WHAT HAS FINANCED GOVERNMENT DEBT?

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Abstract. Equilibrium models imply that the real value of debt in the hands of the public must equal the expected present-value of surpluses. Empirical models of fiscal policy typically do not impose this condition and often do not even include debt. Absence of debt from empirical models can produce non-invertible representations, obscuring the true present-value relation, even if it holds in the data. First, we show that small VAR models of fiscal policy may not be invertible and that expanding the information set to include government debt has quantitatively important implications. Then we impose the present-value condition on an identified VAR and characterize the way in which the present-value support of debt varies across types of fiscal shocks. The role of expected primary surpluses in supporting innovations to debt depends on the nature of the shock. Debt is supported almost entirely by changes in the present-value of surpluses for some fiscal shocks, but for other fiscal shocks surpluses fail to adjust and instead leave a large role for expected changes in discount rates. Horizons over which debt innovations are financed are long—on the order of 50 years.

1. Introduction

The government's present-value budget constraint is an attractive vehicle for rationalizing macroeconomic responses to fiscal policy because the net taxes component of the present-value relation directly impacts forward-looking households through their own present-value constraints. It follows that to understand why households with rational expectations respond in certain ways to fiscal policy shocks, it is valuable to examine how these households perceive their present-value tax burden as evolving. Moreover, to understand why forward-looking households are content to hold government debt at prevailing market prices, it is essential to know how violations of

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the household transversality conditions are avoided, and this means learning how the government present-value relation is satisfied.

Rational expectations implies that economic agents' beliefs about how future fiscal policy will adjust to innovations in government debt play a crucial role in determining the resulting equilibrium. Prominent examples where theoretical conclusions about macro policy hinge on such beliefs include Ricardian equivalence, "Unpleasant Monetarist Arithmetic," and the fiscal theory of the price level.[1] In striking contrast, empirical studies are either mute, as in Blanchard and Perotti (2002) and the identified VAR work that followed, or build in the assumption that net surpluses or total tax revenues clear the government budget constraint [Bohn (1998), Davig and Leeper (2006), or Favero and Monacelli (2005)]. This paper offers some new empirical findings that connect more tightly to theoretical work.

Our desire to examine the historical sources of fiscal financing leads us to include government debt in an otherwise conventional fiscal VAR, like those estimated by Blanchard and Perotti (2002), Perotti (2004), Canova and Pappas (2003), Mountford and Uhlig (2005), and Caldara and Kamps (2006). Including debt has surprisingly important implications for the impacts of fiscal disturbances on macroeconomic variables such as output and inflation. To understand why debt appears to matter so much for the predictions of the VAR, we explore Hansen and Sargent's (1991) caution that estimated VARs may not be invertible, making it impossible to recover structural shocks from current and past information in the VAR. Testing the "poor man's invertibility condition" of Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007), we find that VARs without debt are not invertible relative to a larger model that is rich enough to capture the government's present-value financing. By failing to be invertible, the smaller VARs identify as exogenous fiscal shocks linear combinations of *future* innovations, confounding both the timing and the composition of fiscal disturbances. Adding investment makes the VAR nearly invertible, while adding debt ensures invertibility.[2]

It turns out that inferences about the dynamic impacts of fiscal disturbances depend strongly on the information set of the estimated VAR. Output and investment multipliers associated with government spending, transfers, and taxes tend to be larger in the broader VAR systems. Expansions in expenditure components tend to have stronger price effects in larger systems, while taxes have weaker price effects in

[1]A very partial list of analyses in which intertemporal financing of government debt is pivotal includes Barro (1974), Sargent and Wallace (1981), Leeper (1991), Baxter and King (1993), Sims (1998), Woodford (2001), and Leeper and Yang (2006).

[2]Another potential source of non-invertibility is emphasized by Leeper (1989) and Yang (2005): fiscal news may arrive well before fiscal policies are implemented and their effects show up in fiscal variables. We do not address this source.

those systems. The finding about invertibility and its implications for estimated fiscal impacts constitute the first contribution of the paper.

The second contribution stems from imposing the government's intertemporal budget constraint on the estimated VAR to answer the question posed by the paper's title. The constraint constitutes a set of cross-equation restrictions on the estimated VAR coefficients. With a consistent accounting framework in hand, we examine how innovations in debt produced by exogenous shocks to government spending, transfers, and taxes have been expected to be financed intertemporally.

Fiscal financing has been remarkably shock-dependent in the post-World War II period in the United States. Tax hikes that lower debt have tended to be followed by fiscal adjustments that on net reduced the present value of surpluses to support the reduced value of debt. Discount rates have also moved to support debt, funding about one-quarter of the change in debt. Debt-financed spending increases, in contrast, have been followed by a *lower* present value of surpluses, requiring discount rates to fall by enough to both offset the lower surpluses and support the elevated value of debt. Transfers shocks that raise debt induce a sufficiently strong response of taxes that the present value of surpluses moves with debt; in this case, the present value of discount rates moves strongly against the higher value of debt. Despite the diversity of funding across fiscal disturbances, one common theme emerges: discount rates can move substantially and constitute an important source of fiscal adjustment.

Diversity also marks the dynamics of fiscal adjustment induced by shocks to policy. Fiscal policy is characterized by a high degree of persistence. Present-value balance is seen only after a 50- to 100-year forecast horizon. For intermediate forecast horizons, especially following spending and transfer shocks, the truncated present-value of surpluses can be grossly out of line with the value of debt, making it appear that policy fails to satisfy the government's intertemporal constraint.

Our work is closely related to Giannitsarou and Scott (2006). That paper, however, is concerned with *testing* present-value balance, an effort initiated by Hamilton and Flavin (1986). Instead, we *impose* the linearized intertemporal government budget constraint on an estimated identified VAR and study its implications for fiscal financing.[3] We focus on describing how present-value balance is achieved, contingent on the realization of certain identified fiscal policy shocks. Our paper stresses the need for a consistent accounting framework, under which the VAR estimates of the present-value of surpluses exactly equals the value of outstanding debt.

[3]Favero and Giavazzi (2007) add government debt to an otherwise conventional fiscal VAR and append a non-linear budget identity to accumulate debt. They do not impose the intertemporal budget constraint as a set of cross-equation restrictions on the estimated VAR coefficient.

This paper is also closely related to Roberds (1991). Roberds includes a measure of government debt in his empirical work to test whether the government's present-value condition holds in expectation. Although we impose the condition, both Roberds and we avoid the impossibility result of Hansen, Roberds, and Sargent (1991) by including debt in the information set and applying the present-value condition in expectation.

While the constrained VAR impulse response functions do not lead to rejection of the model at high levels of significance, the point estimates from the constrained VAR are markedly different from the unconstrained model, especially with regard to output and prices, but also, importantly, with regard to the fiscal policy instruments.

The findings of the paper have important implications for dynamic stochastic general equilibrium (DSGE) modeling of fiscal policy. First, fiscal policy is multi-dimensional, with elements of the policy block being funded in present-value in diverse ways, over widely varying time horizons. In particular, while tax shocks appear to behave in the intuitive fashion (temporary tax cuts are rapidly paid for by tax increases), spending and transfer shocks both involve sharp movements in the expected discount rate, which are stabilizing for spending but not for transfers. Second, policy shocks routinely elicit sizeable and persistent responses in all fiscal policy instruments and this dynamic interaction is crucial for understanding how the resulting debt innovation is financed. The result is that present-value balance is not apparent unless forecasts are carried out for more than a century, in the case of spending and transfer shocks. To date, estimated DSGE models specify fiscal sectors in ways that preclude capturing the dynamic interactions present in U.S. data [Coenen and Straub (2004), Forni, Monforte, and Sessa (2006), Kamps (2007)].

2. An Illustrative Model

This section uses a conventional DSGE model—a standard real business cycle model—to derive a typical model's implications for fiscal financing dynamics and to illustrate the computations we perform in the identified VARs below. Although bare-bones, the model is adequate to our focus on the long-run aspects of fiscal finance. More sophisticated versions of this model which are being fit to data largely consist of modifications of the bare-bones model that are designed to capture short-run dynamics in data. Their long-run implications are likely to match closely those of the simpler model we examine.

The model shows that in general the impacts of fiscal disturbances depend on how the government budget constraint is expected to be satisfied in the long run, a point that dates back at least to Christ (1968) and has found recent voice in Baxter and King (1993), Sims (1998), and Leeper and Yang (2006). With a simple model in hand, we derive the sources of intertemporal financing of government debt and the horizons at which that funding occurs.

2.1. **Model Specification.** Consider the following real business cycle model. The representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t), \qquad 0 < \beta < 1 \tag{1}$$

with $u(C, 1 - N) = C^{1-\gamma}/(1-\gamma) + \theta(1-N)^{1-\gamma_N}/(1-\gamma_N)$, subject to

$$C_t + K_t - (1-\delta)K_{t-1} + B_t = (1-\tau_t)Y_t + (1 + r_{t-1})B_{t-1} + Z_t, \tag{2}$$

where Z_t is lump-sum transfers (or taxes when $Z_t < 0$). Goods are produced using a technology that is constant returns to scale in labor, N, and capital, K, jointly

$$Y_t = (A_t N_t)^{\alpha} K_{t-1}^{1-\alpha}. \tag{3}$$

$\{A_t\}$ is the serially correlated technology process.

The aggregate resource constraint is

$$C_t + K_t - (1-\delta)K_{t-1} + G_t = Y_t. \tag{4}$$

G_t is government purchases of goods at t, and the government budget constraint is

$$B_t + \tau_t Y_t = G_t + Z_t + (1 + r_{t-1})B_{t-1}, \tag{5}$$

where B_t is the amount of one-period debt outstanding at t, which pays $(1 + r_t)B_t$ at $t + 1$. We let $T_t \equiv \tau_t Y_t$ denote total tax revenues.

Following existing empirical work on fiscal policy, we posit that policy obeys simple rules that make fiscal variables respond contemporaneously to output and with a lag to the state of government debt (written in log deviations from steady state):

$$\hat{g}_t = \varphi_G \hat{y}_t - \gamma_G \hat{b}_{t-1} + u_t^G, \tag{6}$$

$$\hat{t}_t = \varphi_\tau \hat{y}_t + \gamma_\tau \hat{b}_{t-1} + u_t^\tau, \tag{7}$$

and

$$\hat{z}_t = \varphi_Z \hat{y}_t - \gamma_Z \hat{b}_{t-1} + u_t^Z. \tag{8}$$

The u's follow AR(1) processes. The output elasticities, the φ's, are borrowed from the empirical studies of Blanchard and Perotti (2002), Perotti (2004), and Leeper and Yang (2004). We use the baseline parameter values described in table 1.

