 / 31.57		[45.45, 88.30]	[8.51, 45.44]	[0.44, 19.58]
20	35.29	64.66	0.05	68.45	20.81	10.74
		37.41 / 27.25		[41.75, 87.08]	[7.48, 42.04]	[0.52, 37.41]
36	35.15	64.80	0.05	64.34	20.00	15.66
		37.27 / 27.53		[32.62, 85.83]	[7.23, 40.91]	[0.62, 52.12]
∞	33.12	66.84	0.05	63.95	19.99	16.06

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A15: (MEDIUM-RUN RESTRICTION, k=20) VARIANCE DECOMPOSITION OF HOURS, AND OUTPUT (%)
(Means and 95% Bounds over 1000 Estimates, 5-Shock Model)

	Fraction of hours explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	67.76	32.14	0.10	56.40	18.16	25.44
		6.98 / 25.16		[6.99, 97.71]	[0.05, 66.06]	[0.04, 74.24]
4	71.90	28.05	0.05	60.28	16.88	22.84
		6.72 / 21.34		[10.85, 96.81]	[0.34, 61.31]	[0.38, 68.80]
8	76.44	23.53	0.03	66.40	15.10	18.50
		6.31 / 17.22		[17.78, 97.10]	[0.36, 56.80]	[0.40, 57.11]
12	79.99	19.99	0.02	71.86	13.10	15.03
		5.88 / 14.11		[26.83, 97.70]	[0.30, 50.65]	[0.35, 46.44]
20	84.8	15.19	0.02	79.17	10.05	10.77
		5.04 / 10.15		[43.97, 98.45]	[0.19, 40.91]	[0.26, 34.08]
36	89.44	10.55	0.01	85.30	7.32	7.38
		3.76 / 6.79		[58.19, 98.99]	[0.13, 29.86]	[0.21, 23.86]
∞	94.75	5.24	0.01	85.83	7.07	7.10
	Fraction of output explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	26.45	73.52	0.04	27.31	45.36	27.33
		28.04 / 45.47		[0.08, 77.12]	[7.85, 91.44]	[0.07, 78.46]
4	28.14	71.84	0.01	29.03	46.37	24.60
		29.84 / 42.00		[0.50, 75.49]	[10.02, 91.60]	[0.40, 72.01]
8	30.22	69.78	0.01	31.81	48.17	20.02
		32.04 / 37.74		[0.86, 75.32]	[12.09, 91.68]	[0.35, 60.32]
12	32.07	67.93	0	34.50	49.22	16.28
		34.00 / 33.92		[1.23, 75.64]	[14.04, 91.87]	[0.28, 50.61]
20	35.16	64.84	0	38.76	49.76	11.49
		37.28 / 27.57		[1.81, 77.22]	[15.81, 92.27]	[0.19, 37.53]
36	39.31	60.69	0	43.51	48.83	7.66
		41.68 / 19.01		[2.03, 79.71]	[16.00, 92.96]	[0.13, 26.37]
∞	44.85	55.15	0	44.04	48.62	7.34

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Table A16: (MEDIUM-RUN RESTRICTION, k=20) VARIANCE DECOMPOSITION OF PRICE LEVEL, & INTEREST RATE (%)
(Means and 95% Bounds over 1000 Estimates, 5-Shock Model)

	Fraction of prices explained by shock to					
	TRUE			Estimated by VARs		
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	8.53	12.27	79.21	25.07	28.54	46.39
		9.04 / 3.23		[0.05, 85.01]	[0.15, 87.31]	[0.50, 93.01]
4	7.61	11.45	80.94	24.34	27.83	47.83
		8.07 / 3.38		[0.40, 80.89]	[0.59, 84.09]	[1.83, 92.59]
8	8.16	12.62	79.22	25.48	28.75	45.76
		8.65 / 3.97		[0.51, 81.54]	[0.76, 85.62]	[2.11, 91.39]
12	9.01	13.97	77.02	26.41	29.25	44.34
		9.55 / 4.42		[0.50, 81.23]	[0.92, 84.94]	[2.28, 90.72]
20	10.65	16.06	73.28	27.70	29.33	42.96
		11.30 / 4.77		[0.51, 81.05]	[1.01, 84.34]	[2.38, 90.79]
36	13.21	18.21	68.58	29.00	28.32	42.68
		14.01 / 4.20		[0.54, 80.80]	[0.94, 84.29]	[1.98, 92.01]
∞	17.39	20.18	62.43	29.12	28.11	42.77
	Fraction of real interest rates explained by shock to					
Period	Labor supply	Technology	Aggregate demand	Labor supply	Technology	Aggregate demand
1	26.45	73.52	0.04	27.40	45.61	26.99
		28.04 / 45.47		[0.07, 77.18]	[8.01, 91.59]	[0.07, 77.96]
4	28.66	71.29	0.04	29.63	46.94	23.43
		30.39 / 40.90		[0.59, 75.63]	[10.64, 91.55]	[0.44, 69.91]
8	31.22	68.73	0.05	32.74	48.34	18.92
		33.11 / 35.63		[1.25, 75.71]	[12.65, 91.24]	[0.47, 57.87]
12	33.19	66.76	0.05	35.26	48.23	16.52
		35.19 / 31.57		[1.99, 75.87]	[14.82, 90.34]	[0.57, 50.30]
20	35.29	64.66	0.05	37.82	45.96	16.21
		37.41 / 27.25		[2.67, 75.49]	[14.92, 88.72]	[0.89, 48.44]
36	35.15	64.80	0.05	38.19	42.60	19.21
		37.27 / 27.53		[2.72, 75.02]	[13.66, 86.63]	[1.25, 49.73]
∞	33.12	66.84	0.05	38.09	42.31	19.60

