Demographics and Asset Returns in Japan

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Motivation

The causal relation between the aging of Japanese society and the realized equity premium

Demographic variables predict expected excess returns in theory, is there any quantitatively large predictable variation in practice?

Analysis

Test the forecasting power of regression based on principal components from demographic variables by

1) obtaining the $\overline{R}^2$, Theil ratio and the correlation between the forecast and actual returns

2) deriving the optimal asset allocation based on principal components from demographic variables and comparing the mean excess returns and Sharpe ratios from this asset allocation with those from simple buy and hold strategy.

Conclusion

Principal components from demographic variables do predict future excess stock and bond returns in both pre-war and post-war period, especially for bond excess returns in that

1) demographic model which forecast excess stock and bond returns from principal components of demographic variables (log population between 0 and 9, 10 and 19, 20 and 29, e.t.c.) has a Theil ratio of less than 1 and

2)the mean excess return and the Sharpe ratio of active trading strategy which forecasts excess stock and bond returns from principal components of the demographic variables is higher than those from a simple buy and hold strategy of 100% stock or 100% bond.

Literature Survey

1. Age and Asset Returns

Theoretical Evidence

Bakshi and Chen (1994): “life-cycle risk aversion hypothesis”

Risk aversion is linear in average age

>pricing kernel =f (aggregate consumption, average age)

expected excess return = f (aggregate consumption, average age)

Empirical Evidence

Poterba (2001): risk aversion = f (age)

>average age is not a good test statistic to explain returns

2. Demography and Asset Returns

Theoretical Evidence

Partial Equilibrium

Jagannathan and Kocherlakota (1996): Investors whose labor income is uncorrelated with stock return should buy stocks.

Campbell and Viceira (2002): lifecycle asset allocation with variable labor supply

General Equilibrium

Donaldson and Maddaloni (2002): risk premium is inversely related to population growth rate

Storresletten et al. (2001): hump shaped risky asset demand with variable labor supply

Empirical Evidence

Ang and Maddaloni (2003): long term predictability between excess returns and demographic variables (fraction of people over 65 years old, percentage of people in the age class 20-64, average adult age) for U.S., Japan, U.K., Germany & France

Asset return data: Global Financial Data

Population data: Japan Statistical Yearbook 1996

There was no significant relation between excess returns from correct total return data on financial assets estimated for pre-war and post-war periods using the same RHS variables.

We could not find any demographic variables such as fertility and average age that produce significant coefficient or high R^2

Ang and Maddaloni (2003): Japanese Data 1920 - 2001 (GFD)

	log change of average age of 20+	log change of fraction of 65+	log change of fraction of	long − short bond yield	Adj R^2
log (stock return - bond return)	-13.12			0.005	0.04
		-2.56*		0.004	0.04
			9.59	−0.009	0.07

Goyal (2002): U.S. Data 1926-1998 (S&P, Ibbotson Associates)

		average age of 25+	fraction of 65+	fraction of 25-44	fraction of 45-64	Adj.R^2
Log (stock return - T-bill rate)	percent change	29.5				0.15
			-2.87*	4.8*	-11.38*	0.19
	level	-0.01				0.09
			-0.37	1.31*	1.14	0.14

Poterba (2001): U.S. Data 1926 - 1999 (Ibbotson Associates)

	median age	average age of 20+	fraction of 40-64/65+	fraction of 40-64/20+	fraction of 40-64
Treasury Bill	-0.001	0	-0.002	-0.39*	-1.30*
Government	0.004	-0.006	0	-1.16*	-1.73*
Common Stock	0.014	0.003	-0.001	-0.07	1.46

Data (Annual)

Pre-war (1920-1940)
Population: Population Estimate Series
Stock Returns: Rate of Yields on Equity Shares of All Industries
Bond Returns: Corporate Bond Yields
Risk Free Returns: Interest Rate on Postal Ordinary Savings
All asset returns are from Fujino and Akiyama (1977)

Post-war (1952-2001)
Population: Population Estimate Series & Japan Statistical Yearbook
Stock Returns: VW total return on the TSE (1st section)
Bond Returns: Gov Bond Returns (10 year maturity)
Risk Free Returns: overnight call rates
All asset returns data are from Ibbotson Associates Japan