2.2. **Some Accounting.** Let the equilibrium dynamics be characterized by factors f_t, evolving according to $f_t = f_{t-1}A + u_t$, in terms of which the model variables are $x_t = f_t C_x$. In its log-linearized form, the present-value relation is

$$\hat{b}_t = E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{\tau}{B}\hat{t}_{t+j} - \frac{G}{B}\hat{g}_{t+j} - \frac{Z}{B}\hat{z}_{t+j} - \frac{1}{\beta}\hat{R}_{t+j-1} \right), \tag{9}$$

where unsubscripted variables denote deterministic steady state values. This equation gives a decomposition of innovations to real debt into innovations to surplus components, at constant discount rates, and into innovations in the real interest rate. Using the equilibrium law of motion, the infinite sum can be expressed as

$$\hat{b}_t = f_t \left(\mathbf{I} - \beta A\right)^{-1} \left[\left(\frac{\tau}{B}C_T - \frac{G}{B}C_G - \frac{Z}{B}C_Z \right) \beta A - C_R \right]. \tag{10}$$

Denote innovations in x_t by $\delta x_t \equiv x_t - E_{t-1}x_t$. Then

$$\delta\hat{b}_t = \delta f_t \left(\mathbf{I} - \beta A\right)^{-1} \left[\left(\frac{\tau}{B}C_T - \frac{G}{B}C_G - \frac{Z}{B}C_Z \right) \beta A - C_R \right]. \tag{11}$$

It is also possible to compute the horizon over which an innovation to the expected present value of surpluses converges to the innovation in debt. Specifically, suppose that the previous infinite series is truncated at a horizon K. Again using the equilibrium law of motion, the sum is

$$PV_t(K) = \delta f_t \left(\mathbf{I} - (\beta A)^K\right) \left(\mathbf{I} - \beta A\right)^{-1} \left[\left(\frac{\tau}{B}C_T - \frac{G}{B}C_G - \frac{Z}{B}C_Z \right) \beta A - C_R \right]. \tag{12}$$

Expression (12) answers the question, "What fraction of a \$1 innovation in debt at time t is financed by period $t+K$." Of course, as $K \to \infty$, expression (12) approaches expression (11).

2.3. **Dynamic Impacts of Fiscal Shocks.** Many DSGE models follow the public finance literature in studying the impacts of fiscal disturbances by assuming the government budget clears in some "neutral" manner, for example, through adjustments in lump-sum taxes or transfers. That assumption implies setting $\gamma_Z > 1/\beta - 1$ and $\gamma_G = \gamma_\tau = 0$ in policy rules (6)-(8). Solid lines in figure 1 report the well-known implications of a standard RBC model. Persistently higher government spending reduces wealth, which reduces consumption and induces greater work effort, initially raising output. Higher taxes reduce output, consumption, and investment. Ricardian equivalence implies that lump-sum taxes do nothing to the real variables reported in the figure.

Table 2 reports how an increase in debt brought forth by each of the three fiscal shocks is expected to be financed when transfers adjust. The table reports, for each type of fiscal shock, what fraction of the resulting increase in debt is financed by a present-value change in future fiscal variables or the discount rate. Not surprisingly, serially correlated spending and tax shocks create expectations of present values of spending and taxes that move in opposite directions from the initial change in debt. Transfers move with debt, as the fiscal rule would suggest. Discount rate changes account for a trivial fraction of the financing of debt, a result that is ubiquitous in this plain-vanilla specification of the RBC model. The discount rate also moves against the change in debt.

When government spending adjusts to clear the budget—$\gamma_G = 1, \gamma_\tau = \gamma_Z = 0$—important differences emerge in the impulse response functions, as dotted lines in figure 1 show. An expectation that higher spending will reduce future spending eliminates the expansionary effects of higher spending and ameliorates the negative wealth effects on consumption, while it raises the capital stock in the future. When higher current taxes portend higher future government spending, consumption falls more markedly. Higher transfers now create the expectation of lower future spending, which reduces work effort and output, but raises consumption.

Finally, suppose that taxes adjust to ensure fiscal sustainability—$\gamma_\tau = 1, \gamma_G = \gamma_Z = 0$. Positive spending or transfer shocks, which are expected to generate higher taxes in the future, now sharply reduce output, consumption, and capital [dashed lines in figure 1]. Tax hikes, on the other hand, after initially reducing these variables, raise them with a lag.[4]

The simple policy rules produce monotonic adjustments in funding over horizons after which the serial correlation of the shocks has decayed. Figure 2 illustrates this phenomenon in the case when only taxes adjust to debt. The smaller is the adjustment parameter, the more prolonged is the adjustment process. At horizons beyond about 10 periods, the innovation to the expected present value of surpluses converges monotonically to the innovation in debt for each of the three fiscal disturbances.

3. The VAR Specification

This section discusses the identification of the VAR models and the data used in the estimation.

3.1. **Identifying Fiscal Policy Shocks.** As is well known, the reduced-form residuals do not have straightforward economically meaningful interpretations. In order to identify the linear combinations of reduced-form residuals that reflect exogenous fiscal policy disturbances, we follow the method of Blanchard and Perotti (2002), as extended by Perotti (2004).

[4]When either spending or taxes adjust to clear the budget, as in table 2, the present value of discount rates accounts for a trivial share of the value of debt.

Suppose that the reduced form VAR is $f_t = B_0 + f_{t-1}B + u_t$ and that one is interested in recovering the structural form $f_tA_0 = \bar{A} + f_{t-1}A + \epsilon_t J$, where J is a block diagonal matrix with the first block at the upper left corner a k-dimensional matrix coupling the k fiscal policy shocks. Using the approach of Blanchard and Perotti, one assumes that the reduced-form innovations to a fiscal policy instrument, for example, taxes, $u_t(T)$, can be modeled as

$$u_t(T) = u_t(Y)A_{Y,T} + u_t(\pi)A_{\pi,T} + u_t(R)A_{R,T} + \epsilon_t(T) + \epsilon_t(G)J_{G,T} + \epsilon_t(Z)J_{Z,T}, \quad (13)$$

where $u_t(Y)$ is the residual associated with output, $u_t(\pi)$ the inflation residual, $u_t(R)$ the interest rate residual and $\epsilon_t(T)$, $\epsilon_t(G)$, and $\epsilon_t(Z)$ are the identified exogenous fiscal shocks.

The coefficients of the first k columns in A in (13) are identified from institutional information about automatic responses of the policy instrument in question. It then is possible to obtain the covariance matrix of the fiscal policy matrix J. In order to determine impact responses to the individual shocks, it is necessary to make assumptions concerning the relations among the fiscal shocks themselves. We assume that the fiscal shocks are recursively ordered with taxes first, followed by spending and transfers. We shall refer to the shocks so ordered as "tax", "spending" and "transfer" shocks. However, it is important to bear in mind that each shock will entail substantial movements in the other fiscal policy instruments via endogenous propagation mechanisms.

Previous work in this literature has typically entered taxes net of transfers into the VAR. We prefer to disaggregate taxes and transfers, because, on theoretical grounds, distortionary taxes may lead to behavioral responses not characterized simply by their impact on the present-value of lifetime resources. We follow the assumptions in Perotti (2004); specifically, the price elasticity of real transfers is -1 and the output elasticity of transfers is $-.15$. The output elasticity of taxes is therefore given by $\alpha_{TY} = (1 - \frac{Z}{T})\alpha_{netTY} + \alpha_{ZY}\frac{Z}{T}$, with a similar equation for the inflation elasticity, where Z/T is the steady state ratio of transfers to taxes. The calibrated elasticities in the fiscal rules analogous to (13), which also draw on unpublished results from Leeper and Yang (2004), appear in table 3.

3.2. **The Data.** The empirical model is a quarterly VAR using U.S. data on the following variables in log levels: real GDP, the GDP deflator, gross private domestic investment, the three-month Treasury bill rate, the 10-year Treasury bond yield, the monetary base, and fiscal variables. Fiscal data, which are for the Federal government only, include taxes, transfers, spending, and debt (all NIPA).[5] The data cover

[5]Ideally, fiscal variables would include Federal and state and local variables, as is typical in this literature. But state and local governments generally have balanced-budget rules, while debt financing is permitted only for certain capital expenditures. This suggests that fiscal-financing decisions

the period from 1947:2 to 2006:2. Federal spending is defined as the sum of Federal consumption expenditure, gross investment and consumption of fixed capital. Federal taxes include all current tax receipts and contributions for social insurance. Finally, net transfers include net current transfers, capital transfers, income from assets and subsidies. The three-month T-bill rate is used for the sake of consistency with the theoretical model, while the monetary base is necessary to complete the specification of the government budget constraint. Finally, the surplus components are adjusted to better match the conceptual model described above. In particular, adjustments are made to convert corporate income taxes from accrual to cash basis, to include spending and revenue from U.S. territories and Puerto Rico and to include contributions to Federal employee retirement funds. The quantitative importance of these adjustments is small.[6]

To obtain a Federal debt series which obeys a flow budget constraint, we accumulate debt with the NIPA-defined Federal net borrowing figure using the equation $V_t - V_{t-1} = Net\ Borrowing - Seignorage$. We validate this series by comparison to the market value data produced by Cox and Hirschhorn (1983).[7]

The VAR features 5 lags of each endogenous variable and a constant, in order to maintain, as much as possible, the framework of Perotti (2004), which serves as our point of comparison to the existing literature.

4. The Invertibility of VAR Systems

The government present-value condition ties innovations in the market value of debt together with changes in agents' expectations concerning future primary surpluses and discount rates. As has been recognized in other contexts (Hansen and Sargent (1991), Lippi and Reichlin (1994), Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007)), however, there is no guarantee that the history of the shocks recovered from an econometric model can capture the history of information flows to households. It turns out that non-invertibility wrecks havoc on present-value restrictions.

are likely to differ substantially across Federal and state and local governments, so separating the two levels of government seems reasonable.

[6]These adjustments are derived from NIPA table 3.18B. The data in this table are not seasonally adjusted, unlike the data series that we employ elsewhere. We have used these corrections without seasonal adjustment largely because they slightly improve the fit between our generated series and the Dallas Fed's debt data set, as discussed in footnote 7.

[7] The Cox and Hirschhorn data are available at http://www.dallasfed.org/data/data/natdebt.htm. They construct a market value of debt series by computing $V_t \equiv \sum_{j=1}^{J} B_t(j)Q_t(j)$ for maturities $j = 1, 2, \ldots, J$. Because our empirical model includes NIPA-based measures of receipts and expenditures, the Cox-Hirschhorn debt series will not generally be consistent with net borrowing as defined by NIPA.

Suppose that households receive information before the econometric model represents this flow as occuring. Because saddle-path conditions typically front-load adjustment, the estimated impacts of identified shocks may be biased downward. Because we use timing restrictions to identify fiscal policy, we have a further motive for investigating invertibility, since non-invertibility makes the timing restrictions inapplicable to the estimated shocks.

4.1. **Explaining and Testing for Invertibility.** Let the encompassing dynamical system be given, in reduced form, by $f_t = f_{t-1}B + u_t$, where $u_t = \epsilon_t J A_0^{-1}$.[8] We take the information set composed of current and past f_t to reflect private agents' information at t. We are interested in describing the dynamical system formed by projecting f_t onto a smaller space via a matrix P, which is to represent the information set available to the econometrician; at time t the econometrician observes the history of $y_s \equiv f_s P$ for $s \leq t$. The P matrix could, for example, simply select a subset of the f_t.

We report a simple test devised by Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007) which determines whether or not shocks to the encompassing system (f_t) forecast the projected system's ($f_t P$) Wold innovations—that is, whether the projected system is invertible. In the case where the projected system is not invertible, it is possible to express the encompassing system's innovations as a function of the infinite past and future of the projected system's innovations. We rehearse this derivation and then apply it to several choices of information set for the VAR.