NOTES: The true variance decomposition is derived from the theoretical model. 1000 artificial datasets of length 200 periods are generated from the model. For each database, I estimate the parameters by the VAR method with long-run restrictions, and compute the variance decompositions. From 1000 estimates, the means are reported. The number in square brackets indicate the range of estimates after eliminating the bottom 2.5% and the top 2.5%.

Figure A1: (Case 1) Impulse Responses to One−Standard−Deviation Shocks

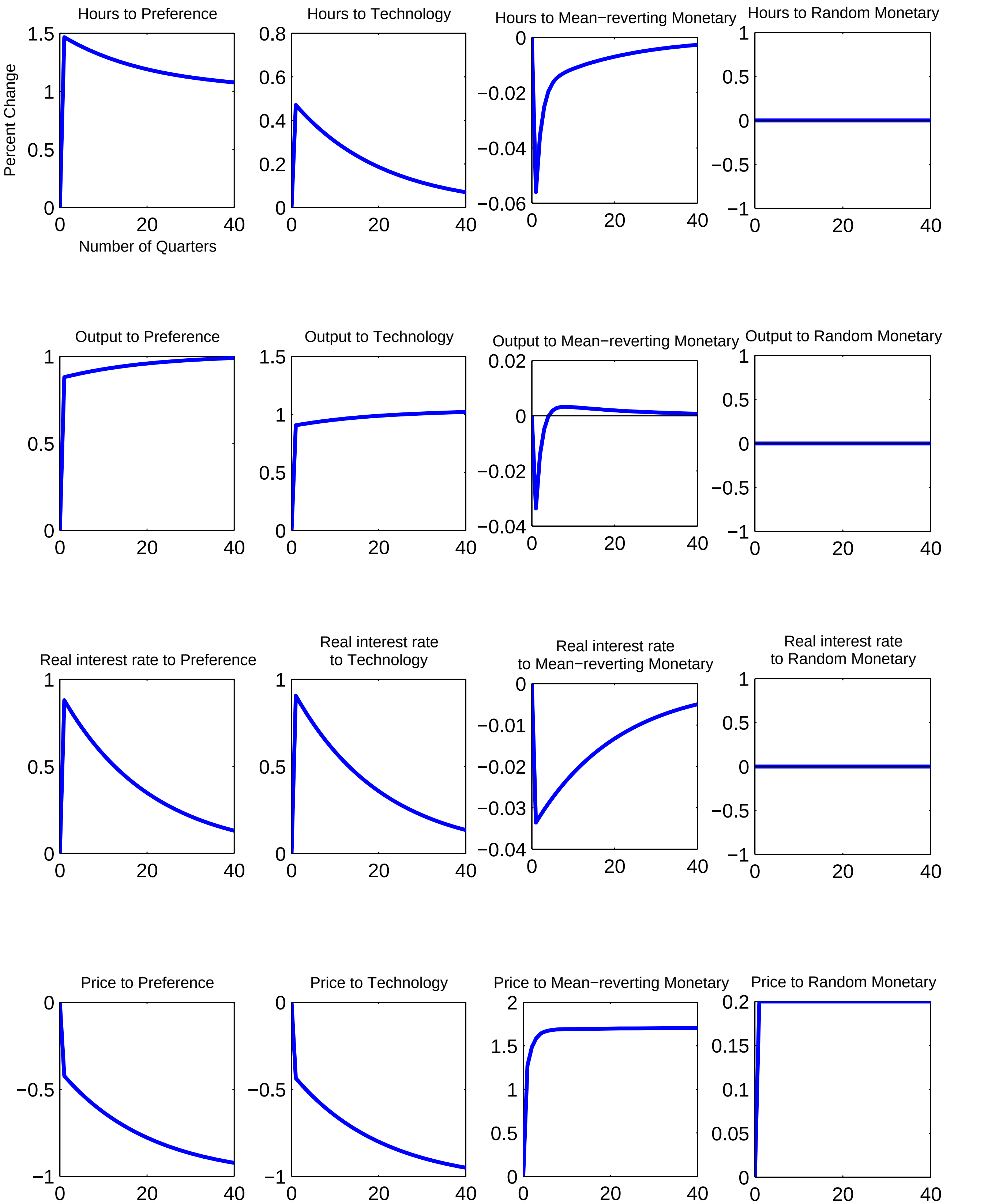

Figure A2: (Case 1) Impulse Responses to One-Standard-Deviation Shocks (long−run identification)