Figure 1
Excess Returns in Pre WWII Period

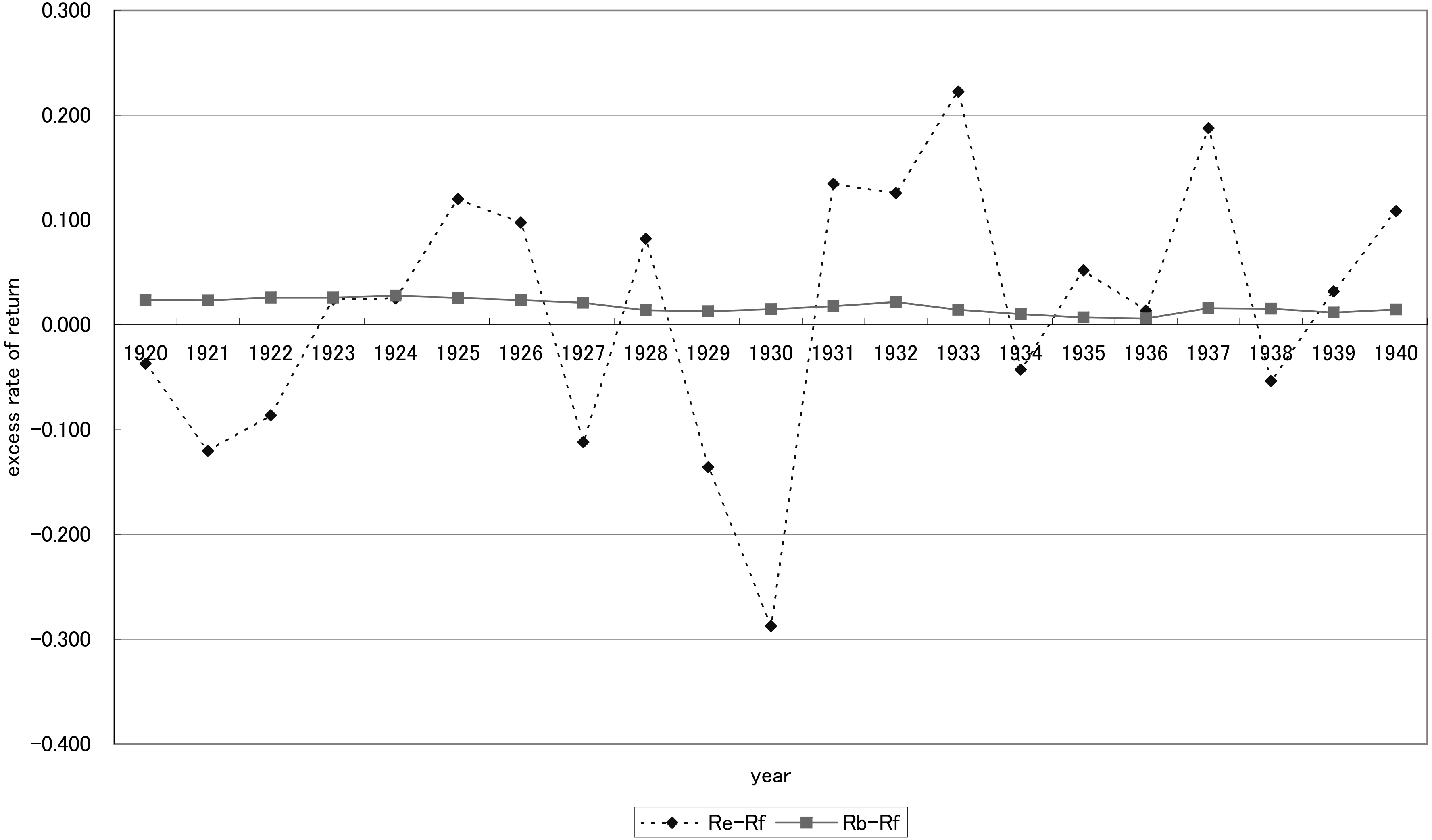
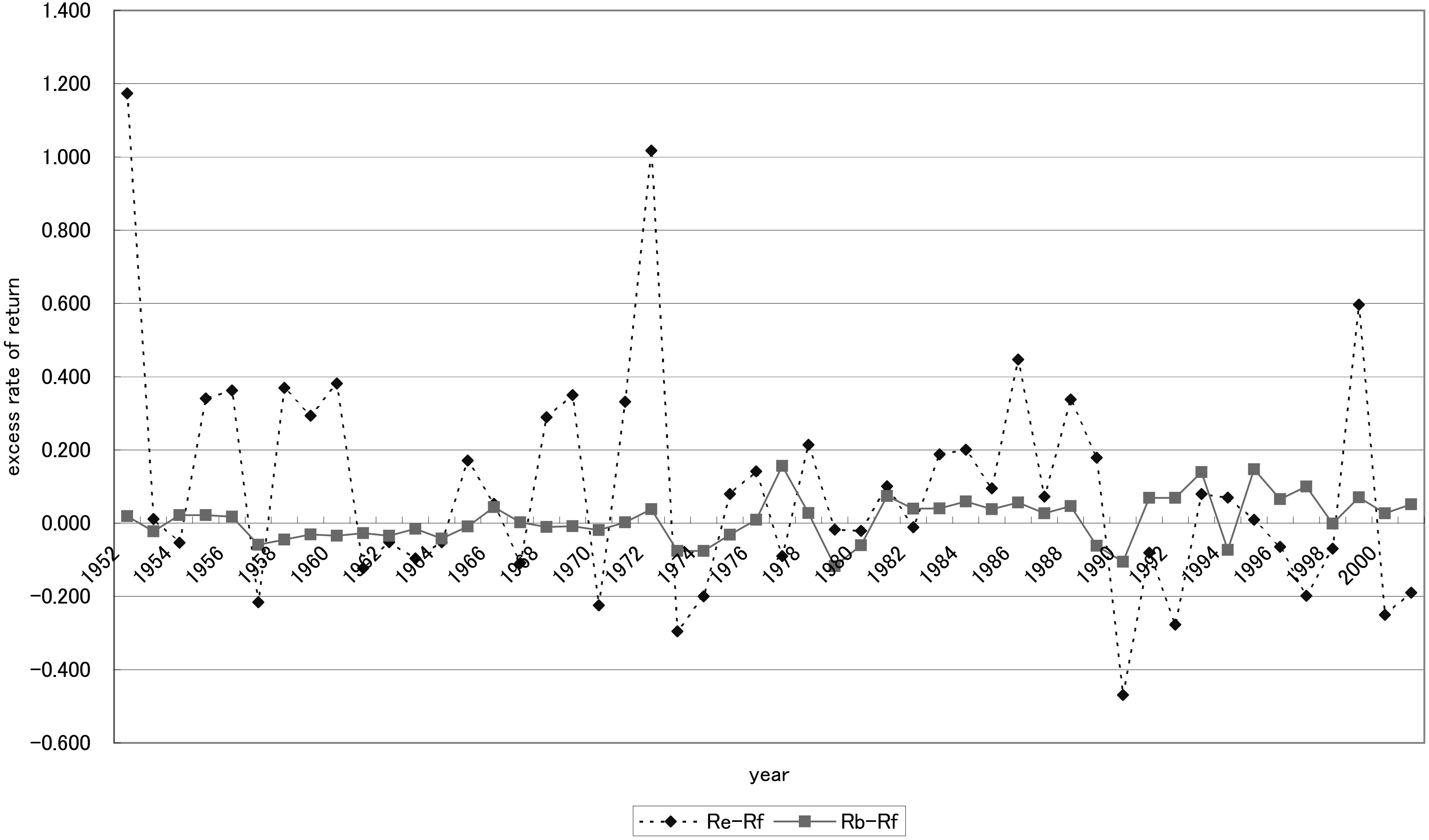
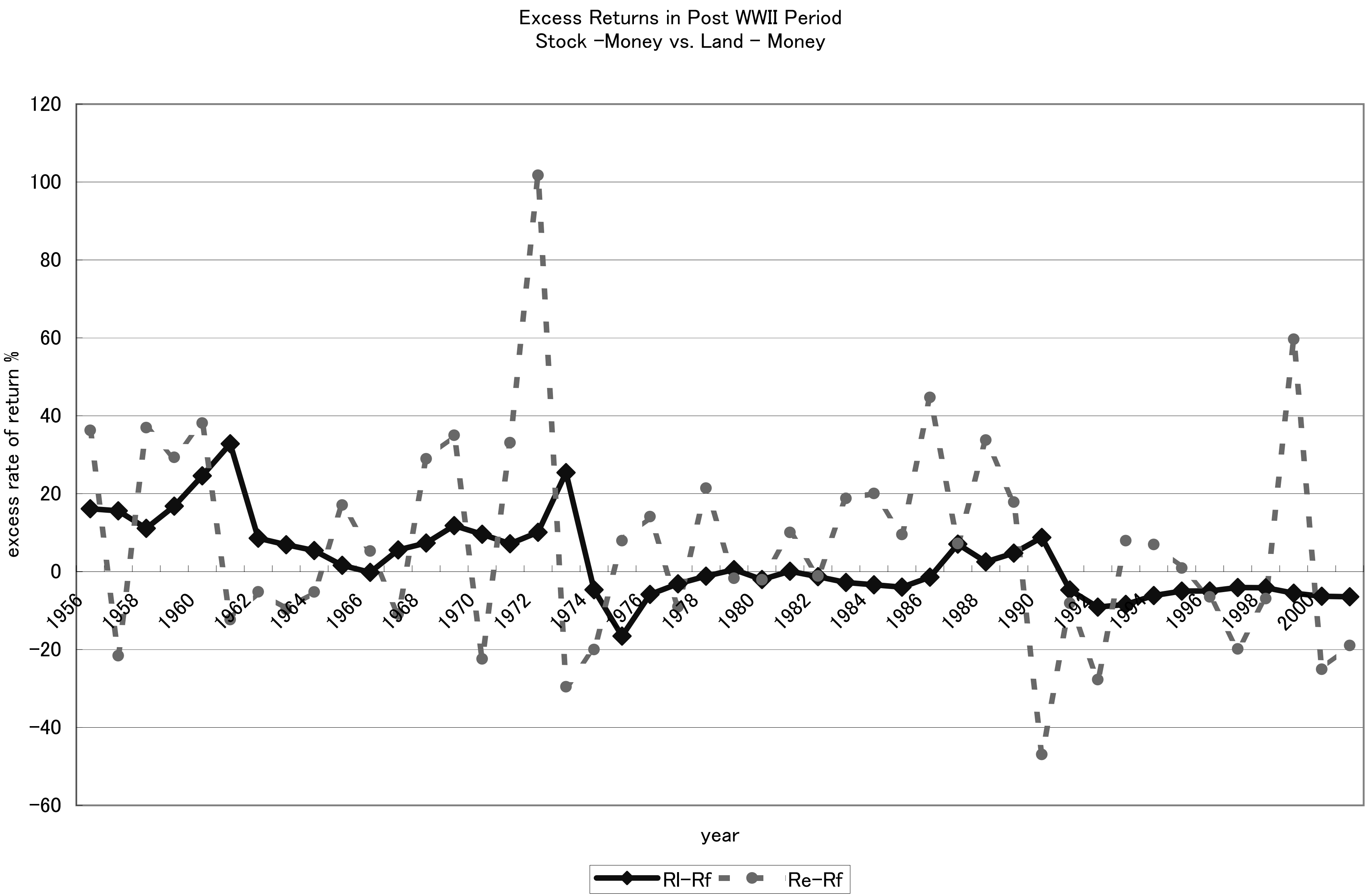


Figure 2
Excess Returns in Post WWII Period




Excess Returns in Post WWII Period
Stock -Money vs. Land - Money
120
100
80
60
40
20
0
-20
-40
-60
excess rate of return %
1956
1958
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
year
Rl-Rf
Re-Rf

Amounts of Savings and Liabilities Held per Household (All Households)

(Value in thousands of yen)

	No. of tabulated households	Yearly income	Total Savings	Financial institutions	Stocks & shares	Public & corporate bonds	Total Liabilities
1959	...	432	302	302	78	5	40
1960	...	453	359	339	84	6	69
1961	...	573	462	440	118	7	109
1962	...	583	441	419	89	8	68
1963	...	626	647	609	129	13	88
1964	...	706	689	642	115	13	116
1965	...	784	764	721	128	16	131
1966	...	882	910	852	120	19	127
1967	...	976	995	934	113	29	159
1968	4,305	1078	1126	1069	130	37	190
1969	4,847	1235	1395	1326	191	34	225
1970	5,091	1394	1603	1524	188	36	284
1971	4,313	1621	1829	1755	218	41	339
1972	4,630	1816	2150	2057	312	54	462
1973	5,173	2124	2426	2327	271	55	623
1974	5,180	2598	2704	2576	232	65	756
1975	5,185	2990	3168	3027	244	72	850
1976	5,172	3428	3768	3618	281	103	874
1977	5,231	3769	4271	4091	334	133	1078
1978	5,676	3932	4511	4328	342	118	1404
1979	6,065	4314	5212	5027	445	146	1708
1980	6,045	4643	5794	5597	429	196	1772
1981	6,051	5017	6500	6291	515	273	1758
1982	5,909	5051	6972	6772	557	319	1858
1983	5,964	5235	7263	7038	495	354	2077
1984	5,981	5297	7697	7458	602	315	2404
1985	5,974	5557	8528	8265	708	382	2721
1986	5,893	5710	9095	8822	902	351	2843
1987	5,695	5923	10452	10170	1441	377	3113
1988	5,732	6075	11198	10880	1434	366	3096
1989	5,734	6413	13110	12760	2335	380	3742
1990	5,627	6773	13530	13212	1829	360	3592
1991	5,701	7189	14654	14289	1594	346	3753
1992	5,395	7505	15368	14914	1331	359	3926
1993	5,449	7510	14982	14539	1140	316	3998
1994	5,548	7552	15921	15455	1145	377	4391
1995	5,481	7618	16035	15620	991	320	4599
1996	5,496	7545	16553	16127	917	340	5100
1997	5,350	7548	16345	15952	898	331	4985
1998	5,419	7584	16607	16134	853	268	5347
1999	5,458	7550	17377	16947	1071	339	5773
2000	5,466	7213	17812	17414	915	309	5382

Percentages of Household Holding Savings and Liabilities (All Households)

(Holding percentage in percent)