A simple theoretical example illustrates what is a stake. Consider a model economy with constant real interest rates, so that the stock of real debt b_t evolves as $b_t = rb_{t-1} - s_t$, where $r > 1$ and s_t is the real surplus. Further suppose that there is a policy rule that sets $s_t = \gamma b_{t-1} + e_t$ with $|r - \gamma| < 1$. Although agents know the model and at time t observe current and past e_t, the econometrician observes only s_t. Applying the operator $1 - rL$ to the law of motion for s_t implies that the surplus behaves like $s_t = (r - \gamma)s_{t-1} + e_t - re_{t-1}$. From the error term in this equation, $h_t \equiv e_t - re_{t-1}$, one can recover the structural shock e_t by solving forward to obtain $e_t = \sum_{j=1}^{\infty} r^{-j} h_{t+j}$. However, it is impossible to represent e_T as a function purely of the history of the h_t for $t \leq T$, as is required for conventional identification schemes to recover the structural error from current and past information.

As is well known, the single equation residual h_t also has another representation in terms of the Wold innovations for s_t, $\tilde{e}_t \equiv \sum_{j=0}^{\infty} r^{-j}(e_{t-j} - re_{t-j-1})$.[9] Unfortunately, the Wold representation does not exhibit the present-value relation between debt and surpluses, in the following sense. Substituting the present-value relation into the flow

[8]It is convenient to suppress the constant in this section.

[9]See Lippi and Reichlin (1994) for a systematic method for obtaining such a representation.

constraint, one sees that $(E_t - E_{t-1})\sum_{j=0}^{\infty} r^{-j} s_{t+j} = 0$. Suppose that one replaces the expectations in this expression with conditional expectations with respect only to the history of surpluses up to time t, denoted $E_t^{(S)}$. Then

$$E_t^S \sum_{j=0}^{\infty} r^{-j} s_{t+j} - E_{t-1}^S \sum_{j=0}^{\infty} r^{-j} s_{t+j} = (E_t^S - E_{t-1}^S) \sum_{j=0}^{S} \sum_{k=1}^{j+1} (r-\gamma)^{j-k} (\tilde{e}_{t+k-1} - \tilde{e}_{t+k-2}/r) = \sum_{j=0}^{\infty} r^{-j} (r-\gamma)^{j-1} \tilde{e}_t - \sum_{j=1}^{\infty} r^{-j} (r-\gamma)^{j-2} \tilde{e}_t / r = \tilde{e}_t \frac{(r/\gamma)(1-r^{-2})}{r-\gamma} \quad (14)$$

Thus, $(E_t^{(S)} - E_{t-1}^{(S)})\sum_{j=0}^{\infty} r^{-j} s_{t+j} = \tilde{e}_t \frac{(r/\gamma)(1-r^{-2})}{r-\gamma}$, which is generally not zero, as the present-value relation requires. Because we want to explore the implications of present-value balance, we must take some care to avoid invertibility problems.

Following Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007), define matrices $C = BP$ and $D = JA_0^{-1}P$ and suppose that D is invertible. The invertibility of D says that the innovations to f_t and f_tP, relative to history of ϵ_t—the identified exogenous shocks—both span the same space. Consistent with this assumption, when we assess the invertibility of a given projected model, we will approximate the encompassing model residual covariance matrix with the covariance matrix of the first K principal components of the encompassing model's residuals, if K is the dimension of the small model. Given the invertibility of D, one can solve for ϵ_t in the definition of y_t to obtain $\epsilon_t = y_t D^{-1} - f_{t-1} C D^{-1}$. Substituting this expression into the law of motion for f_t, write $f_t = f_{t-1}(B - CD^{-1}JA_0^{-1}) + y_t D^{-1} J A_0^{-1}$, from which it follows immediately that the projected model is invertible if and only if $B - CD^{-1}JA_0^{-1}$ is stable. If this matrix is not stable, then one cannot express ϵ_t in terms of the history of f_tP up to time t alone. Rather, as shown in appendix A, one can express ϵ_t as a function of past and future realizations of the projected model's residuals.

Our approach is informal. We seek to ascertain whether the relatively small models used, for example, in Blanchard and Perotti (2002) and Perotti (2004) are invertible with respect to a larger model that is constructed to be sufficiently rich to capture the government's present-value budget constraint. The smallest model is that of Perotti (2004), consisting of taxes net of transfers, spending, output, the three-month Treasury-bill rate, and inflation (denoted P2004). We also consider a variant of this model in which investment is included (denoted P2004I). Finally, the largest model considered is a 10-variable VAR, consisting of all variables in P2004I plus it splits net taxes into taxes and transfers separately and it adds the 10-year nominal interest rate and debt.

Figure 3 displays spending shock multipliers for all models mentioned above.[10] For those models in which both tax and transfers shocks are separately identifiable, figure 4 exhibits tax and transfer shock multipliers respectively. Multipliers for the government spending shock are shown for all models, since this shock can be identified in each model. However, there is no unique way to specify the effects of a net tax shock in models with disaggregated taxes and transfers.

The figures suggest that the smallest model, P2004, is perhaps too small, as the addition of transfers, investment or debt all appear to lead to dynamical systems which are relatively similar to each other, but notably different from P2004. For all shocks, the addition of either debt or investment to the information set results in sharp increases in the output and price responses to fiscal policy shocks, relative either to P2004 (the smallest model) or to the version of P2004 where taxes and transfers are disaggregated. For example, the P2004 system has output become strongly negative after a spending shock within around five years, while all larger systems have output far above its initial condition for more than 40 years. Similarly, in response to a transfer shock, the smallest system shows a strong, persistent, positive impact on output, while the larger systems all show output rapidly falling off from its initial positive impact response. The intuition that P2004 is too small is confirmed by the test of Fernández-Villaverde, et al., which reveals the presence of explosive eigenvalues in the criterion matrix.

The addition of investment to the P2004 model brings the impulse response functions much closer in line with those exhibited by the largest system. This behavior is particularly clear in the case of the output response following a spending shock, in which case the addition of investment rather sharply increases the output response. Nevertheless, the Fernández-Villaverde, et al. test again shows that these smaller models are not invertible with respect to the encompassing system.

Given the qualitative closeness of the basic P2004I model to the encompassing model, we now investigate how serious is the resulting non-invertibility. When the projected system is not invertible, its residuals may be expressed as an infinite series in the encompassing system's innovations. Alternatively, non-invertibility implies it is possible to write the encompassing system's innovations, ϵ_t, as an infinite series containing the projected model residuals' past and future realizations, ϵ_t^P. This second representation is worked out in appendix A and figure 5 displays the first 100 years

[10]Multipliers are calculated from log-responses by scaling all quantity variables by their average share in GDP and then choosing the magnitude of the initial shock so that the associated policy instrument has unit impact response.

of coefficients for the P2004 projected system. Schematically, if the projected system has residuals ϵ_t^P conditional on the history of the projected observables, the figure shows the magnitudes of the vector of coefficients q_j in[11]

$$\epsilon_t = \sum_{j=1}^{\infty} \epsilon_{t+j}^P q_j + \phi_t \tag{15}$$

where ϕ_t refers to terms dated t or earlier.

Intuitively, the coefficients q_j measure the information flow regarding the true structural innovation to an econometrician who observes only the history of ϵ_t^P. A long train of non-negligible coefficients indicates that the limited information set lags behind the information set possessed by private agents by a considerable degree. Such an information lag is, of course, particularly a problem for us, as we depend on timing restrictions to identify fiscal policy. Furthermore, as fiscal policy shocks are identified both in the projected models and the encompassing model, and we are primarily concerned about fiscal policy impacts, we use the above equation to calculate the loadings of the 10-variable system's fiscal policy shocks on the projected model's identified fiscal policy shocks.[12] Fiscal policy shocks from both models are weighted by their standard deviations.

Figure 5 suggests a fairly substantial weighting on future realizations of the projected system's residuals, over a considerable horizon, for the P2004 system. In particular, leads of the projected system's fiscal policy variables only gradually reveal the larger model's fiscal policy shocks: information about the true spending shock is still being revealed by the projected model's residuals several decades after the true shock is realized. By contrast, the net tax shock appears relatively uninformative about spending and taxes at any but the shortest horizons.

5. A Consistent Accounting Framework

5.1. **The Intertemporal Budget Constraint.** Let government debt in the hands of the public at time t consist of zero coupon bonds with nominal face value $B_t(j)$ maturing at $t+j$, for all $j \geq 1$. Further, let the total nominal value of debt outstanding be $V_t \equiv \sum_{j=1}^{\infty} B_t(j)Q_t(j)$ and let the nominal primary surplus be S_t. The surplus is defined as $S_t = T_t - G_t - Z_t$, where T_t is tax receipts, G_t is government spending, and

[11] Appendix A also explains how the q_j's are computed.

[12] In principle, the ϵ_t^P vector includes all the shocks in the projected system. Of course, a finding that the q_j's are non-zero for the fiscal subvector of ϵ^P is sufficient to conclude non-invertibility of the projected system.

Z_t is transfer payments. The government budget identity is then[13]

$$\sum_{j=1}^{\infty}(B_t(j) - B_{t-1}(j+1))Q_t(j) = B_{t-1}(1) - S_t. \tag{16}$$

In real terms the identity is

$$\frac{V_t}{P_t} = \frac{1}{P_t}\sum_{j=1}^{\infty} B_t(j)Q_t(j) = \frac{P_{t-1}}{P_t Q_{t-1}(1)}\frac{1}{P_{t-1}}\sum_{j=1}^{\infty} B_{t-1}(j)Q_{t-1}(j) - \frac{S_t}{P_t} + \omega_t \tag{17}$$

where $P_t\omega_t \equiv \sum_{j=1}^{\infty}\left(Q_t(j) - \frac{Q_{t-1}(j)}{Q_{t-1}(1)}\right)B_{t-1}(j+1)$ and P_t is the price level.

The Euler equation for a nominal discount bond implies

$$Q_t(j) = \delta^j E_t \frac{\lambda_{t+j}}{\lambda_t}\frac{P_t}{P_{t+j}}, \tag{18}$$

where $\delta \in (0,1)$ is the subjective rate of discount.

Express ω_t as

$$\omega_t \equiv \frac{1}{P_t}\sum_{j=1}^{\infty}\left(Q_t(j) - \frac{Q_{t-1}(j+1)}{Q_{t-1}(1)}\right)B_{t-1}(j+1),$$

from which it follows, after imposing the Euler equation for bond prices, that

$$\omega_t = \frac{1}{\lambda_t}\sum_{j=1}^{\infty}\delta^j\left(E_t\frac{\lambda_{t+j}}{P_{t+j}} - \frac{\lambda_t}{P_t}\frac{E_{t-1}\frac{\lambda_{t+j}}{P_{t+j}}}{E_{t-1}\frac{\lambda_t}{P_t}}\right)B_{t-1}(j+1). \tag{19}$$

Thus, $\lambda_t\omega_t = \eta_t$, where $E_t\eta_{t+1} = 0$. To anticipate slightly, when discounted, the ω_t term in (17) will disappear in expectation, so it will not contribute to the present-value expressions below. Innovations in ω_t can nonetheless play an important role by revaluing debt. Note that the presence of this term eliminates the stochastic singularity which is at issue in Hansen, Roberds, and Sargent (1991) and also present in Favero and Giavazzi (2007).