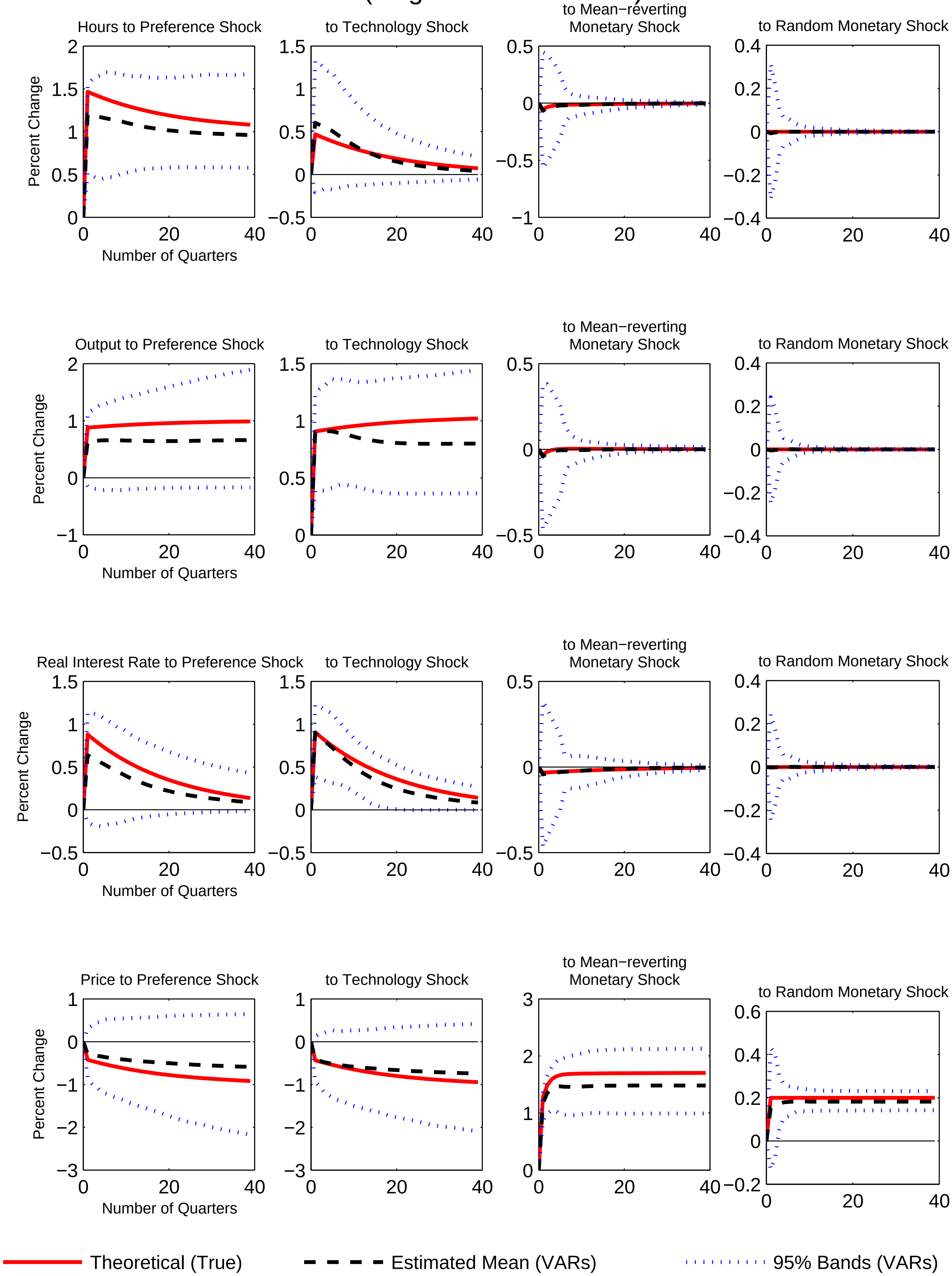

Figure A3: (Case 1) Impulse Responses to One-Standard-Deviation Shocks (short−run identification)