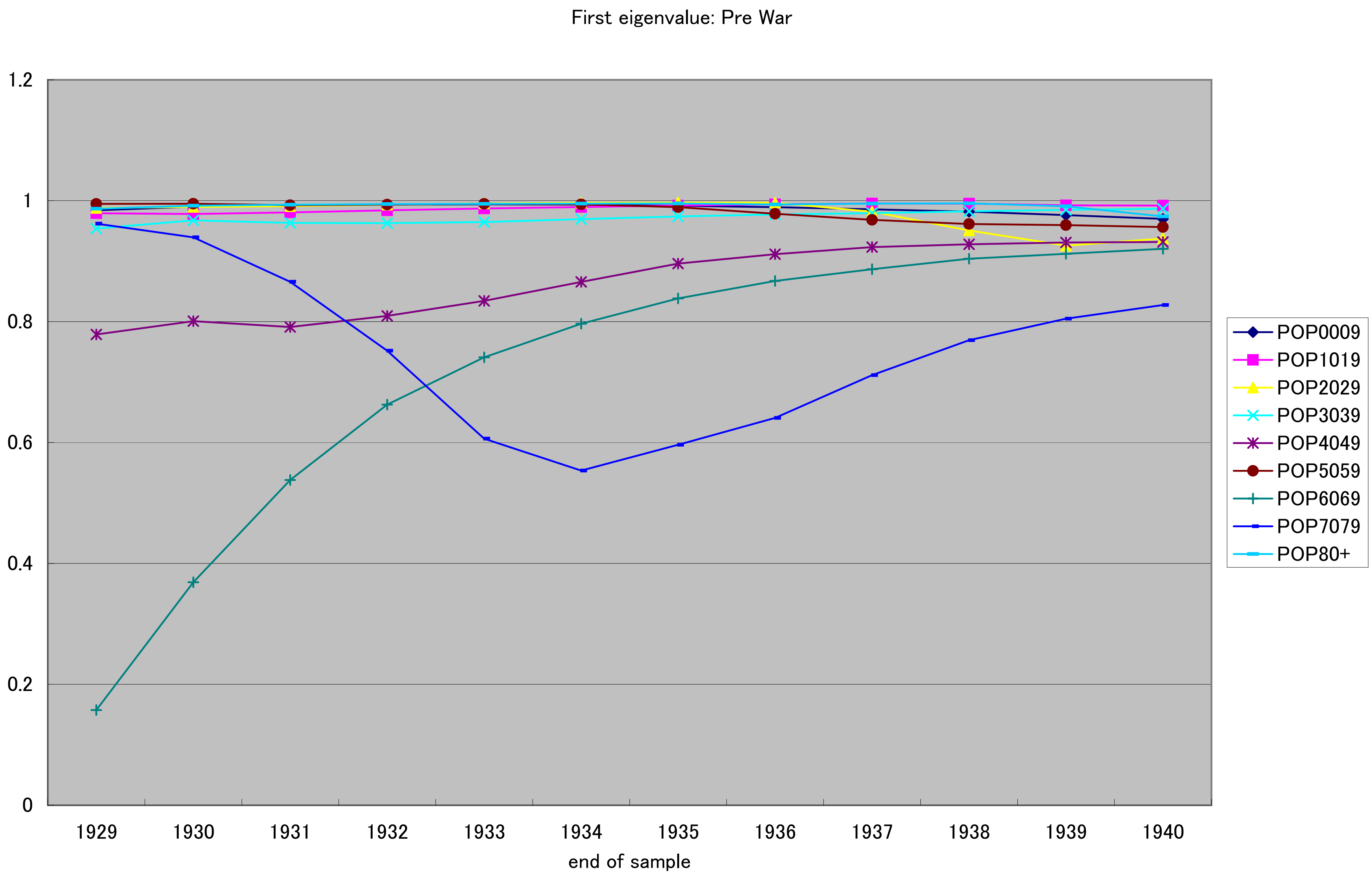
	Yearly income	Savings	Demand deposits	Time deposits	Stocks & shares	Public & corporate bonds	Liabilities 4)
1959	...	92.2	69.6	56.5	15.9	5.1	20.2
1960	...	93.5	68.4	58.6	18.0	5.6	27.7
1961	...	94.8	69.9	56.8	19.8	5.6	31.0
1962	598.0	94.3	70.9	57.6	19.8	5.6	32.5
1963	638.4	98.8	80.1	62.3	20.5	8.2	39.2
1964	719.9	99.6	80.8	65.3	22.0	8.5	34.7
1965	799.3	97.5	79.5	65.9	19.9	8.8	35.6
1966	891.8	97.5	83.7	69.4	20.7	11.4	34.8
1967	...	97.5	84.6	70.7	18.5	11.7	37.6
1968	1,095.2	99.4	87.6	74.1	18.7	12.7	38.4
1969	1,237.2	99.7	90.0	76.3	20.2	12.7	38.9
1970	1,397.7	98.9	89.5	77.4	18.7	12.1	40.6
1971	1,625.2	98.9	90.1	79.1	17.3	12.7	41.3
1972	1,818.7	98.8	90.7	79.7	16.2	11.9	43.8
1973	2,123.4	98.8	90.8	81.3	15.3	10.8	44.8
1974	2,597.7	99.0	91.7	82.1	15.3	10.5	44.4
1975	2,991	99.3	91.5	83.4	15.8	10.8	44.3
1976	3,438	99.3	92.3	85.8	16.3	9.9	42.2
1977	3,776	99.7	94.1	87.6	16.4	9.9	46.0
1978	3,932	99.7	93.9	88.5	15.6	9.1	47.4
1979	4,314	99.3	92.9	88.1	15.3	9.0	49.1
1980	4,643	99.5	92.6	89.8	16.3	9.4	49.9
1981	5,017	99.5	91.3	89.5	16.9	10.2	49.4
1982	5,051	99.5	90.8	89.8	15.6	10.3	49.0
1983	5,234	99.2	90.4	89.5	15.4	11.0	49.9
1984	5,297	99.0	89.9	89.7	15.7	10.5	50.3
1985	5,557	99.3	90.3	89.0	15.7	11.2	51.9
1986	5,710	99.2	89.2	89.7	15.5	10.7	49.0
1987	5,923	99.1	88.0	90.2	18.4	10.3	50.0
1988	6,075	99.2	89.3	90.3	18.5	10.1	48.6
1989	6,413	99.2	89.2	89.9	19.2	9.6	49.7
1990	6,773	99.5	90.8	90.6	20.5	10.2	48.2
1991	7,189	99.4	91.0	91.0	20.1	9.0	47.4
1992	7,505	99.4	90.6	89.2	19.5	8.4	46.4
1993	7,510	99.5	90.0	88.0	19.9	7.7	48.0
1994	7,552	99.3	90.5	88.3	19.3	7.8	46.7
1995	7,618	99.2	90.4	87.8	19.0	7.0	46.9
1996	7,545	99.2	90.3	87.9	17.5	6.2	45.3
1997	7,548	98.9	89.9	86.4	17.5	5.5	45.4
1998	7,584	98.9	90.6	86.8	18.5	5.9	45.7
1999	7,550	99.1	91.4	87.0	19.6	6.5	45.2
2000	7,213	98.7	91.5	86.0	18.8	6.6	43.0