In equilibrium a transversality condition holds such that

$$\lim_{s\to\infty} E_t\lambda_{t+s}\frac{V_{t+s}}{P_{t+s}} = 0.$$

[13]For expository clarity, we abstract from seigniorage in this section of the paper. The empirical work, however, involves imposing the full budget constraint, including seigniorage, which for the flow budget constraint is defined as $(M_t - M_{t-1})/P_t$, where M is the monetary base. The addition of seigniorage terms introduces no new conceptual issues.

Iterate forward on (17) to obtain

$$\frac{V_t}{P_t} = E_t \sum_{j=1}^{\infty} \delta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{S_{t+j}}{P_{t+j}}. \tag{20}$$

where, as usual, the transversality condition implies the absence of a bubble term in (20).

For the purposes of linearization, it is convenient to express (20) and (17) in terms of output. For any nominal variable X_t, let the corresponding variable $x_t \equiv \frac{X_t}{Y_t}$, where Y is nominal output, and define γ_t as $\frac{Y_t/P_t}{Y_{t-1}/P_{t-1}}$, the growth rate of real output. Scaled versions of the flow and intertemporal government budget constraints are:

$$v_t = \frac{1}{\gamma_t \pi_t Q_{t-1}(1)} v_{t-1} - s_t + \frac{P_t \omega_t}{Y_t} \tag{21}$$

and

$$v_t = E_t \sum_{j=1}^{\infty} \delta^j \frac{\lambda_{t+j}}{\lambda_t} \left(\prod_{k=1}^{j} \gamma_{t+k} \right) s_t, \tag{22}$$

where v_t is the market value of debt as a share of output.

If it were feasible to estimate an empirical model that included government bonds and bond prices at *all* maturities, we would work directly with the flow constraint in (16). Because such a model is not practicable, we introduce ω_t to express the budget constraint in terms of the value of debt, v, the one-period interest rate, $Q_{t-1}(1)$, the surplus, and other variables, as in (21). In the estimated model, ω_t is a residual, which is a linear combination of the VAR errors, that clears the period-by-period government budget constraint.

5.2. **The VAR and Its Constraints.** Now consider log-linearizations of (21) and (22) around fixed values for $(v, \gamma, s, P\omega/Y)$ and the growth rate of λ which satisfy both (21) and (18). Denote $\hat{x}_t \equiv \log(x_t) - log(x)$. Defining the one-period nominal interest rate at date $t-1$ as $R_{t-1} = Q_{t-1}^{-1}(1)$, the linearized form of the flow constraint is:

$$\hat{v}_t = \frac{1}{\beta} \left(\hat{v}_{t-1} - \hat{\gamma}_t - \hat{\pi}_t + \hat{R}_{t-1} \right) - \frac{s}{v} \hat{s}_t + \frac{1}{v} d \left(\frac{P_t \omega_t}{Y_t} \right) \tag{23}$$

where for steady state values of variables we use sample means. (Because ω can be negative, we do not log-linearize it—the term $d\left(\frac{P_t\omega_t}{Y_t}\right)$ represents deviations of $\frac{P_t\omega_t}{Y_t}$ from its linearization point.) The linearized form of the present-value constraint is:

$$\hat{v}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{s}{v} \left(\sum_{s=1}^{j} -\hat{R}_{t+s-1} + \hat{\gamma}_{t+s} + \hat{\pi}_{t+s} + \hat{s}_{t+j} \right) \tag{24}$$

where $\beta \equiv \delta\gamma\frac{\lambda_{t+1}}{\lambda_t}$, $\frac{\lambda_{t+1}}{\lambda_t}$ is the constant steady state growth rate of the marginal utility of consumption, and the deviations of the net surplus are given by $s\hat{s}_t = \tau\hat{\tau}_t - g\hat{g}_t - z\hat{z}_t$.

Note that $\sum_{j=1}^{\infty}\beta^j\sum_{s=1}^{j}X_{t+s} = \frac{1}{1-\beta}\sum_{j=1}^{\infty}\beta^j X_{t+j}$. Therefore, equation (24) can be written

$$\hat{v}_t = E_t\sum_{j=1}^{\infty}\beta^j\frac{s}{v}\left(\frac{-\hat{R}_{t+j-1}+\hat{\gamma}_{t+j}+\hat{\pi}_{t+j}}{1-\beta}+\hat{s}_{t+j}\right). \tag{25}$$

Ultimately, we wish to express the real quantity variables in levels, rather than as fractions of output. Define $\tilde{v}_t \equiv log(V_t/P_t)$, $\tilde{\pi}_t \equiv log(\pi_t)$, and $\tilde{R}_t \equiv log(R_t)$. Using the steady-state relations and eliminating output growth, equation (25) implies that

$$\tilde{v}_t = k + E_t\sum_{j=1}^{\infty}\beta^j\frac{s}{v}\left(\frac{-\tilde{R}_{t+j-1}+\tilde{\pi}_{t+j}}{1-\beta}+\tilde{s}_{t+j}\right). \tag{26}$$

where $k = -\frac{\beta}{1-\beta}\left(ln(1/\beta)/\beta + \tau ln(\tau)/v - gln(g)/v - zln(z)/v\right) + ln(v)$ and $s\tilde{s}_t = \tau\tilde{\tau}_t - g\tilde{g}_t - z\tilde{z}_t$. Thus, innovations to real debt must be balanced by innovations in the present-value expression on the right-hand side. Moreover, this present-value can itself be thought of as consisting of two components: the present-value of surpluses at constant steady-state discount rates and a term which measures changes in the expected path of those discount rates.

Suppose that the state of the model economy is characterized by the M-dimensional factors f_t which, in companion form, evolve according to the VAR process

$$f_t = B_0 + f_{t-1}B + u_t. \tag{27}$$

Let a model variable x_t be related to the underlying factors by a coefficient matrix C_x such that

$$\tilde{x}_t = f_t C_x \tag{28}$$

The government budget constraint in equation (26) implies the following restrictions on the VAR in (27):

$$\beta B\left(C_v + \frac{1}{\beta}C_\pi + \frac{\tau}{v}C_\tau - \frac{g}{v}C_g - \frac{z}{v}C_z\right) - (C_v + C_R) = 0 \tag{29}$$

and

$$k + B_0\frac{\beta}{1-\beta}(1-\beta B)^{-1}\left(\frac{1}{\beta}C_\pi + \frac{\tau}{v}C_\tau - \frac{g}{v}C_g - \frac{z}{v}C_z - C_R\right) = 0 \tag{30}$$

Expression (30) imposes restrictions on the deterministic growth components of the VAR. Because our focus is on innovation accounting and the deterministic components are irrelevant, we do not impose (30) on the estimated VAR.

In addition, if the matrix βB possesses explosive eigenvalues, we must impose conditions which guarantee that the infinite sum in (26) exists. Specifically, let W be the matrix of right eigenvectors of βB and suppose that λ_j is an eigenvalue such that $|\lambda_j| \geq 1$. If $W^{-1}(j)$ denotes the corresponding row of the inverse of W, we require that

$$W^{-1}(j)\left(\frac{1}{\beta}C_\pi - C_R + \frac{\tau}{v}C_\tau - \frac{g}{v}C_g - \frac{z}{v}C_z\right) = 0 \tag{31}$$

Note that the VAR does not contain the term ω_t in equation (17). Variations in this variable thus implicitly maintain the government's flow budget constraint. As is evident above, however, the present-value constraint implies that the flow constraint holds in expectation.

5.3. **Estimation Procedure.** We conduct estimation and inference in a least-squares framework. As before, let the VAR be $f_t = B_0 + f_{t-1}B + u_t$, for $t = 0, \ldots, T$. Then the objective function is $min \sum_t u_t' \Sigma u_t$. Let f be a data matrix whose rows consist of the variables f_t, for each $t > 0$ and define f_- as the corresponding lagged data matrix. Define $W_- \equiv [\mathbf{1}, f_-]$ and $b \equiv vec([B_0', B']')$. The part of the objective function relating to b can be re-written $(b - \hat{b})'S(b - \hat{b})$, where $\hat{b} \equiv I \otimes (W_-'W_-)^{-1}(W_-'f)$ and $S = (\Sigma \otimes Z'Z)$. We maximize this objective function subject to a constraint $C_0 b = C_-$. The first-order condition for this problem is $S(b - \hat{b}) = C_0'\lambda'$. It follows that $C_0 b = C_- = C_0\hat{b} + C_0 S^{-1} C_0' \lambda'$, or that $b = \hat{b} + S^{-1}C_0'(C_0 S^{-1} C_0')^{-1}(C_- - C_0\hat{b})$.

From equation (29), we have that $[\mathbf{0}, \mathbf{I}][B_0', B']'C_0 = C_-$, so that the constraint is $(C_0' \otimes [\mathbf{0}, \mathbf{I}])b = vec(C_-)$, where, explicitly,

$$C_0 \equiv \beta\left(C_v + \frac{1}{\beta}C_\pi + \frac{\tau}{v}C_\tau - \frac{g}{v}C_g - \frac{z}{v}C_z\right) \tag{32}$$

and

$$C_- \equiv (C_v + C_R) \tag{33}$$

Estimates presented here are from a feasible GLS procedure, iterated until convergence, in which Σ^{-1} is a consistent estimate of the residual covariance matrix.

The distribution of the estimated quantities is obtained by Monte-Carlo simulation, using the unconstrained VAR as the data generating process. Least-squares estimates of persistence parameters are known to be downward-biased in small samples. We correct for the downward bias in the Monte-Carlo studies using the technique of Kilian (1998).

The discount factor β plays an important role in computing present values. We compute this from the steady state government budget constraint after imposing that $1/\beta = R/\pi$ and using the sample means for taxes, spending, transfers, and debt as a share of GDP. The calculated value is $\beta = .9967$.

5.4. **Impulse Response Functions and Multipliers.** Figure 6 reports multipliers for both the constrained and unconstrained VAR. Sixty-eight percent confidence intervals are computed for the unconstrained baseline VAR. Responses of fiscal variables and output are in dollar units, so the output responses are conventional multipliers relative to the initial unit shock to the fiscal variable. Price level and real interest rate responses are in logs and may be interpreted as percentage changes.

Over horizons of 10 or more years, output responses from the constrained model fall outside the 68 percent intervals of the unconstrained model. For prices, differences emerge only over still longer horizons. Dynamic interactions among fiscal variables are also different in the two systems, as figure 7 shows.[14]

The baseline VAR reproduces many of the standard findings in the literature. Upon a surprise tax increase, output falls, while a spending increase generates a rise in output. Moreover, also consistent with the literature, spending shocks generate short-lived crowding out of investment and a prolonged period of lower prices.

The most dramatic effect of imposing the present-value constraint is on output. After about 10 years, the output response leaves the confidence interval for every fiscal policy shock. Following a tax increase, output is persistently lower by an additional 10 percent, while, following a spending increase, output is higher by roughly the same amount. The persistent impact of fiscal policy shocks on prices is also noteworthy. After 40 years, the constrained VAR forecasts prices as 20 percent below their starting point, while, after an initial period of deflation following a spending shock, after 40 years, the price level is a startling 30 percent above its initial position.