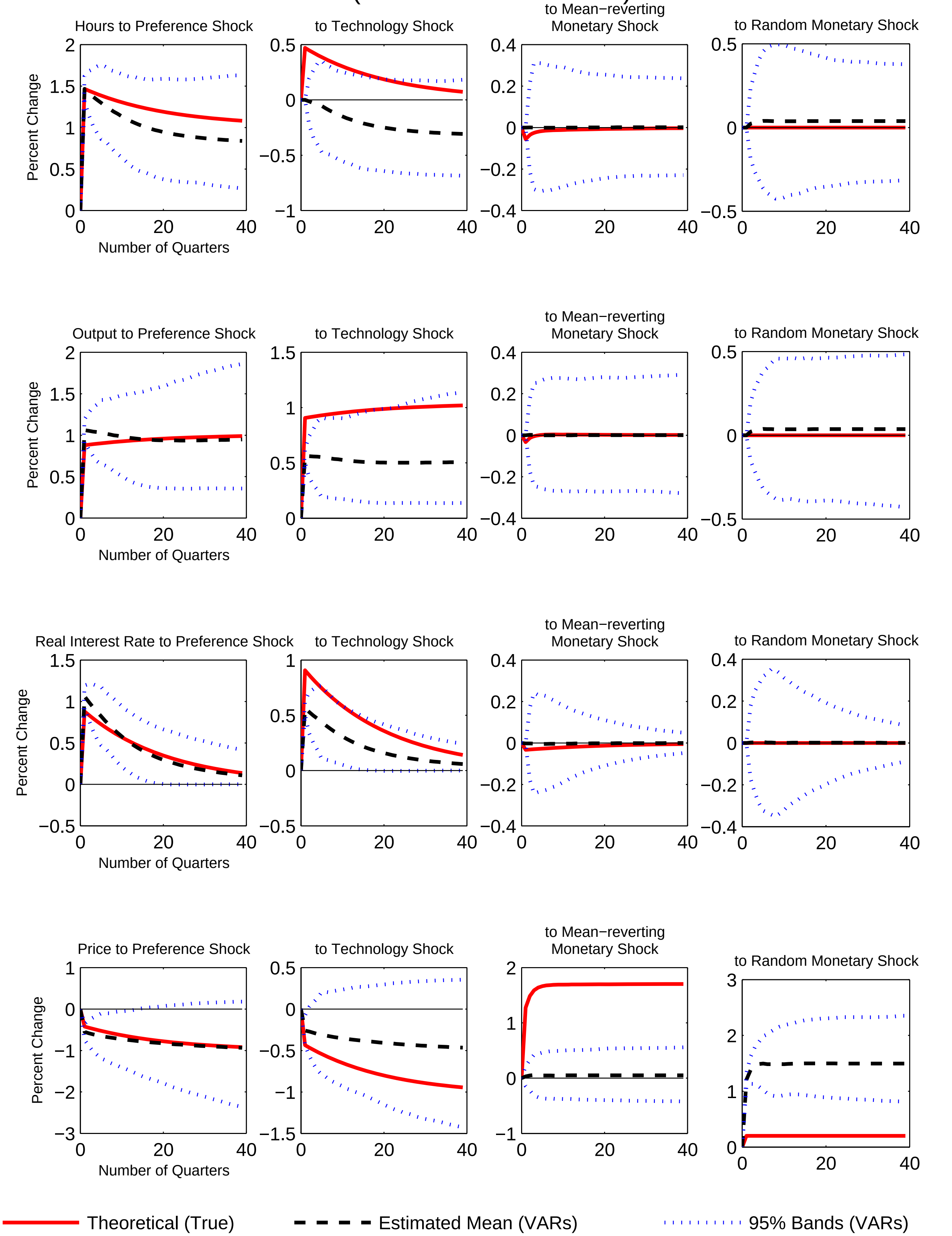

Figure A4: (Case 1) Impulse Responses to One-Standard-Deviation Shocks
(medium−run identification, k=4)