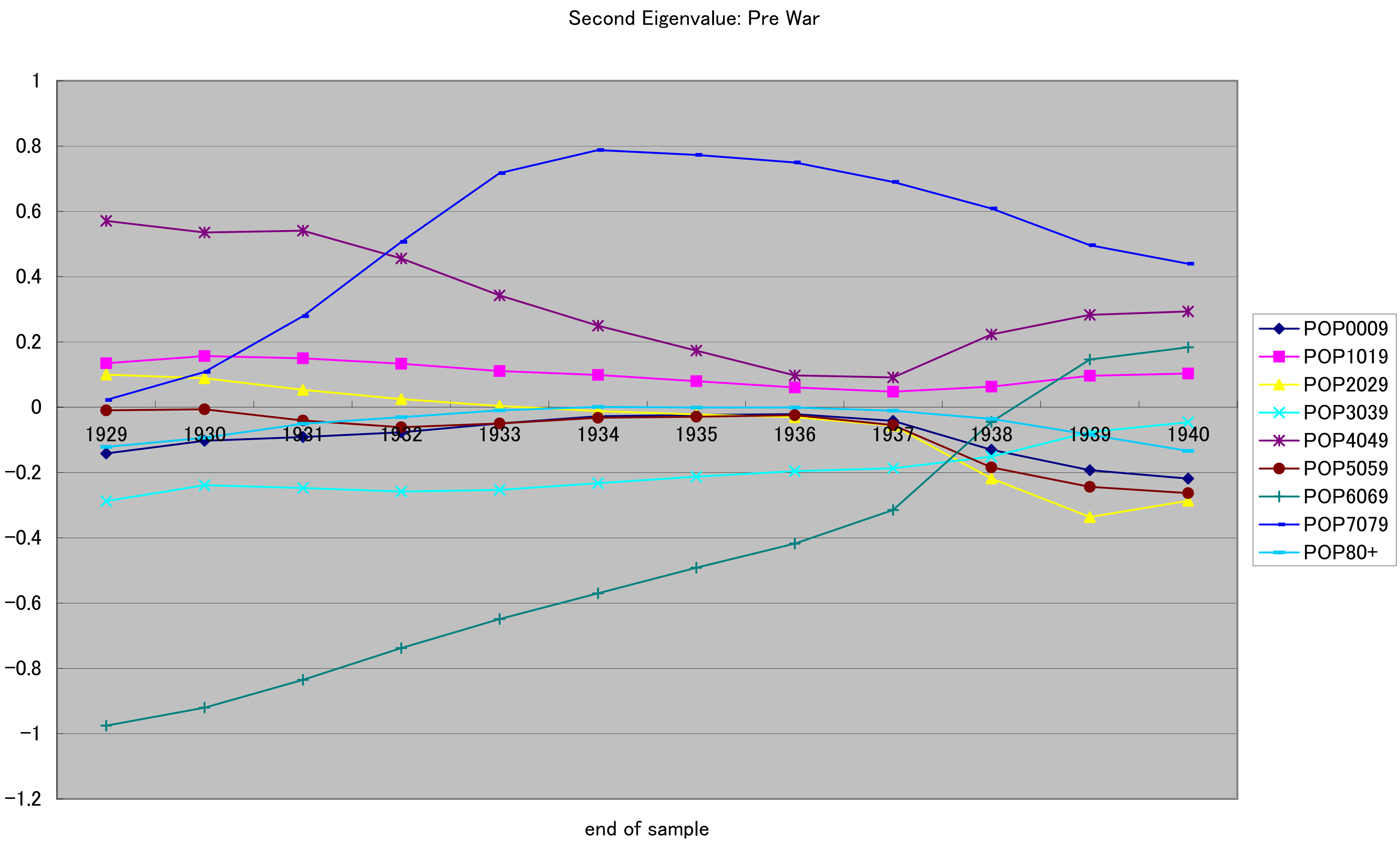
Construction of 5 principal Components

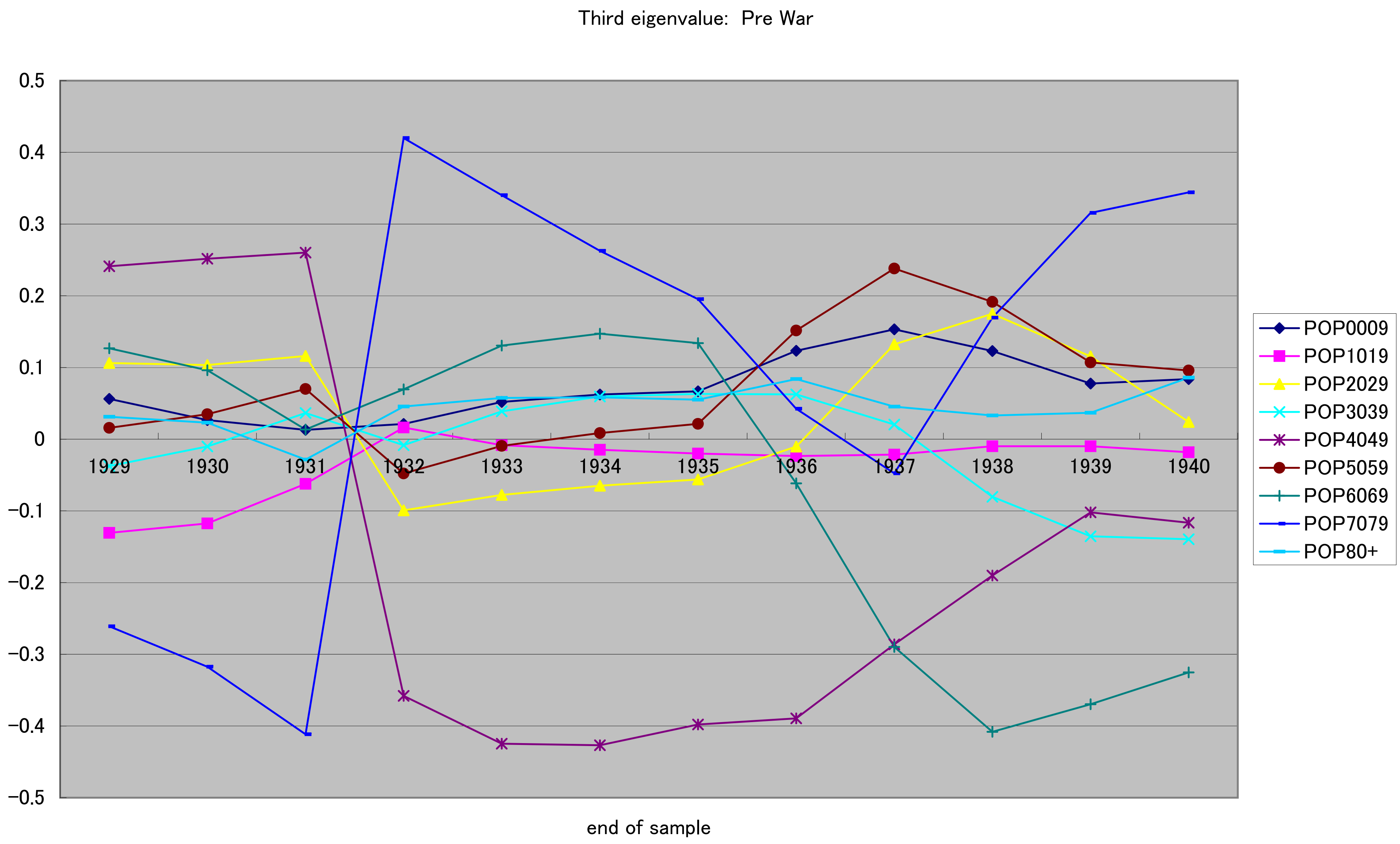
$$\underset{10x9}{\begin{bmatrix} \log(pop09_1) & \cdots & \log(pop80p_1) \\ \vdots & \ddots & \vdots \\ \log(pop09_{10}) & \cdots & \log(pop80p_{10}) \end{bmatrix}} \Rightarrow \underset{10x5}{\begin{bmatrix} PC1_1 & \cdots & PC5_1 \\ \vdots & \ddots & \vdots \\ PC1_{10} & \cdots & PC5_{10} \end{bmatrix}}$$

$$\underset{11x9}{\begin{bmatrix} \log(pop09_1) & \cdots & \log(pop80p_1) \\ \vdots & \ddots & \vdots \\ \log(pop09_{11}) & \cdots & \log(pop80p_{11}) \end{bmatrix}} \Rightarrow \underset{11x5}{\begin{bmatrix} PC1_1 & \cdots & PC5_1 \\ \vdots & \ddots & \vdots \\ PC1_{11} & \cdots & PC5_{11} \end{bmatrix}}$$

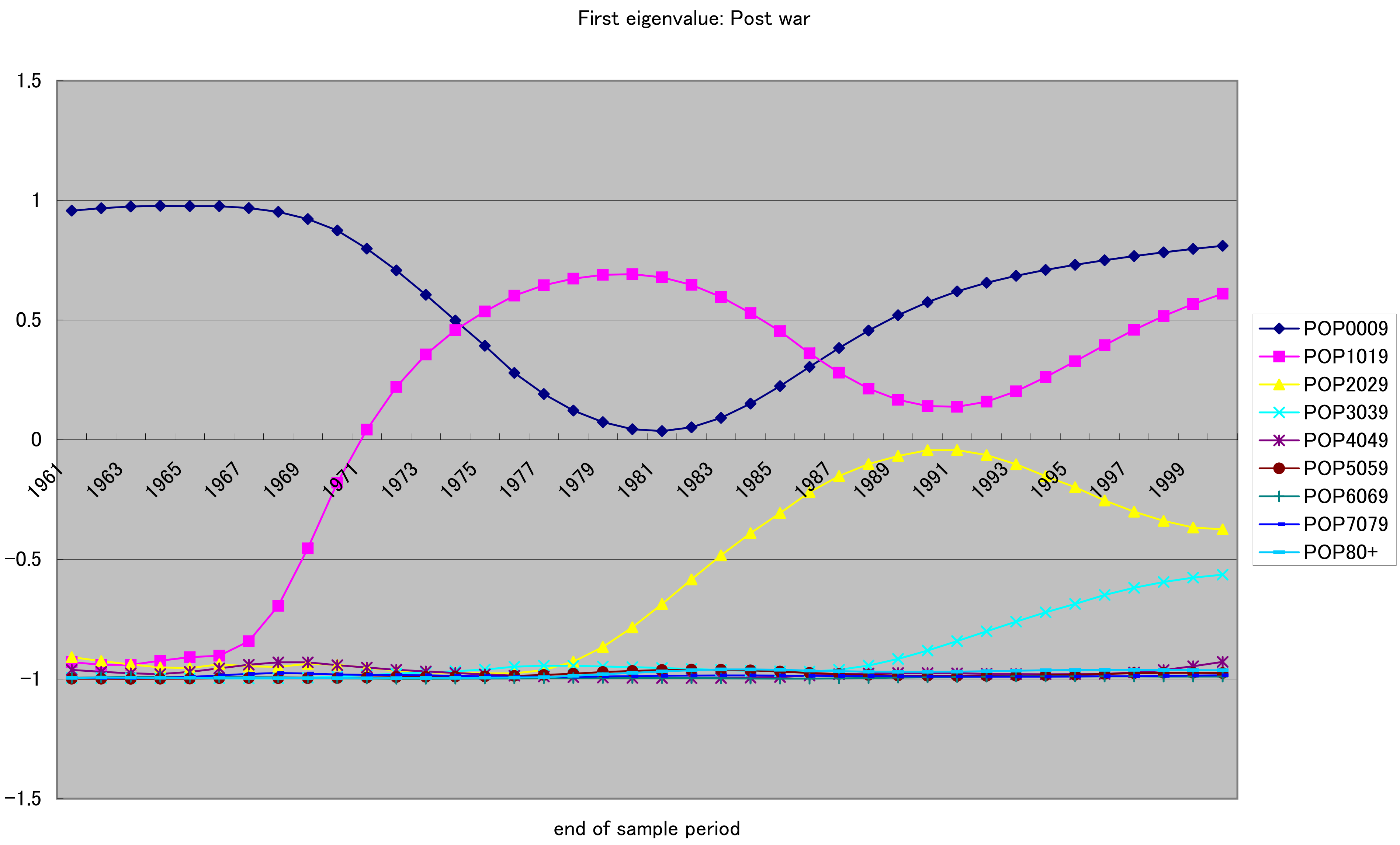
$$\underset{Tx9}{\begin{bmatrix} \log(pop09_1) & \cdots & \log(pop80p_1) \\ \vdots & \ddots & \vdots \\ \log(pop09_T) & \cdots & \log(pop80p_T) \end{bmatrix}} \Rightarrow \underset{Tx5}{\begin{bmatrix} PC1_1 & \cdots & PC5_1 \\ \vdots & \ddots & \vdots \\ PC1_T & \cdots & PC5_T \end{bmatrix}}$$