In light of our coming discussion of present-value financing, it is worth keeping track of the path of the surplus components (see figure 7). Especially after tax and spending shocks, and especially at longer horizons, the imposition of the present-value constraint produces large effects. By around 25 years after the initial tax shock, the constrained VAR predicts an almost 40 percent fall in transfers. This fall in transfers is so persistent that by 40 years, transfers have still only recovered to 16 percent below their initial level. By this point, taxes themselves, after experiencing a positive unit shock, have fallen by 12 percent. Perhaps surprisingly, the net effect of these differences between the two systems does not appear much to affect the forecast of debt itself, at least, within the first 40 years. Rather, the imposition of the constraint tends to exaggerate the responses of both taxes and transfers, so that the net effect is muted, as seen in the primary surplus responses in figure 6. Indeed, the impact of the present-value constraint on spending appears slight.

[14]Impulse response functions from the identified fiscal policy shocks always lie within the 95 percent confidence bands for the first 40 years and these impulse response functions are also typically within the 90 percent confidence intervals as well.

6. How Debt-Financed Fiscal Shocks Have Been Financed

We turn now to the forward-looking aspect of government finance. In particular, we wish to ascertain what combination of adjustments in the expected path of fiscal policy instruments and discount rates rationalizes the decision of private agents to hold government debt at prevailing market prices. We also ask some questions about the dynamics of fiscal adjustment. How does adjustment depend on the nature of the fiscal policy shock? Over what horizon must one forecast in order to see present-value balance?

6.1. **Fiscal Finance: Present Values.** The basic present-value decomposition previously described is displayed in table 4. Consistent with the convention for impulse response functions, the table shows present-value components following unit responses from the policy instruments associated with each type of shock. The components are scaled so that they add to the initial debt innovation, shown in the third column. The standard deviations of the fiscal policy shocks are also shown to indicate the typical magnitude of the present-value adjustment associated with each shock. While spending shocks have a far lower variance than do tax shocks, they generate much more violent movements in present values and in some periods may dominate the effect of the tax shocks. As we shall see, the turn of the millennium features a number of such episodes.

Different fiscal policy shocks are financed very differently in present-value terms. For tax shocks, both the discount rate and the present value of surpluses at constant rates move to support debt, with the lion's share of the work done by changes in the present value of surpluses. This coincides with the usual picture of how debt is financed in present-value. By contrast, the role of the discount rate is much less intuitive for spending and transfer shocks. In the case of surprise spending increases, in fact, while taxes do rise sharply and persistently, their contribution is swamped by the combination of higher spending and transfers. The present-value of surpluses at constant discount rates actually *falls*. Present-value balance is maintained only by drastic and prolonged fall in real interest rates, the bulk of which is accounted for a fall in the nominal interest rate.

The story is reversed for surprise increases in transfers. The initial transfer increase is quite transient and, in the longer horizon, lower expected transfers account for most of the present-value of surpluses, with taxes and spending offsetting each other. The discount rate resists present-value balance, again largely due to changes in the path of nominal interest rates, in this case a sharp rise.

For each type of fiscal policy shock, taxes and transfers experience sizeable but offsetting movements in present value, as was apparent from the impulse response functions. Moreover, except in the case of transfers, the present value of taxes moves to support the innovation in debt, while, consequently, the present-value of transfers moves against it. In the case of transfers, it is spending and taxes which move in offsetting fashion, while transfers bears the burden of adjustment.

6.2. **Distribution of Present-Value Components.** In this section, we report and discuss Monte Carlo simulation results concerning the distribution of the present-value components. For this purpose, we draw 14,000 times from the residuals of the constrained VAR, using the constrained VAR as the data generating process.

Tables 5 and 6 show that the sign of the initial debt innovation is quite precise for tax and spending shocks, while the small impact magnitude of the debt response for transfers prevents us from making similar statements regarding the impact of a transfer shock. Substantial probability mass appears to be on the event that the primary surplus moves with the debt innovation, following a tax shock, while against, following a spending shock, although we cannot reject the contrary possibility at the five percent level. This conclusion is supported by table 7, which reports the fraction of Monte Carlo draws in which the listed components move in the same direction as the debt innovation. In slightly more than 90 percent of draws, the present-value of surpluses supports the debt innovation following a tax shock, while the opposite is true in more than 90 percent of cases, for spending shocks.

Regarding the role of the present-value surplus components taken separately, the confidence bands are wide in all cases, except following the spending shock. In that case, however, in over 97 percent of cases, the present value of taxes moves to support debt, but is more than offset by counter-moves in the present values of spending and transfers, both of which move against debt. In the case of the other shocks, while there is tendency for taxes to support the debt innovation following a tax shock, this occurs in only 75 percent of cases and, indeed, the 95 percent confidence interval is quite broad. Finally, note that the addition of seigniorage to the primary surplus does not materially affect any of these conclusions.

6.3. **Fiscal Finance: Dynamics.** The summary accounting in the previous section, while informative, does not reveal much of the dynamic structure which support present-value balance. In this section, we aim to illuminate this topic by examining the horizon over which present-value balance is attained. The funding horizon addresses the issue of how far into the future one must forecast in order to see present-value balance. For some shocks, the answer is "quite a long time."

The forecast horizons for each type of fiscal policy shock are illustrated by figure 8. The solid lines represent truncated present-values of discounted surpluses, up to the date indicated. Each series is scaled by the initial debt innovation, so that asymptotically, each must converge to plus or minus one. Of the three types of shocks, the tax shock generates by far the mildest variations in funding, but even here the truncated present-value repeatedly over- and under-shoots its asymptotic target by modest amounts. Indeed, after the 10-year mark, the truncated present value for tax shocks is essentially a scaled down version of the present value for transfer shocks.

Most dramatic, however, are the responses of the present value of surpluses to shocks in transfers and spending. First, the time over which one must forecast in order to see present-value balance is on the order of 100 years or so. Second, at intermediate points the truncated present-values experience wide swings. For example, a truncated forecast following a transfer shock would, at 15 years, show a present-value deficit of 16 times its asymptotic value, while a 40-year forecast would reveal a surplus of over four times the asymptotic value.

7. Conclusion

Our findings may be split into two parts. First, we find that very small VAR systems, in particular, systems which include neither debt nor investment, may fail to be invertible with respect to systems which do include these variables. Second, we have imposed a linearized version of the Federal government's present-value constraint and investigated how present-value balance is achieved following identified fiscal policy shocks.

Two historical episodes nicely illustrate our central findings regarding present-value balance. Consider first the tax shock created by the tax rebate in the second quarter of 1975. Figure 9 displays debt innovations attributable to fiscal policy shocks and the means of their present-value funding between 1972 and 1978. According to the constrained VAR, in quarter 2 of 1975, a large fiscal-policy-induced rise in debt occurs. Consistent with table 4 and with the idea that this innovation represents a tax shock, the bulk of the surprise rise in debt is supported, in present-value, by expected increases in the primary surplus, while expected changes in the path of real interest rates plays a minor, but supporting role. Indeed, one sees that the lead role in supporting the surprise tax cut is the expectation of higher future tax revenue, albeit partially offset by higher future transfers and spending. Moreover, from figures 4 and 8, the tax cut should have been expected to be highly transient, as a pure tax cut would have been enduringly reversed after only two-and-a-half years.[15] By the same token, the public would have foreseen very persistent effects from this seemingly transient tax cut—effects which would propagate 50 years or more, before quiescence.

This pattern, however, is not the only one in the sample. Now consider the present-value support of the last seven years of fiscal policy shocks, shown in figure 10. During

[15]Blinder (1981) studies the effects of this episode on consumption.

this period, there are multiple instances in which the innovation to debt has *not* been supported by corresponding movements in the present value of primary surpluses, at constant discount rates. Rather, as this period features several large spending shocks, the present value of surpluses has, on a number of occasions, moved against the debt innovation, due to the extreme persistence of the spending response, which remains far above its initial level for decades. Again, to see present-value balance in forecasts, one would have to extrapolate out 50 to 100 years, while truncated forecasts before this horizon would have produced wildly misleading impressions of imbalance.

The essential lessons of this paper are therefore these: whether or not the primary surplus will adjust to support innovations in debt depends on nature of the shock to debt. For taxes and transfers, it appears as though the surplus will so adjust, while, with spending, it will not. Furthermore, the impact of fiscal policy shocks is highly persistent and forecasts must be carried out well beyond the 50-year mark to see present-value balance.

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Parameters						
β	γ	γ_N	θ	A	α	δ
.99	1	1	3.48	1	2/3	.025
Policy Rule Output Elasticities						
		φ_G	φ_τ	φ_Z		
		0	1.5065	$-.15$		
Steady State Policy Variables						
		G/Y	T/Y	Z/Y	B/Y	
		.0839	.1771	.0882	.495	

Table 1. Parameter settings in the real business cycle model. Steady state policy variables calibrated to match U.S. data used to estimated identified VARs. Policy shocks have first-order serial correlation parameter of .80. Policy rule output elasticities calibrated from Perotti (2004) and steady state policy variables.

	Financed by (Present Value)				
Shock to	T	G	Z	R	S
G	$-.03$	-4.03	5.14	$-.08$	1.08
T	-3.24	0	4.25	$-.004$	1.004
Z	0	0	1	0	1

Table 2. Real business cycle model. The fraction of positive government debt innovations, due to shocks listed in the first column, that are financed by each of the components of the government budget. Simulation assumes that transfers clear the government budget: $\gamma_Z = 1, \gamma_G = \gamma_\tau = 0$. R denotes the stochastic discount factor; S denotes net surplus, derived by summing columns labeled T, G, and Z.

	Y	π
Tax Elasticity	1.5065	.3248
Spending Elasticity	0	$-.5$
Transfer Elasticity	$-.15$	-1

Table 3. Calibrated elasticities in identified VAR with taxes and transfers separated.

Shock to	Std Dev	ΔB	T	G	Z	S	R
Taxes	.0464	−.2365	−1.9339	.4260	1.3782	−.1296	−.0835
Spending	.0090	.0375	2.3501	−0.8115	−1.6848	−.1462	.1701
Transfers	.0056	.0035	−.1976	.1891	.1114	.1028	−.1187

Table 4. Policy Block Present-Value Funding Decomposition. Present-values calculated following a unit shock.

Present-Value Component	Shock to Taxes	Shock to Spending	Shock to Transfers
ΔB	$[-.2498, -.2241]$	$[.0229, .0519]$	$[-.0056, .0121]$
T	$[-1.6841, .7103]$	$[.7657, 2.3250]$	$[-.4920, -.0071]$
G	$[-.4301, .3920]$	$[-.7947, -.2804]$	$[.1012, .2978]$
Z	$[-.4385, 1.2244]$	$[-.1720, -.5546]$	$[-.0424, .3411]$
S	$[-.2320, -.0517]$	$[-.2085, -.0395]$	$[.0564, .1449]$
$S + Seigniorage$	$[-.2503, -.0407]$	$[-.2197, -.0221]$	$[.0666, .1707]$

Table 5. Monte Carlo Distribution for Present-Value Components. Reports 68 percent confidence intervals, computed from 14,000 Monte Carlo draws, using the constrained VAR as the data generating process.