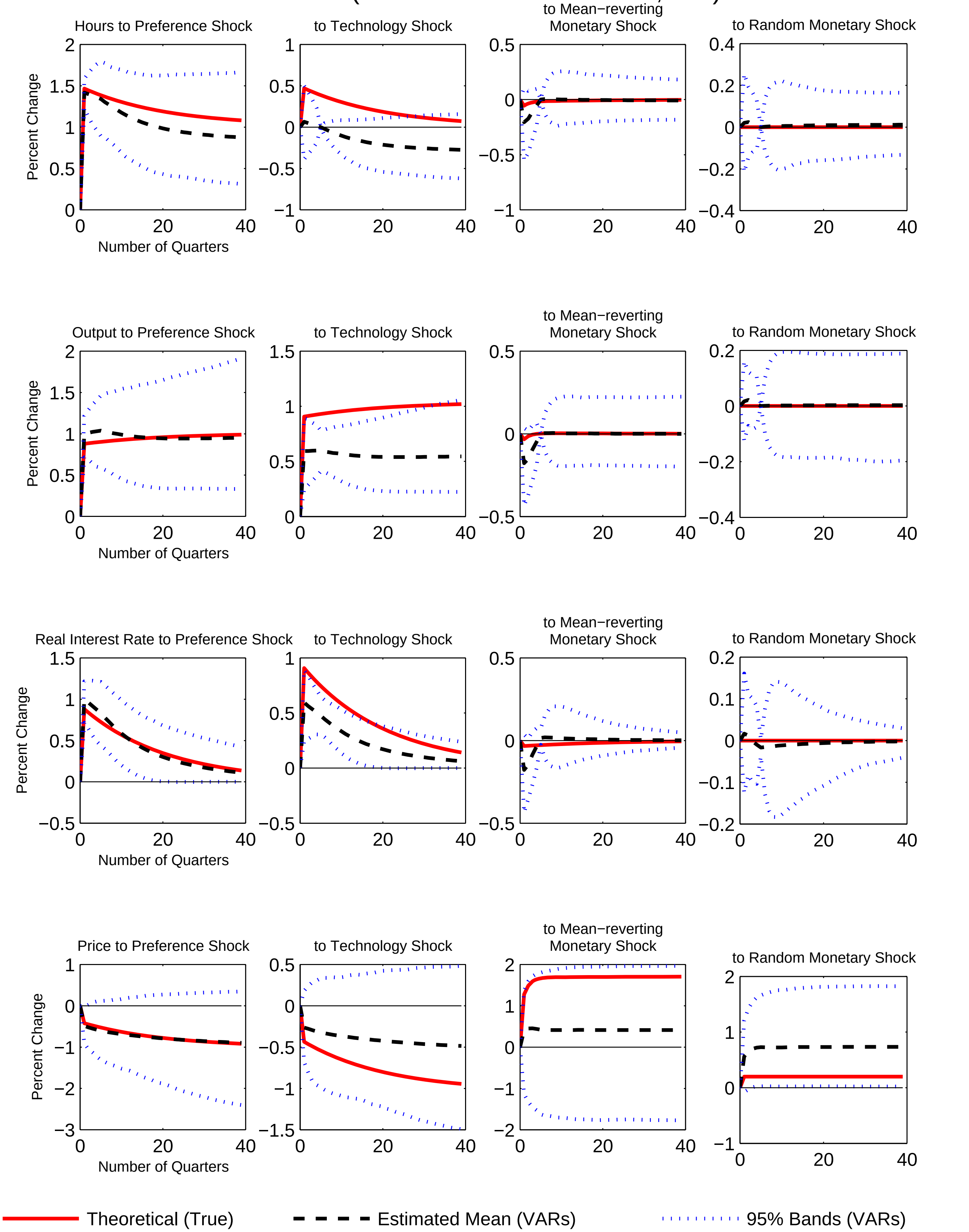

Figure A5: (Case 1) Impulse Responses to One−Standard−Deviation Shocks (medium−run identification, k=20)