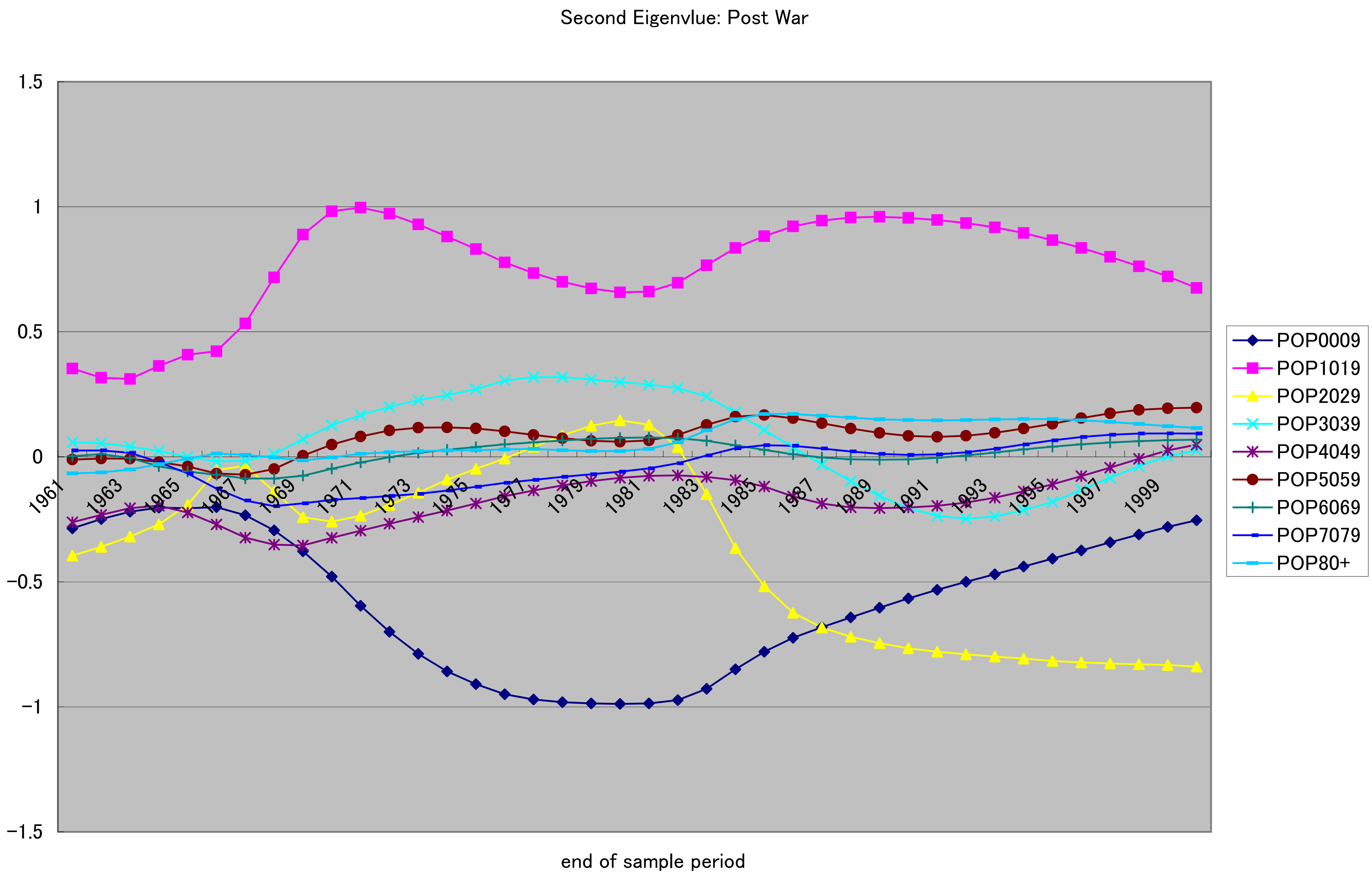

First eigenvalue: Pre War
1.2
1
0.8
0.6
0.4
0.2
0
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
1940
end of sample
POP0009
POP1019
POP2029
POP3039
POP4049
POP5059
POP6069
POP7079
POP80+

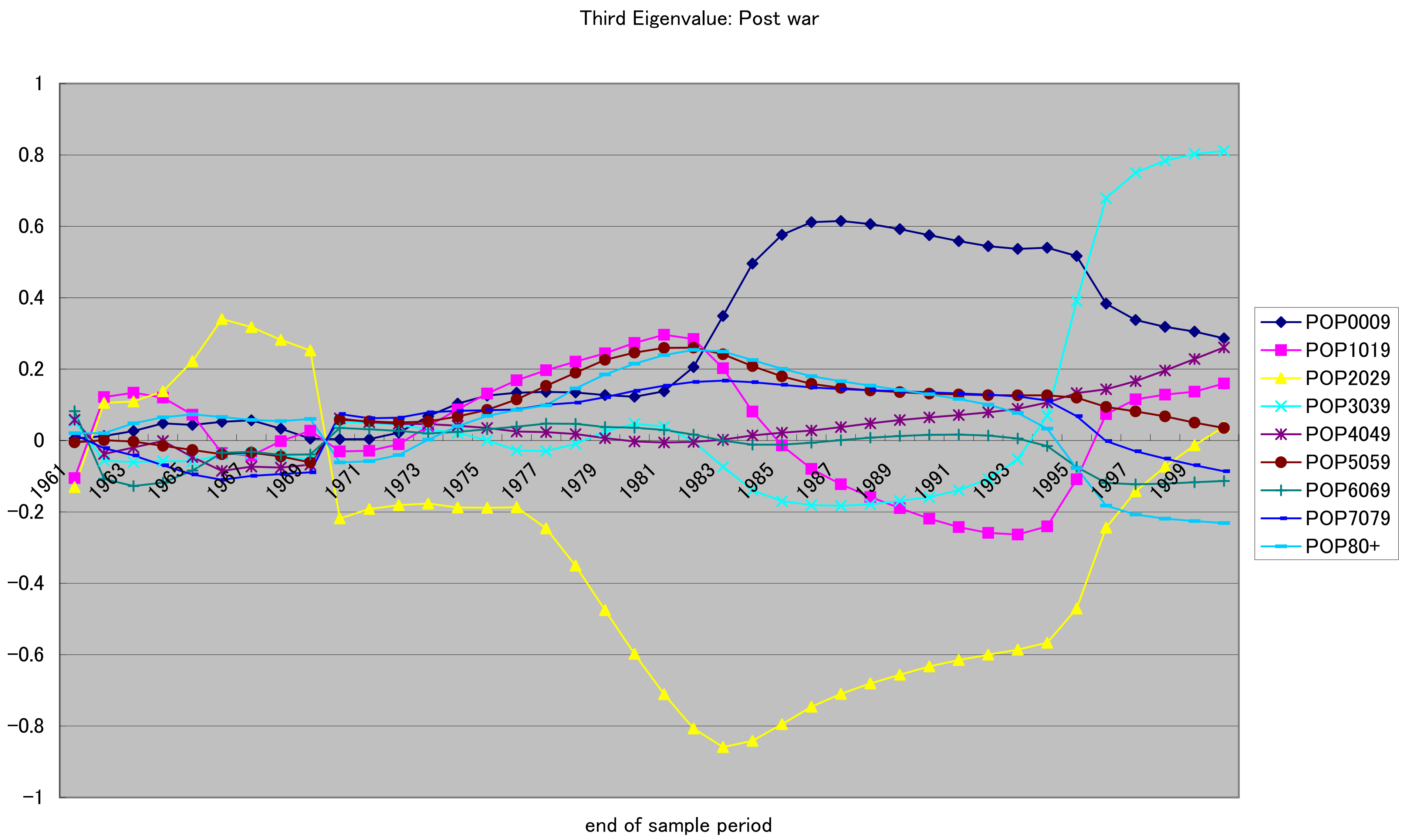

Second Eigenvalue: Pre War
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
1940
end of sample
POP0009
POP1019
POP2029
POP3039
POP4049
POP5059
POP6069
POP7079
POP80+



Third eigenvalue: Pre War
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
1940
end of sample
POP0009
POP1019
POP2029
POP3039
POP4049
POP5059
POP6069
POP7079
POP80+


First eigenvalue: Post war
1.5
1
0.5
0
−0.5
−1
−1.5
1961
1963
1965
1967
1969
1971
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
end of sample period
POP0009
POP1019
POP2029
POP3039
POP4049
POP5059
POP6069
POP7079
POP80+


Second Eigenvlue: Post War
1.5
1
0.5
0
−0.5
−1
−1.5
1961
1963
1965
1967
1969
1971
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
end of sample period
POP0009
POP1019
POP2029
POP3039
POP4049
POP5059
POP6069
POP7079
POP80+



Third Eigenvalue: Post war
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
1961
1963
1965
1967
1969
1971
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
end of sample period
POP0009
POP1019
POP2029
POP3039
POP4049
POP5059
POP6069
POP7079
POP80+

Table 1
Principal components estimation of demographic variables for population levels

Factor sensitivities				
first eigenvector	second eigenvector	third eigenvector	fourth eigenvector	fifth eigenvector
0.973	0.210	0.073	0.052	0.012
0.993	-0.090	-0.006	-0.030	0.001
0.944	0.271	0.036	-0.157	-0.075
0.985	0.051	-0.145	0.048	-0.041
0.928	-0.300	-0.111	-0.175	0.073
0.960	0.253	0.095	0.009	0.016
0.913	-0.178	-0.349	0.108	-0.032
0.829	-0.433	0.346	0.051	-0.047
0.976	0.140	0.089	0.095	0.086
0.895	0.953	0.986	0.995	0.998

Factor sensivitities				
first eigenvector	second eigenvector	third eigenvector	fourth eigenvector	fifth eigenvector
0.792	0.131	0.438	0.389	0.111
0.622	-0.636	0.207	-0.389	0.114
-0.418	0.822	0.183	-0.334	0.069
-0.686	-0.126	0.690	-0.051	-0.183
-0.944	-0.084	0.218	0.072	0.215
-0.977	-0.195	-0.025	0.069	0.002
-0.993	-0.064	-0.086	-0.007	0.043
-0.989	-0.093	-0.061	0.091	-0.006
-0.984	-0.126	-0.112	0.023	0.014
0.716	0.847	0.938	0.987	0.999