Present-Value Component	Shock to Taxes	Shock to Spending	Shock to Transfers
ΔB	$[-.2638 - .2118]$	$[.0656, .0096]$	$[-.0144, .0206]$
T	$[-5.7325, 1.2699]$	$[.1248, 4.5300]$	$[-1.1007, .9158]$
G	$[-.7057, 1.4565]$	$[-1.4197, -.0508]$	$[-.1297, .5091]$
Z	$[-.8787, 4.5243]$	$[-3.4649, -.0628]$	$[-.7640, .7780]$
S	$[-.4194, .1472]$	$[-.3782, .0774]$	$[.0000, .2275]$
$S + Seigniorage$	$[-.4935, .1658]$	$[-.4041, .1300]$	$[.0021, .2713]$

Table 6. Monte Carlo Distribution for Present-Value Components. Reports 95 percent confidence intervals, computed from 14,000 Monte Carlo draws, using the constrained VAR as the data generating process.

Shock to	T	G	Z	S
Taxes	.7479	.3970	.2483	.9157
Spending	.9747	.0235	.0263	.0801
Transfers	.4185	.6242	.5636	.6395

Table 7. Fraction of MC Draws (M = 14,000) in which listed component moves in the same direction as the debt innovation.

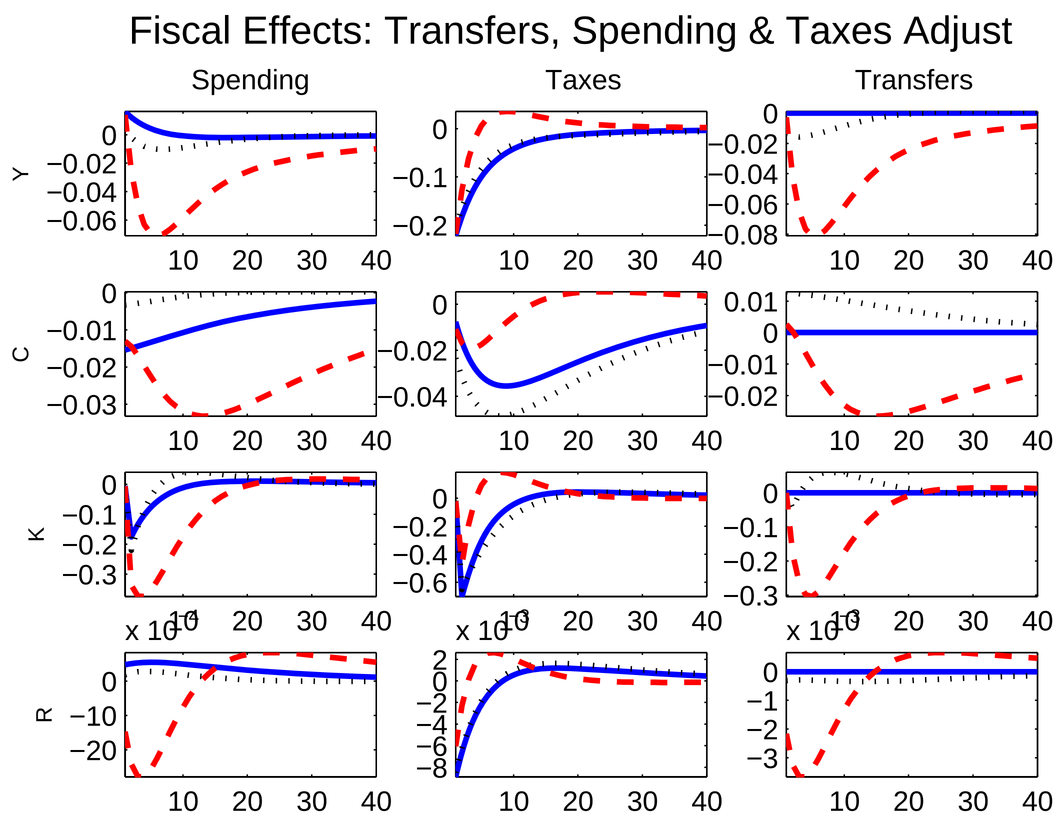

FIGURE 1. Responses to fiscal shocks when taxes clear the budget: $\gamma_\tau = 1, \gamma_Z = \gamma_G = 0$. Solid line: transfers adjust; dashed line: taxes adjust; dotted line: spending adjusts.

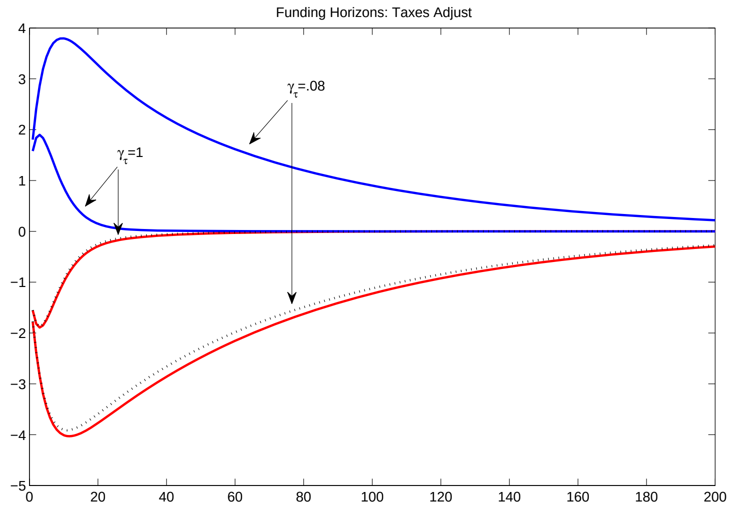

FIGURE 2. Government debt funding horizons for fiscal shocks when taxes clear the budget. Solid line: tax shock; dotted-dashed line: spending shock; dotted line: transfers shock.

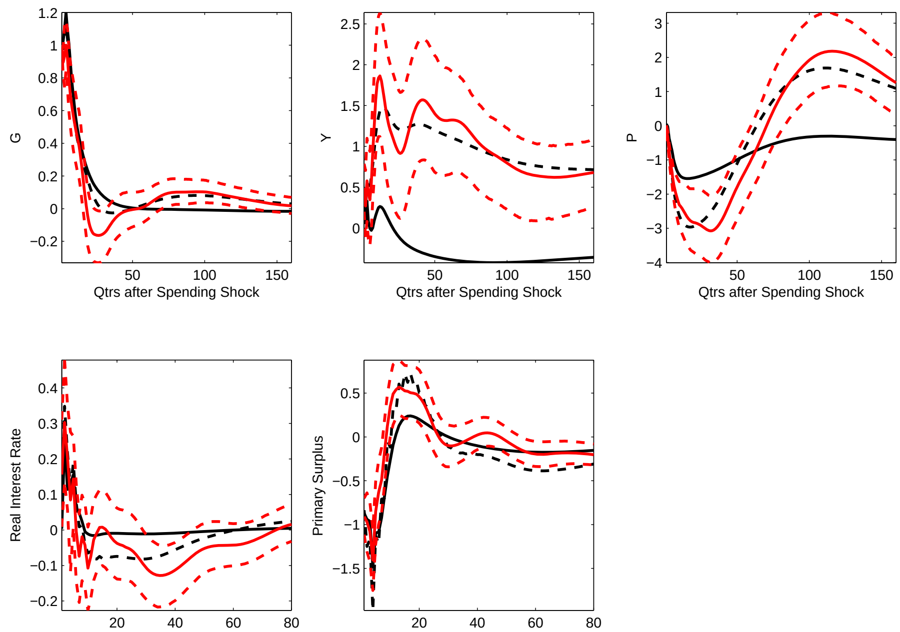

Figure 3. Selected multipliers following a spending shock. Black solid line (P2004): VAR system includes net tax revenue, spending, output, prices and the 3-month T-bill rate. Black dashed line (P2004I): Same as P2004 but adds investment. Grey lines (Baseline VAR and 68 percent error band): VAR includes tax revenue, spending, transfers, output, prices, 3-month T-bill rate, 10-year bond rate, monetary base, investment, debt.

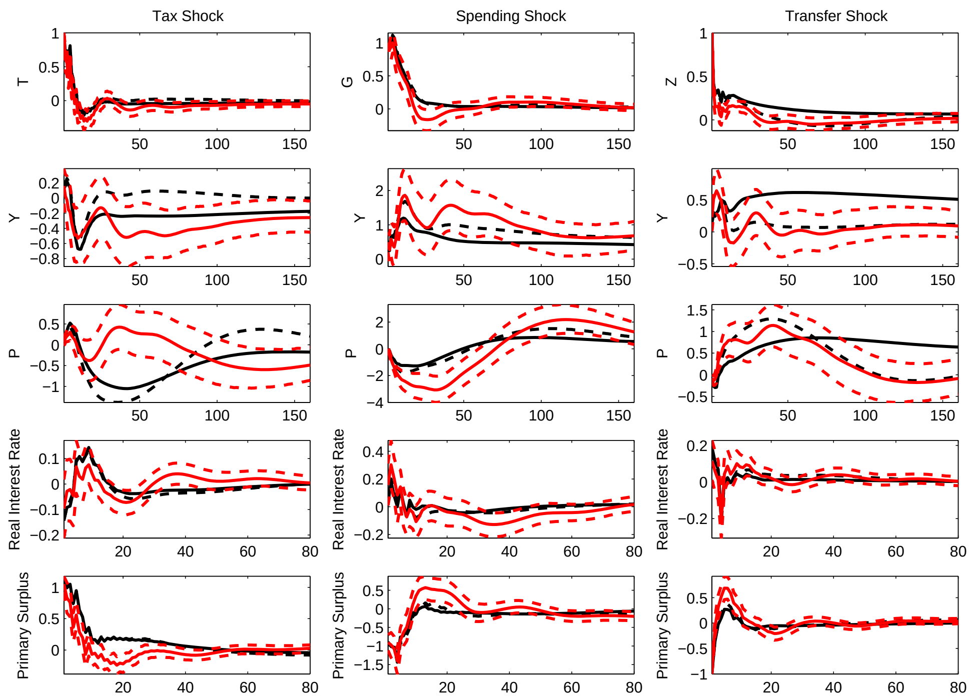

FIGURE 4. Selected multipliers following a tax, spending, and transfers shocks. Black solid line (P2004): VAR system includes net tax revenue, spending, output, prices and the 3-month T-bill rate. Black dashed line (P2004I): Same as P2004 but adds investment. Grey lines (Baseline VAR and 68 percent error band): VAR includes tax revenue, spending, transfers, output, prices, 3-month T-bill rate, 10-year bond rate, monetary base, investment, debt.