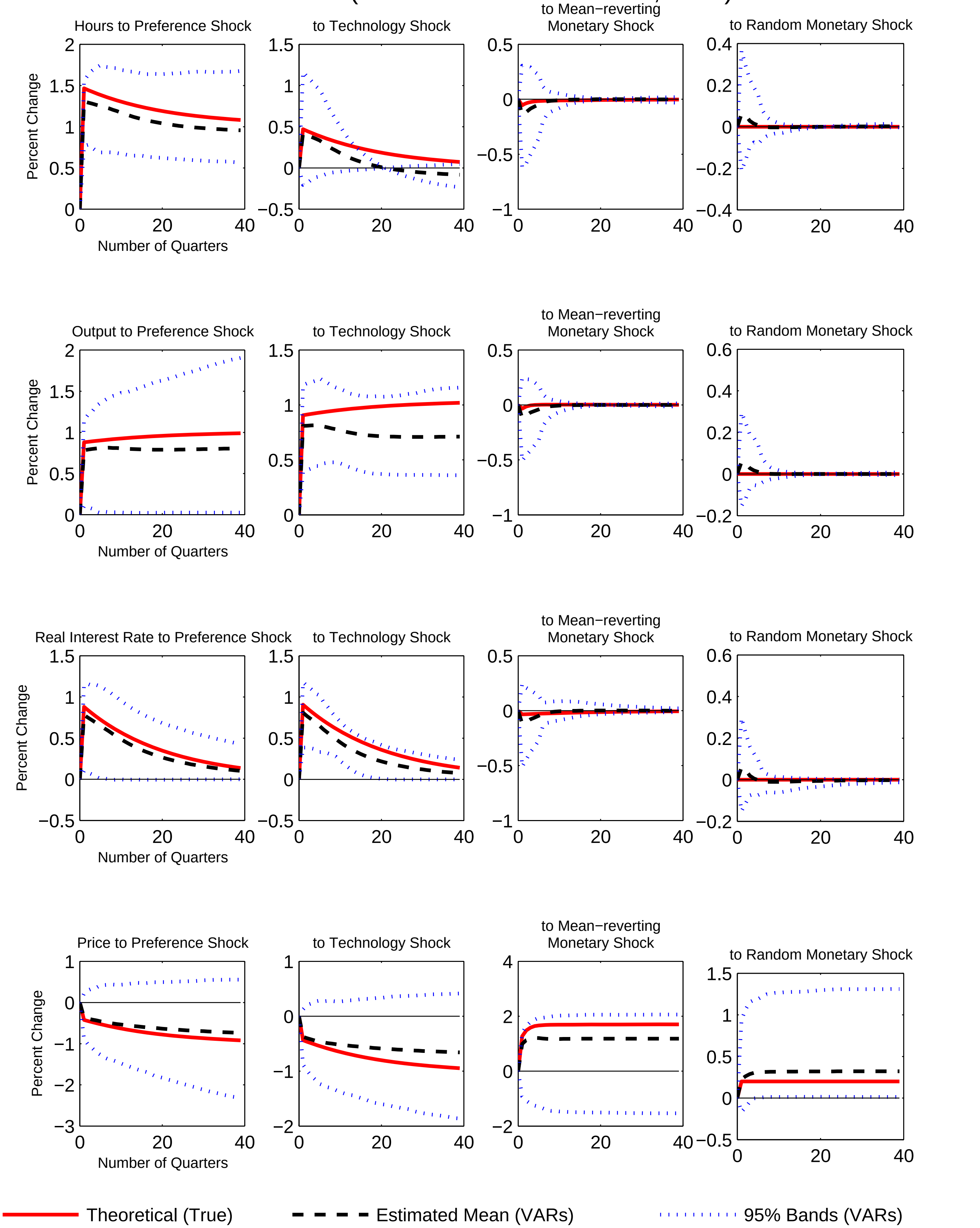

Figure A6: (Case 2) Impulse Responses to One−Standard−Deviation Shocks

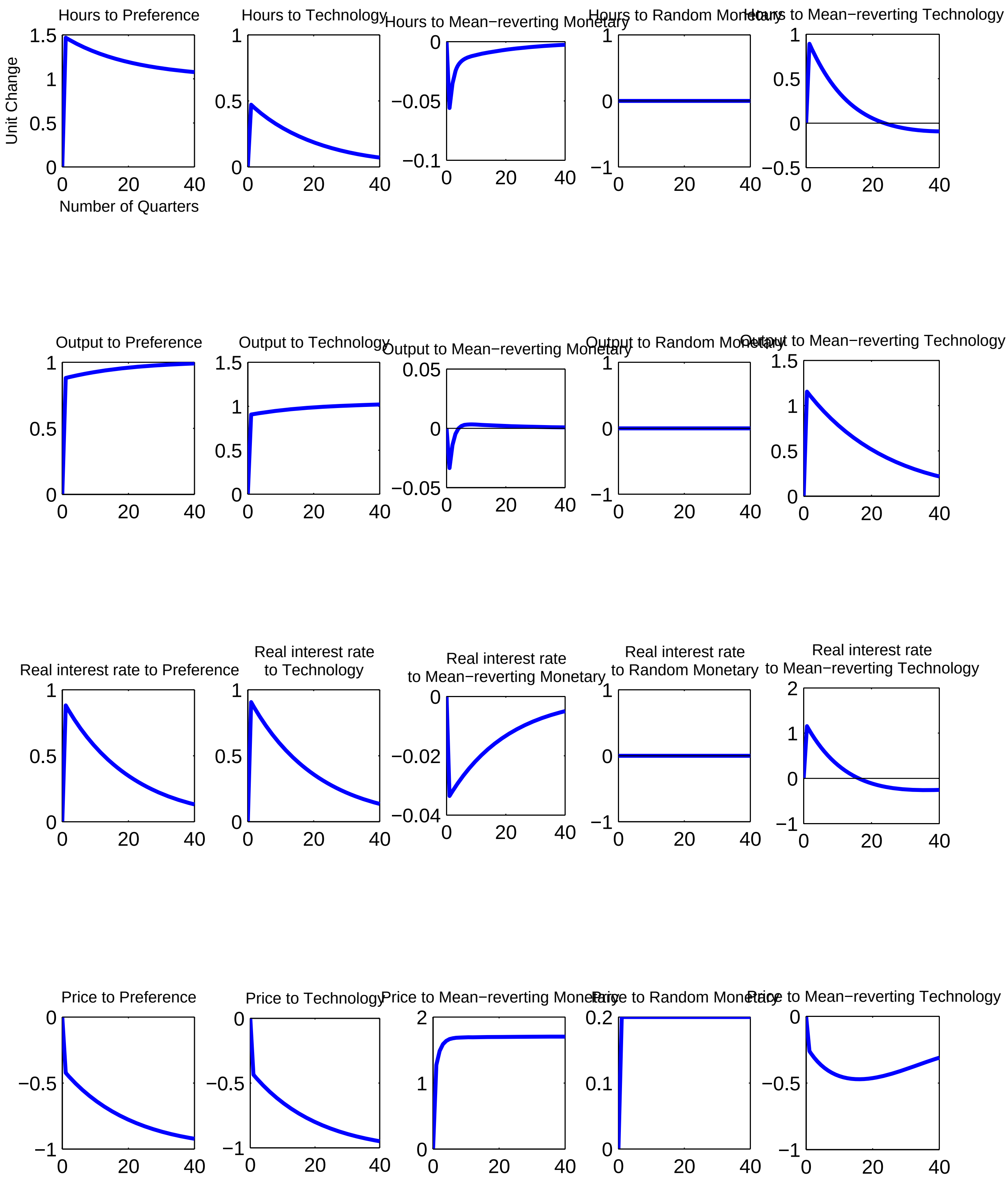

Figure A7: (Case 2) Impulse Responses to One-Standard-Deviation Shocks (long−run identification)