Demographic model for equity excess returns

$$\begin{bmatrix} R^e{}_2 - R^f{}_2 \\ \vdots \\ R^e{}_{11} - R^f{}_{11} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} PC1_1 \\ \vdots \\ PC1_{10} \end{bmatrix} + a_2 \begin{bmatrix} PC2_1 \\ \vdots \\ PC2_{10} \end{bmatrix} + a_3 \begin{bmatrix} PC3_1 \\ \vdots \\ PC3_{10} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{10} \end{bmatrix}$$

$$\Rightarrow (\widehat{a_0}, \cdots, \widehat{a_3}) \Rightarrow \widehat{R^e{}_{12} - R^f{}_{12}} = \widehat{a_0} + \widehat{a_1} PC1_{11} + \widehat{a_2} PC2_{11} + \widehat{a_3} PC3_{11}$$

$$\begin{bmatrix} R^e{}_2 - R^f{}_2 \\ \vdots \\ R^e{}_{12} - R^f{}_{12} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} PC1_1 \\ \vdots \\ PC1_{11} \end{bmatrix} + a_2 \begin{bmatrix} PC2_1 \\ \vdots \\ PC2_{11} \end{bmatrix} + a_3 \begin{bmatrix} PC3_1 \\ \vdots \\ PC3_{11} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{11} \end{bmatrix}$$

$$\Rightarrow (\widehat{a_0}, \cdots, \widehat{a_3}) \Rightarrow \widehat{R^e{}_{13} - R^f{}_{13}} = \widehat{a_0} + \widehat{a_1} PC1_{12} + \widehat{a_2} PC2_{12} + \widehat{a_3} PC3_{12}$$

The same procedure for bond excess returns

Financial model for excess stock returns

$$\begin{bmatrix} R^{e}_{2}-R^{f}_{2} \\ \vdots \\ R^{e}_{11}-R^{f}_{11} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} R^{e}_{1}-R^{f}_{1} \\ \vdots \\ R^{e}_{10}-R^{f}_{10} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{10} \end{bmatrix}$$

$$\Rightarrow (\widehat{a_0}, \widehat{a_1}) \Rightarrow \widehat{R^{e}_{12}-R^{f}_{12}} = \widehat{a_0} + \widehat{a_1}(R^{e}_{11}-R^{f}_{11})$$

$$\begin{bmatrix} R^{e}_{2}-R^{f}_{2} \\ \vdots \\ R^{e}_{12}-R^{f}_{12} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} R^{e}_{1}-R^{f}_{1} \\ \vdots \\ R^{e}_{11}-R^{f}_{11} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{11} \end{bmatrix}$$

$$\Rightarrow (\widehat{a_0}, \widehat{a_1}) \Rightarrow \widehat{R^{e}_{13}-R^{f}_{13}} = \widehat{a_0} + \widehat{a_1}(R^{e}_{12}-R^{f}_{12})$$

Demographic and financial model for excess stock returns:

$$\begin{bmatrix} R^e{}_2 - R^f{}_2 \\ \vdots \\ R^e{}_{11} - R^f{}_{11} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} PC1_1 \\ \vdots \\ PC1_{10} \end{bmatrix} + a_2 \begin{bmatrix} PC2_1 \\ \vdots \\ PC2_{10} \end{bmatrix} + a_3 \begin{bmatrix} PC3_1 \\ \vdots \\ PC3_{10} \end{bmatrix} + a_4 \begin{bmatrix} R^e{}_1 - R^f{}_1 \\ \vdots \\ R^e{}_{10} - R^f{}_{10} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{10} \end{bmatrix}$$

$$\Rightarrow (\widehat{a_0}, \cdots, \widehat{a_4}) \Rightarrow \widehat{R^e{}_{12} - R^f{}_{12}} = \widehat{a_0} + \widehat{a_1} PC1_{11} + \widehat{a_2} PC2_{11} + \widehat{a_3} PC3_{11} + \widehat{a_4} (R^e{}_{11} - R^f{}_{11})$$

$$\begin{bmatrix} R^e{}_2 - R^f{}_2 \\ \vdots \\ R^e{}_{12} - R^f{}_{12} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} PC1_1 \\ \vdots \\ PC1_{11} \end{bmatrix} + a_2 \begin{bmatrix} PC2_1 \\ \vdots \\ PC2_{11} \end{bmatrix} + a_3 \begin{bmatrix} PC3_1 \\ \vdots \\ PC3_{11} \end{bmatrix} + a_4 \begin{bmatrix} R^e{}_1 - R^f{}_1 \\ \vdots \\ R^e{}_{11} - R^f{}_{11} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{11} \end{bmatrix}$$

$$\Rightarrow (\widehat{a_0}, \cdots, \widehat{a_4}) \Rightarrow \widehat{R^e{}_{13} - R^f{}_{13}} = \widehat{a_0} + \widehat{a_1} PC1_{12} + \widehat{a_2} PC2_{12} + \widehat{a_3} PC3_{12} + \widehat{a_4} (R^e{}_{12} - R^f{}_{12})$$

Theil Ratio

Index to compare root mean squared error of the model in question with that of the no change model

$$RMSE = \sqrt{\frac{\sum_{i=12}^{T}\{\overline{R_i^{e} - R_i^{f}} - (R_i^{e} - R_i^{f})\}^2}{T - 12 + 1}}$$

$$RMSE_{nochange} = \sqrt{\frac{\sum_{i=12}^{T}[\{\frac{1}{i-1}\sum_{k=1}^{i-1}(R_k^{e} - R_k^{f})\} - (R_i^{e} - R_i^{f})]^2}{T - 12 + 1}}$$

Theil Ratio = $RMSE / RMSE_{no\ change}$

If Theil Ratio is less than 1, the model in question performs better than the no change model

Table 2: Demographic, financial and demographic and financial model performance

Pre-War Bonds:

1. $\overline{R}^2$ is more than 60% for all the models

2. Theil ratios for all the models are less than one

3. The correlations between the forecast and actual excess returns are more than 25% for all the models

4. Financial model performs best

Pre-War Equity:

1. $\overline{R}^2$ is negative and small for all the models

2. Theil ratios for all the models are more than one

3. The correlations between the forecast and actual excess returns are negative for all the models

4. Financial model performs best

Post-War Bonds

1. $\overline{R}^2$ **is positive but small for all the models**

2. **Theil ratios for all the models are close to one**

3. **The correlations between the forecast and actual excess returns are more about 12% for all the models**

4. **Demographic model performs best**

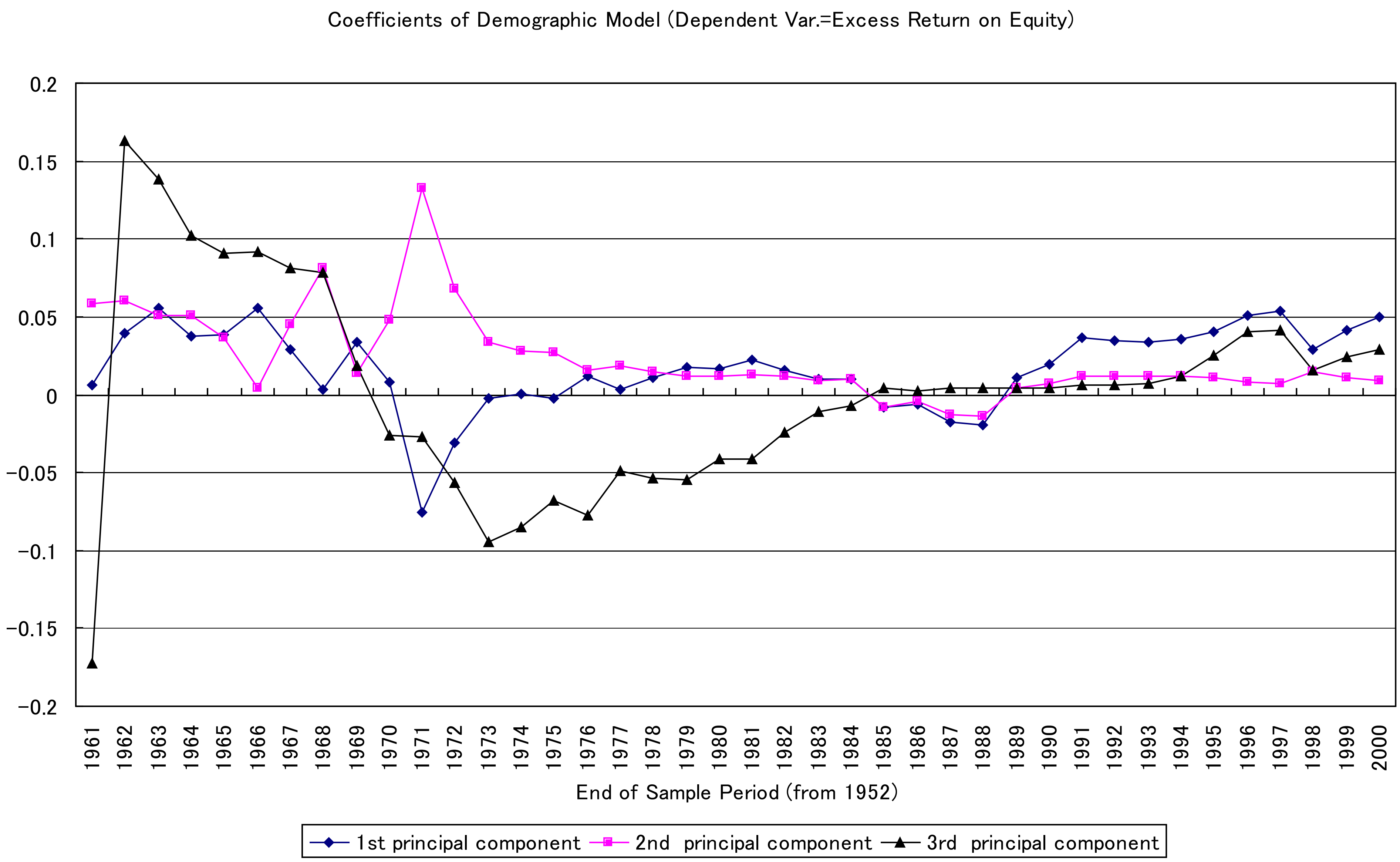
Post-War Equity

1. $\overline{R}^2$ **is negative and small for all the models**

2. **Theil ratios for all the models are close to one**

3. **The correlations between the forecast and actual excess returns are negative for all the models**