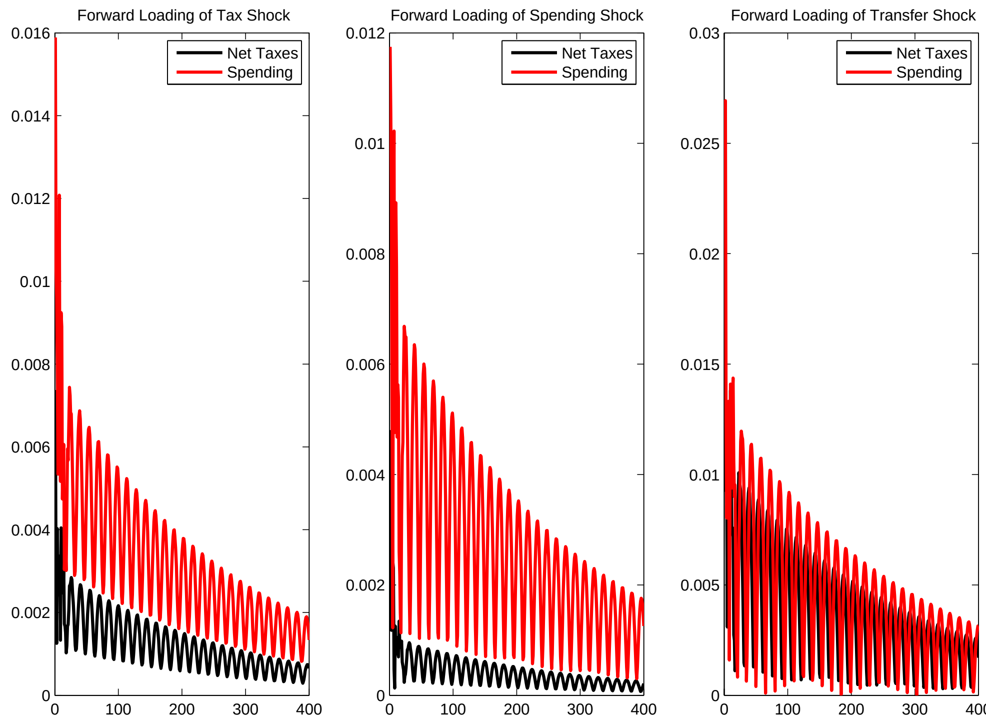

FIGURE 5. Magnitude of coefficients on future projected-model residuals in expansion of encompassing-model fiscal policy shocks. All variables are scaled by their standard deviations. Horizon is 100 years.

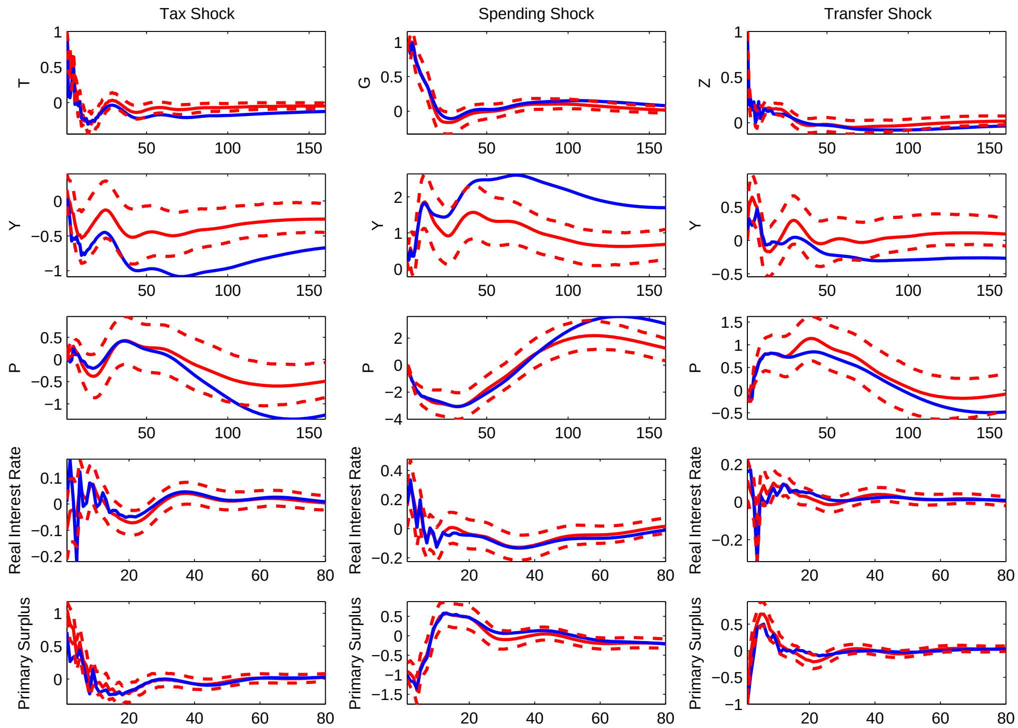

FIGURE 6. Responses to fiscal shocks with and without present-value constraint imposed. Grey solid line and dotted 68 percent error band: VAR includes taxes, government spending, transfers, output, price level, three-month Treasury bill rate, 10-year Treasury bond yield, monetary base, government debt and investment. Darker solid line: VAR system with intertemporal government budget constraint imposed.

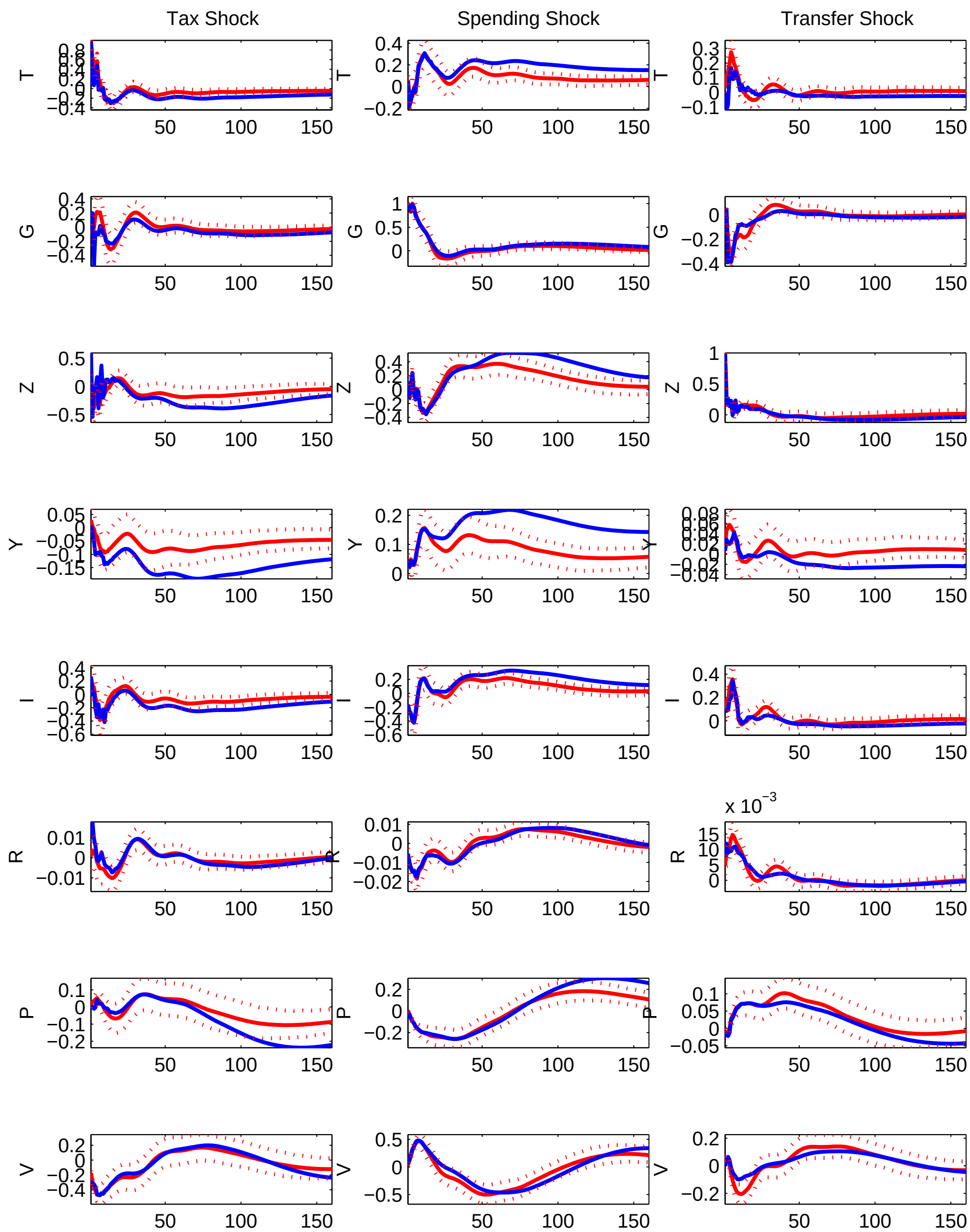

FIGURE 7. Full system responses to fiscal shocks. Solid line and dotted 68 percent error band: VAR includes taxes, government spending, transfers, output, price level, three-month Treasury bill rate, 10-year Treasury bond yield, monetary base, government debt and investment. Dashed line: VAR system with intertemporal government budget constraint imposed.

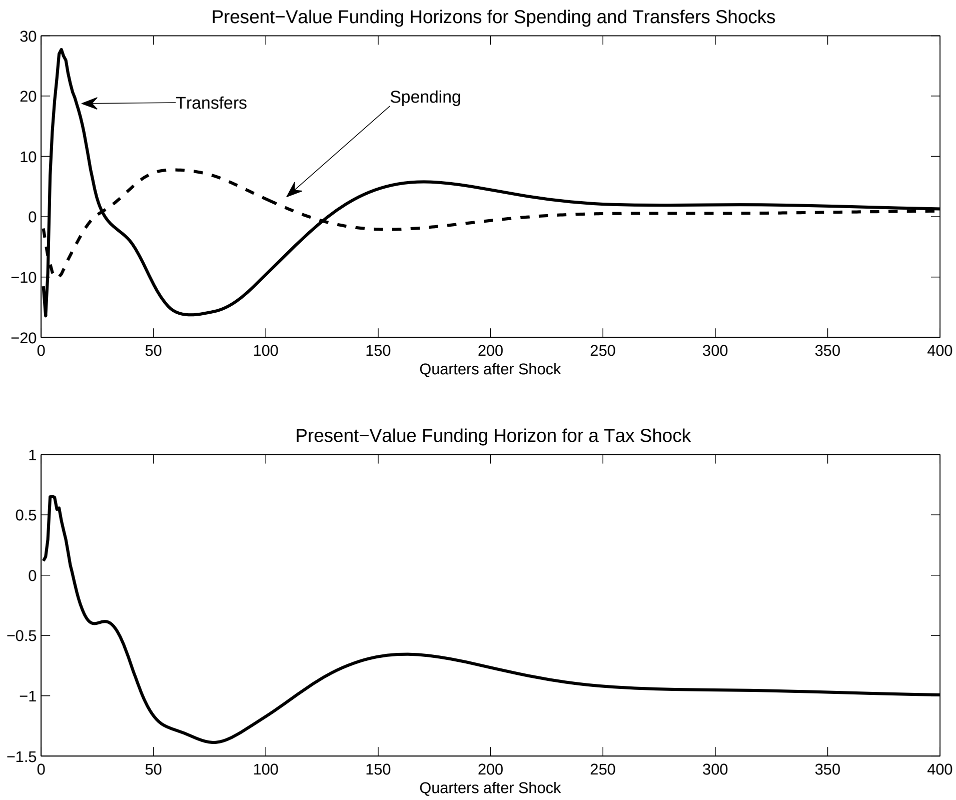

FIGURE 8. Present-Value Budget Balance Horizon. Top panel: Dashed line: truncated present-value following a spending shock; solid line: truncated present-value following transfer shock. Bottom panel: truncated present-value following a policy shock to tax revenue.