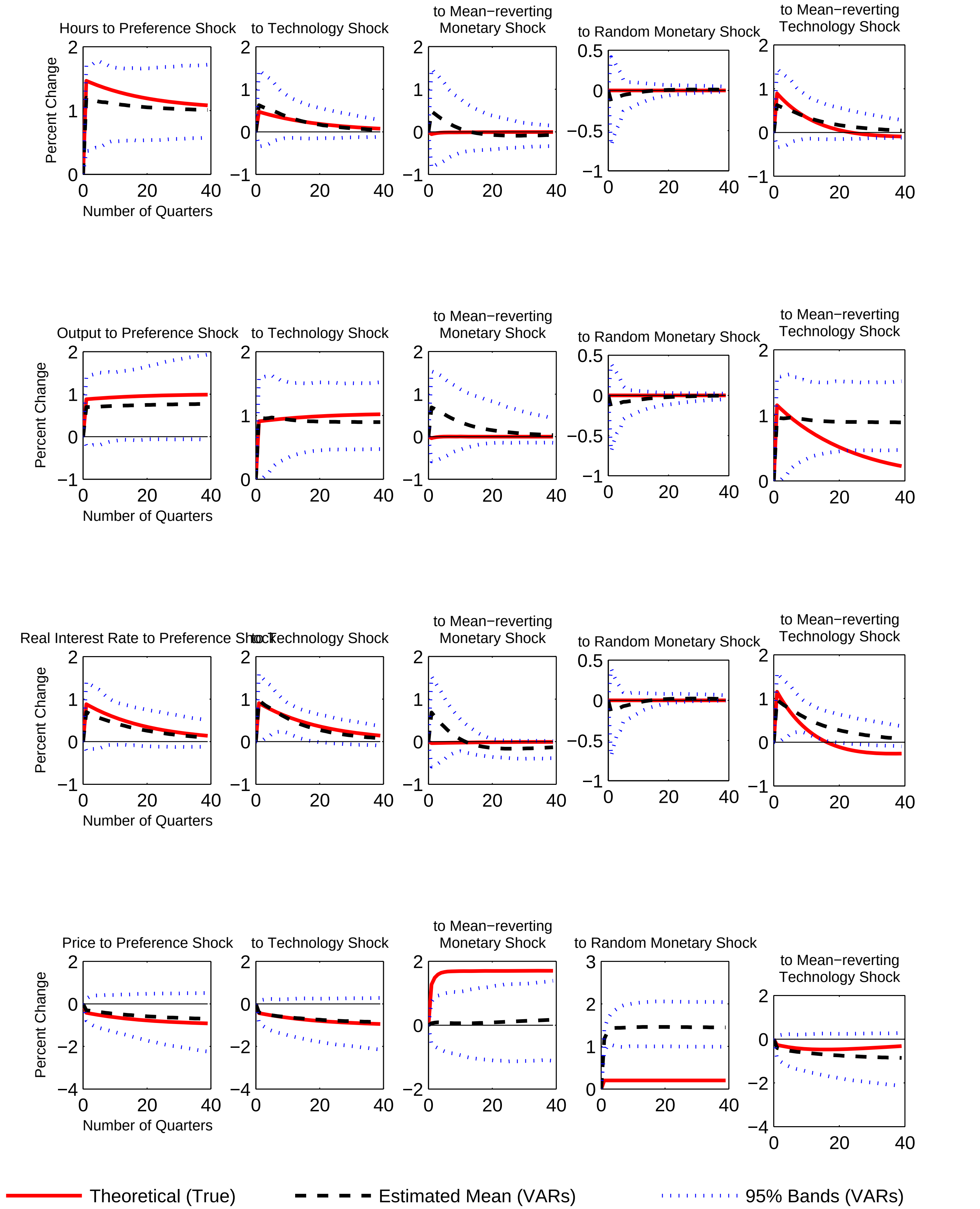

Figure A8: (Case 2) Impulse Responses to One-Standard-Deviation Shocks
(short−run identification)