4. **Financial model performs best**



Coefficients of Demographic Model (Dependent Var.=Excess Return on Equity)
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
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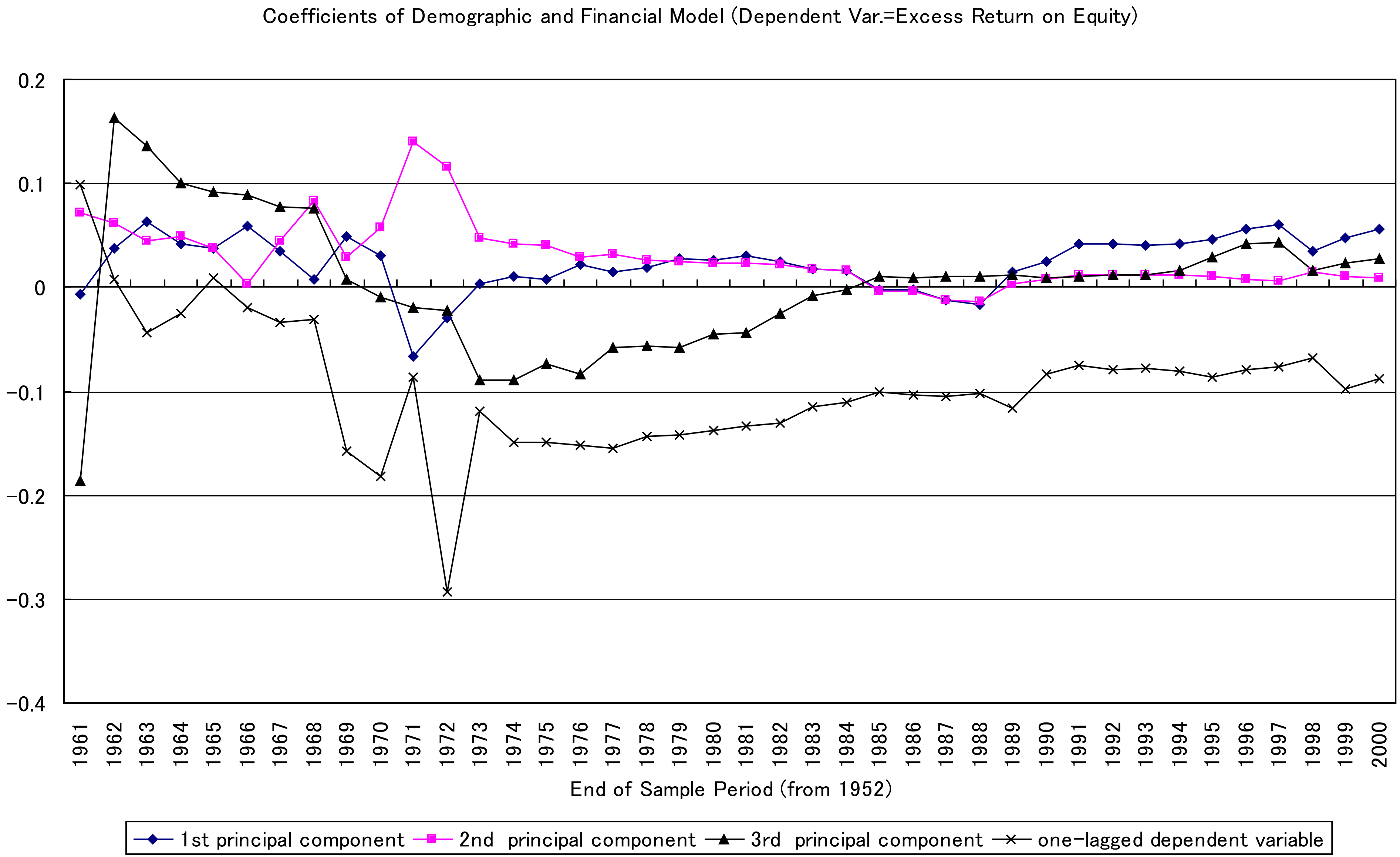
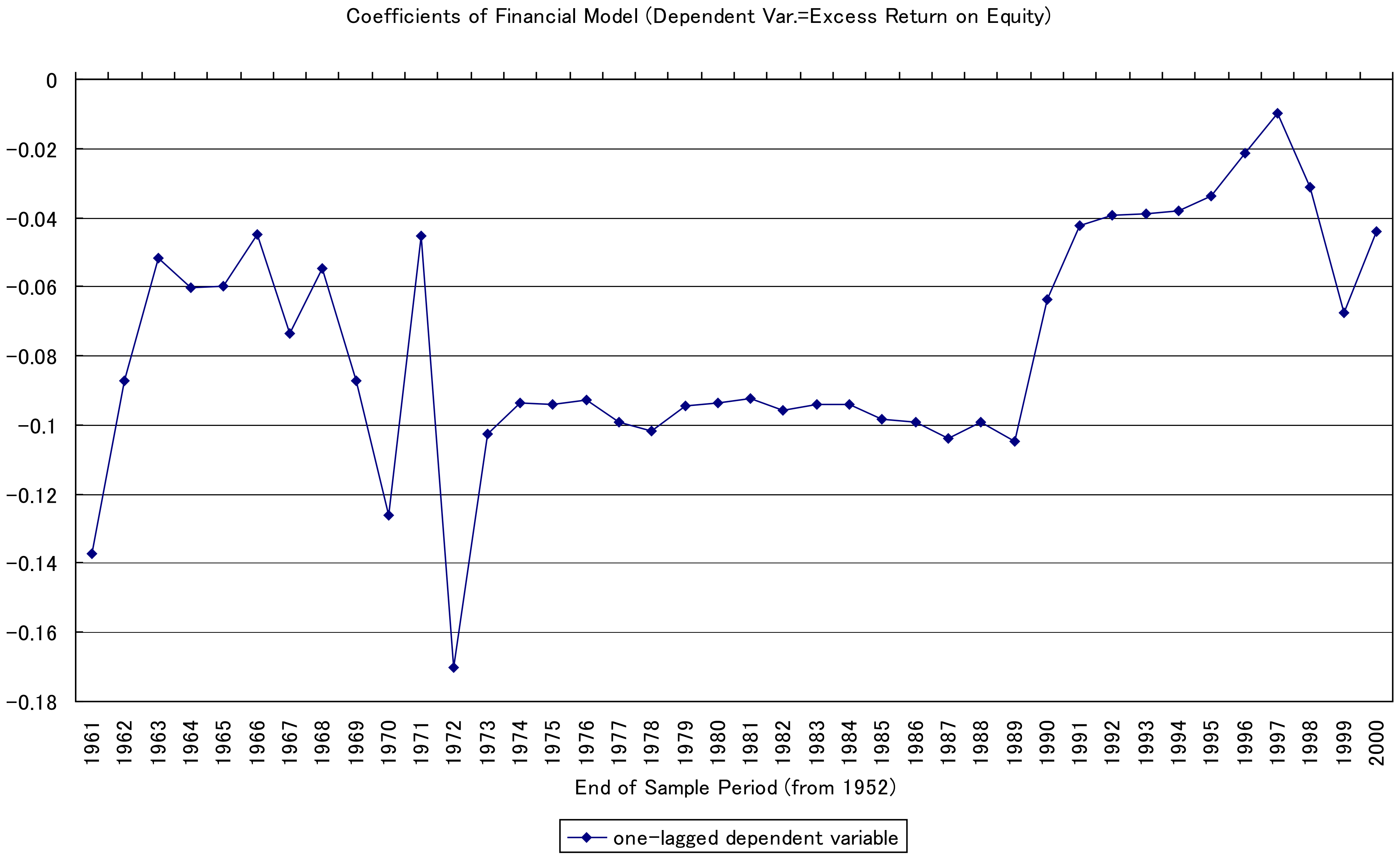
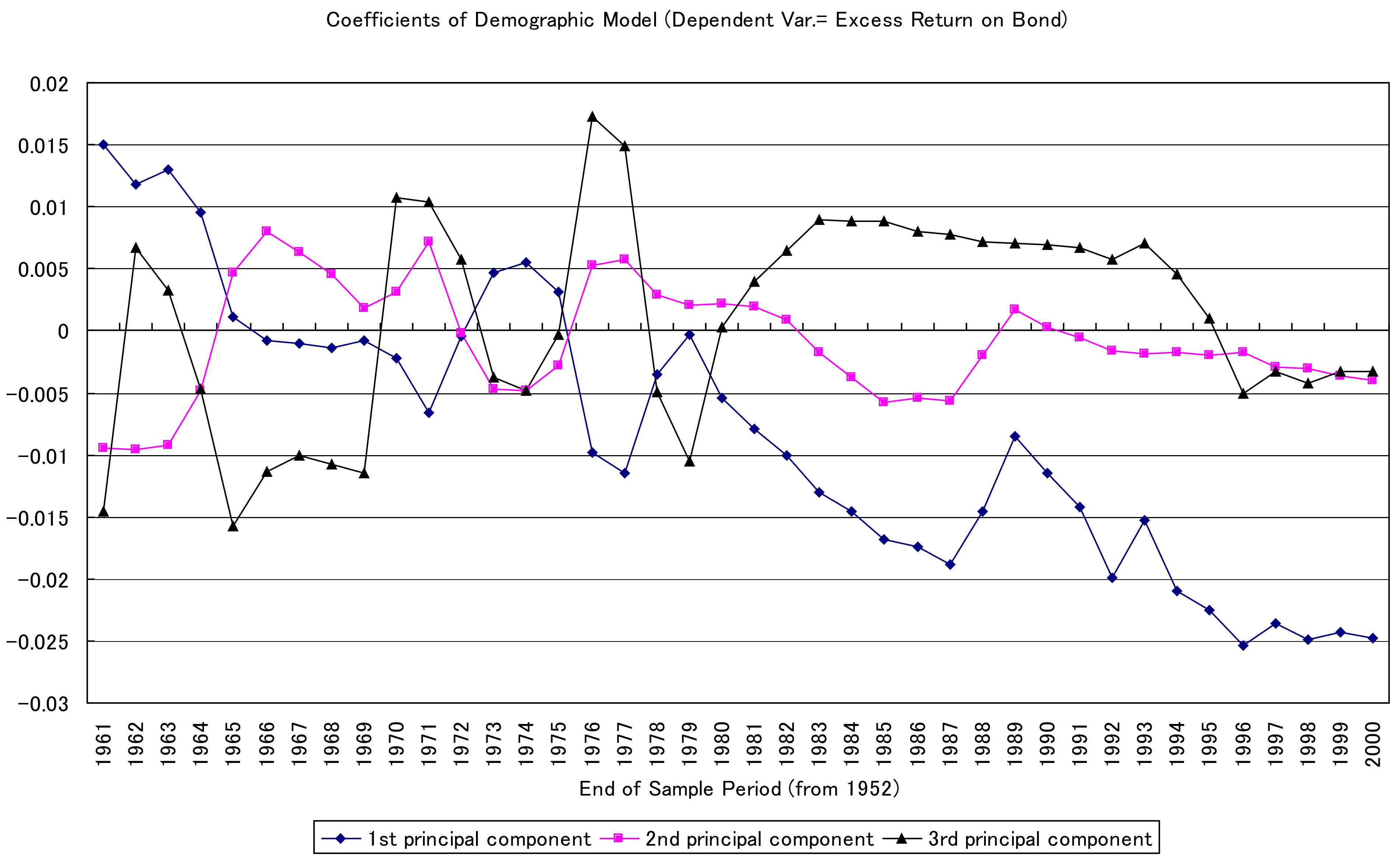
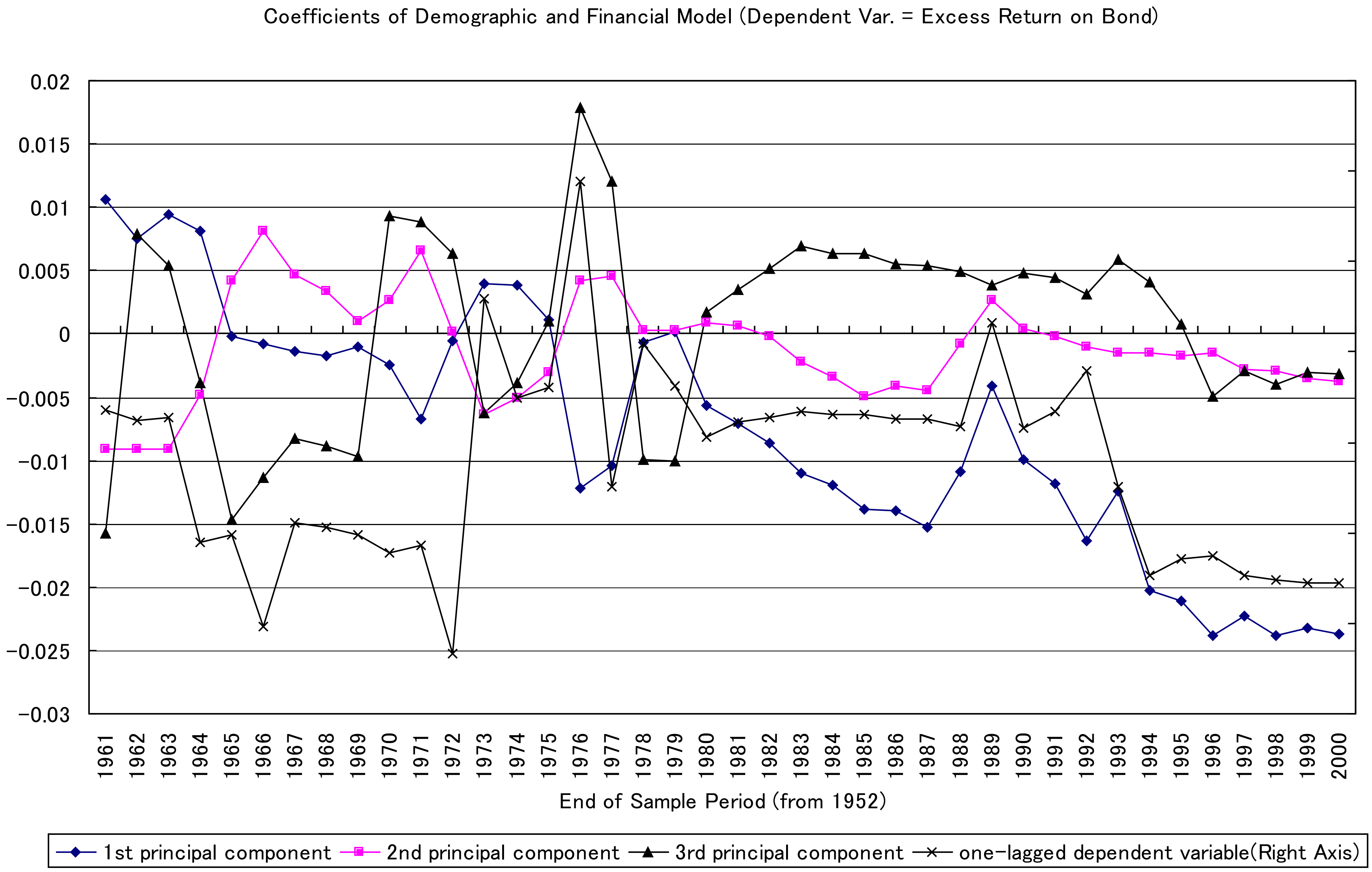
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
End of Sample Period (from 1952)
1st principal component
2nd principal component
3rd principal component