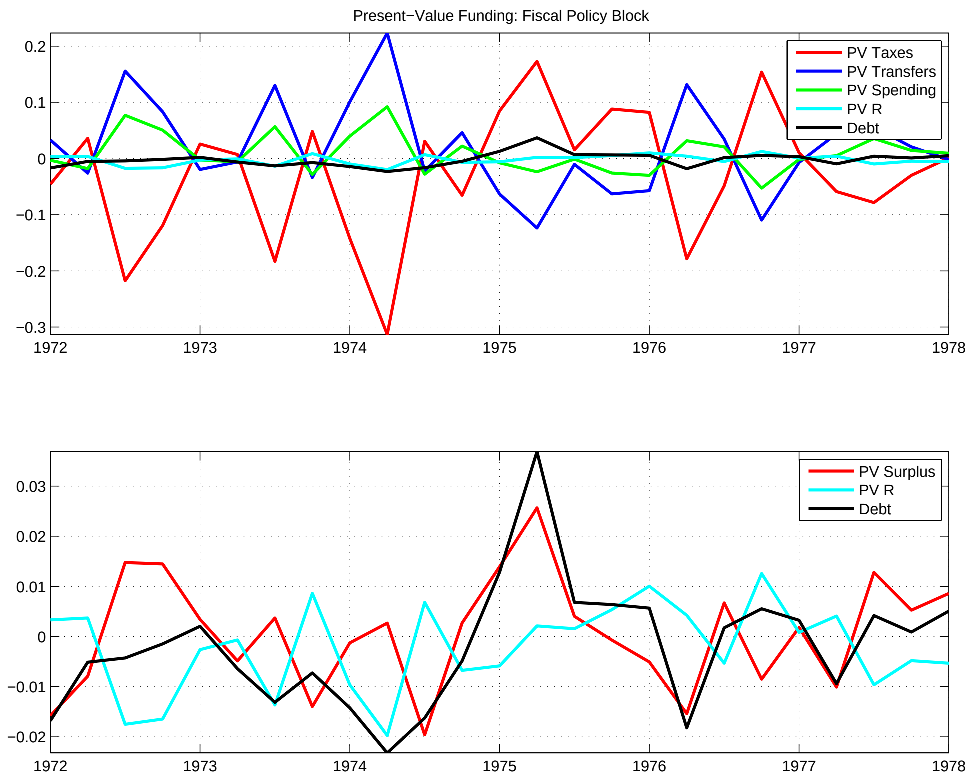

FIGURE 9. Present-value financing of innovations to debt (1972-1978). Top panel: present-value contributions broken down by policy instrument. Bottom panel: present-value components broken down into total surplus contribution and total discount rate contribution. Black line in both panels: debt innovation.

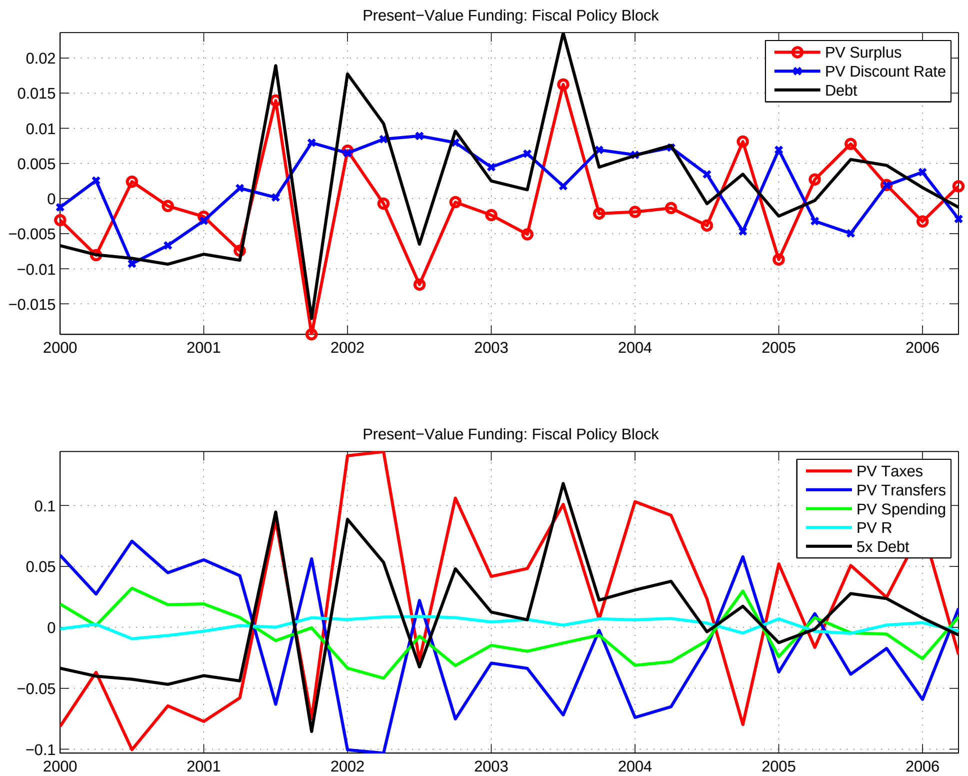

FIGURE 10. Present-value financing of recent fiscal policy shocks. Top panel: present-value components broken down into total surplus contribution and total discount rate contribution. Bottom panel: present-value contributions broken down by policy instrument. Black line in both panels: debt innovation.

Appendix A. Derivation of the Forward-Looking Part of Baseline Model Residuals

Let the baseline dynamical system be given by $f_t = f_{t-1}A + \epsilon_t B$ and $\Gamma \equiv A - CD^{-1}B$. Γ has QZ decomposition $A = Q'_A K_A Z'_A$ where $Q_A Z_A = V_A$. Define $\Lambda \equiv K_A V_A^{-1}$. Then $f_t Q' = f_{t-1} Q' \Lambda + y_t D^{-1} B$. Let the explosive modes be denoted by w_t^+ and stable modes by w_t^0 with a corresponding partition for Q_A, Z_A and Λ. By convention, the explosive modes are in the right lower corners. Solving the explosive modes forward and the stable modes backward yields

$$w_t^+ = -\sum_{s=1}^{\infty} y_{t+s} D^{-1} B Z_A^+ (\Lambda^+)^{-s} \tag{34}$$

$$w_t^0 = -\sum_{s=1}^{\infty} y_{t+s} D^{-1} B Z_A^0 (\Lambda^0)^{s} \tag{35}$$

Using the law of motion for f_t, it follows that

$$\epsilon_t B = \left(w_t^0, w_t^+\right) Q_A (\mathbf{I} - A\mathbf{L}) \tag{36}$$

Suppose now that one is interested in the representation of residuals from a finite order VAR. Without loss of generality in this case, we can take the dynamics to be given by $y_t = \sum_{j=0}^{\infty} u_{t-k} G^k$, where u_t is the residual. As before, let G have QZ decomposition $G = Q'_G K_G Z'_G$ with $Q_G Z_G = V_G$. Then it is possible to write equation (36) as

$$\epsilon_t B = \left(\sum_{s=1}^{\infty} \sum_{k=0}^{\infty} u_{t-k+s} Q'_G (K_G V_G^{-1})^k Q_G D^{-1} B Z_A^+ (\Lambda^+)^{-s}, w_t^0 \right) Q_A (\mathbf{I} - A\mathbf{L}) \tag{37}$$

The infinite geometric sums over k and s can be simplified using the fact that $\sum_{s=0}^{\infty} \sum_{k=0}^{\infty} A_1^s A_0 A_2^k = A_1 \sum_{s=0}^{\infty} \sum_{k=0}^{\infty} A_1^s A_0 A_2^k A_2 + A_1^s A_0 A_2^k$ for arbitrary matrices A_0,A_1, and A_2, and hence that $vec\left(\sum_{s=0}^{\infty} \sum_{k=0}^{\infty} A_1^s A_0 A_2^k A_2\right) = (\mathbf{I} - A'_2 \otimes A_1)^{-1} vec(A_0)$. Applying this equation to our case,

$$\epsilon_t B = \\ \sum_{s=1}^{\infty} u_{t+s} Q'_G vec^{-1} \left((\mathbf{I} - (\Lambda^+)' \otimes (K_G V_G^{-1}))^{-1} vec(Q_G D^{-1} B Z_A^+) \right) (\Lambda_+^{-1})^{-s} Q_A^+ (\mathbf{I} - A\mathbf{L}) \\ + J_t \tag{38}$$

where J_t depends only on the history of u up to date t. Finally, therefore, the "anticipating" part of ϵ_t must given by

$$\begin{aligned} \epsilon_t = \textstyle\sum_{s=1}^{\infty} u_{t+s} Q'_G vec^{-1} \left((\mathbf{I} - (\Lambda^+)' \otimes (K_G V_G^{-1}))^{-1} vec(Q_G D^{-1} B Z_A^+) \right) \\ (\Lambda_+^{-1})^{-s} Q_A^+ (\mathbf{I} - A\mathbf{L}) (B * B')^{-1} + \dots \end{aligned} \tag{39}$$

A.1. **Calculating the forward looking coefficients.** From equation (), given the encompassing system's dynamics (matrices A and B), and a matrix G which represents the projected system, one can calculate explicitly the coefficients in the forward part of the expansion. In our case, the matrices A and G are obtained by simple OLS estimates of the relevant VARs. The matrix B, which represents the encompassing system's covariance matrix, is chosen so that BP is invertible. This is done by choosing B equal to K eigenvectors of the covariance matrix with the largest eigenvalues, where K is the dimension of smaller system's covariance matrix, i.e., we approximate the encompassing system's shocks with its first K principal components. Finally, as we are interested in the loadings of fiscal policy shocks on "fscal policy shocks" identified in the smaller system, we hit both sides of () with a matrix P_{BP}, where P_{BP} is a matrix of Blanchard-Perotti elasticities for the larger system. This operation produces

$$\begin{aligned}\epsilon_t^{FP} = \textstyle\sum_{s=1}^{\infty} u_{t+s} Q_G' vec^{-1}\left((\mathbf{I} - (\Lambda^{+})' \otimes (K_G V_G^{-1}))^{-1} vec(Q_G D^{-1} B Z_A^{+})\right) \\ (\Lambda_{+}^{-1})^{-s} Q_A^{+}(\mathbf{I} - A\mathbf{L})(B * B')^{-1} P_{BP} + \ldots\end{aligned} \tag{40}$$

In turn, it is possible to express the small system's residuals u_t as $u_t = u_t^{FP}\gamma + u_t^{Y}$ where u_t^{FP} is the "identified" fiscal policy shock from the small model and u_t^{Y} is the orthogonal (with respect to the history of the u_t) component. (The projection coefficient γ is easily calculated from the $P_B P$ matrix for the smaller system and that system's covariance matrix.) Finally, in order to give some better sense of the scale of the coefficients, the fiscal policy shocks are both weighted by their standard deviations. That is, let S_ϵ be a diagonal matrix consisting of the inverse standard deviations of each of the ϵ^{FP} and let S_u be the corresponding diagonal matrix for the fiscal policy component of the u_t. Then we report the coefficients in the expansion

$$\begin{aligned}\epsilon_t^{FP} S_\epsilon = \textstyle\sum_{s=1}^{\infty} u_{t+s}^{FP} \gamma Q_G' vec^{-1}\left((\mathbf{I} - (\Lambda^{+})' \otimes (K_G V_G^{-1}))^{-1} vec(Q_G D^{-1} B Z_A^{+})\right) \\ (\Lambda_{+}^{-1})^{-s} Q_A^{+}(\mathbf{I} - A\mathbf{L})(B * B')^{-1} P_{BP} S_\epsilon + \ldots\end{aligned} \tag{41}$$

where "…" refers to terms not of interest.