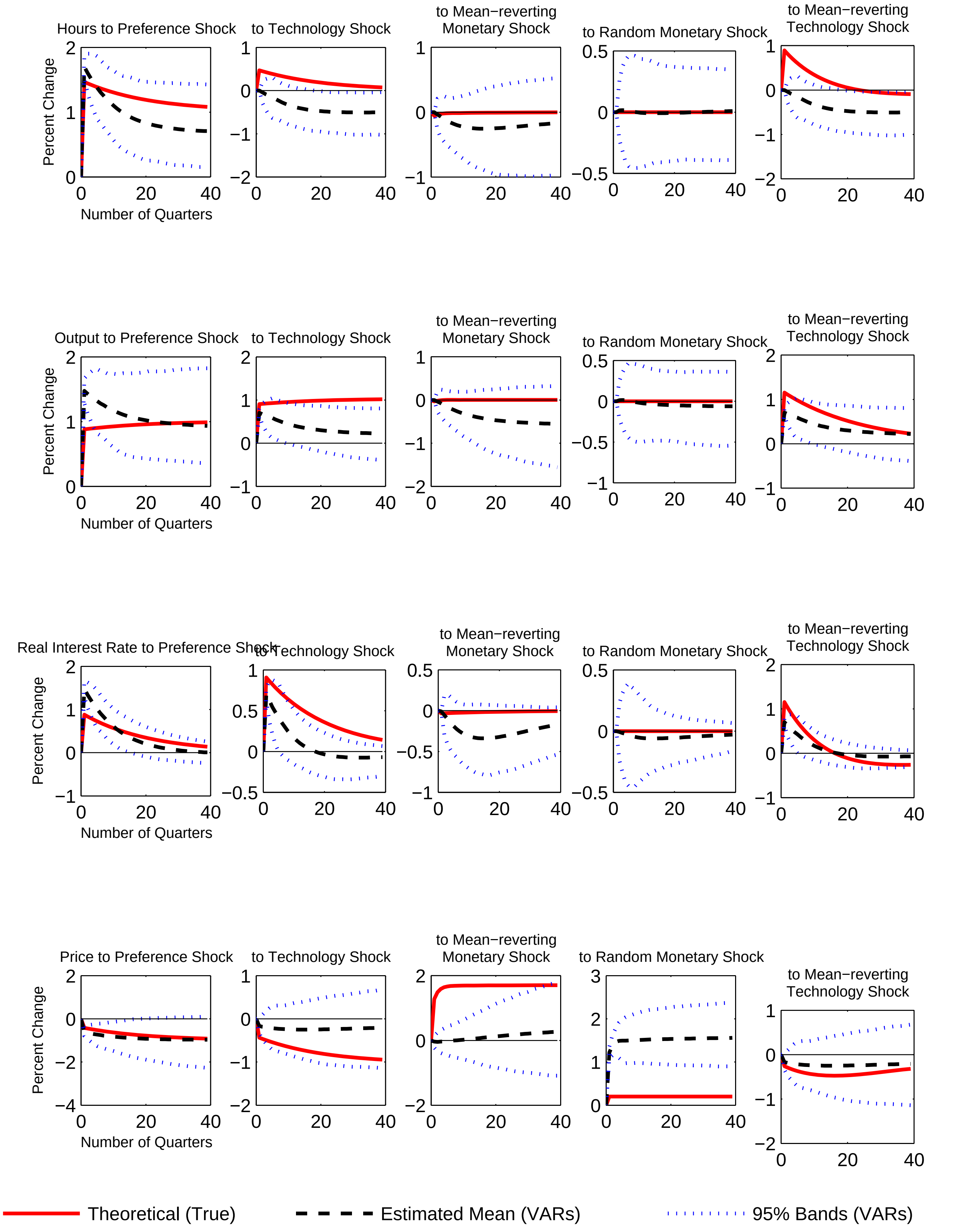

Figure A9: (Case 2) Impulse Responses to One-Standard-Deviation Shocks
(medium−run identification, k=4)

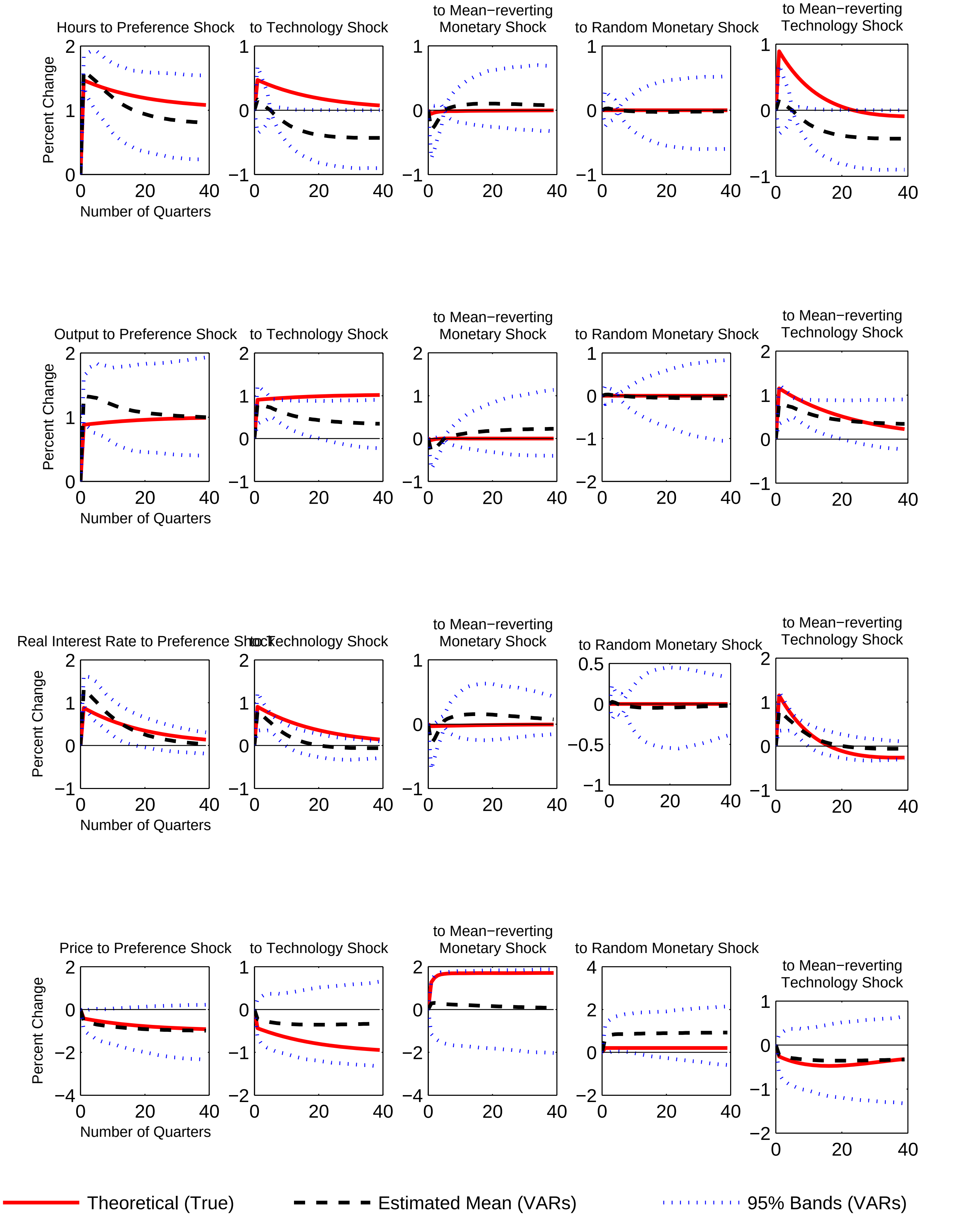

Figure A10: (Case 2) Impulse Responses to One−Standard−Deviation Shocks (medium−run identification, k=20)