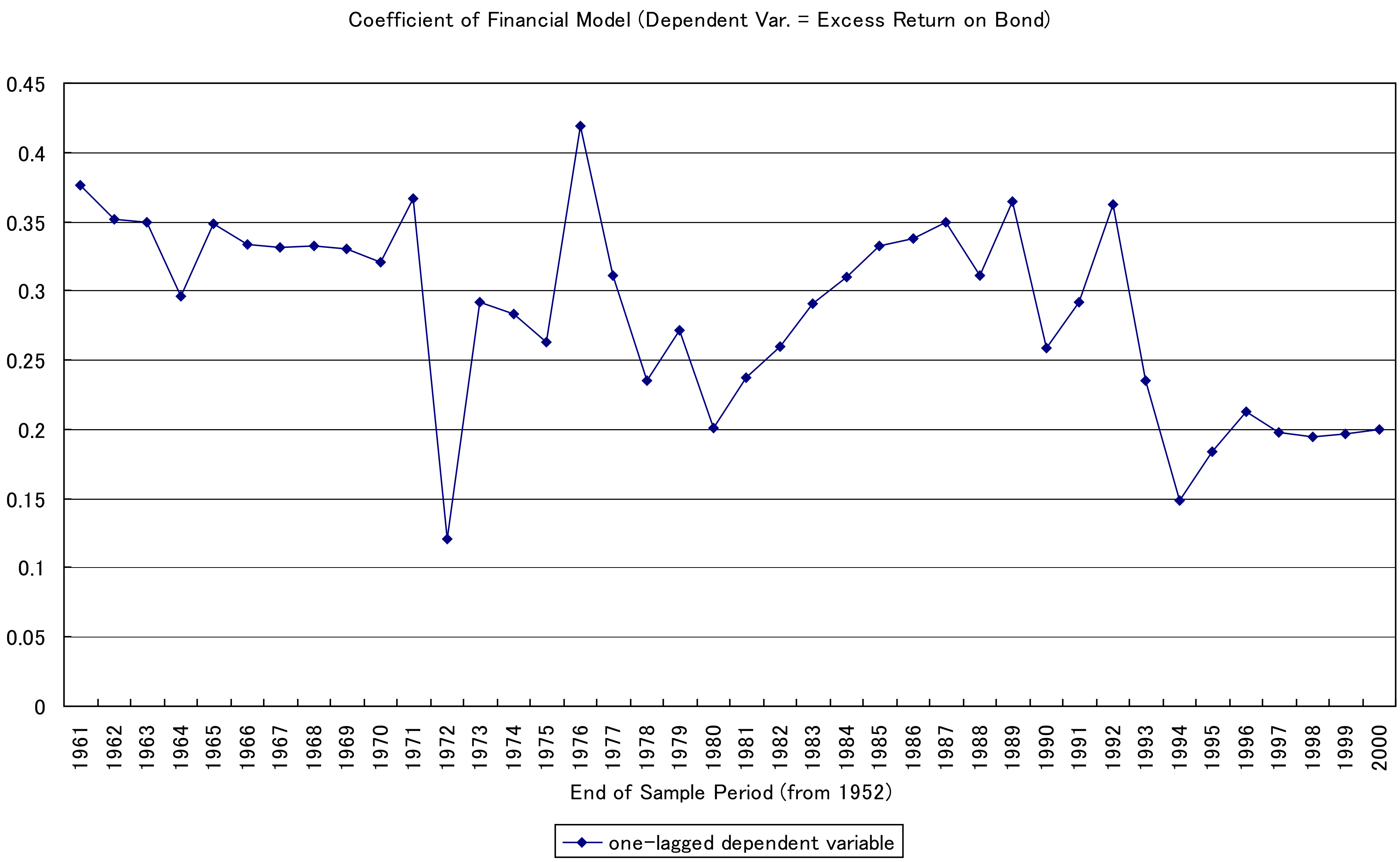

Coefficients of Demographic and Financial Model (Dependent Var.=Excess Return on Equity)
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
End of Sample Period (from 1952)
1st principal component
2nd principal component
3rd principal component
one-lagged dependent variable


Coefficients of Financial Model (Dependent Var.=Excess Return on Equity)
0
−0.02
−0.04
−0.06
−0.08
−0.1
−0.12
−0.14
−0.16
−0.18
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
End of Sample Period (from 1952)
one−lagged dependent variable



Coefficients of Demographic Model (Dependent Var.= Excess Return on Bond)
0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
−0.025
−0.03
1961 1962 1963 1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000
End of Sample Period (from 1952)
1st principal component
2nd principal component
3rd principal component


Coefficients of Demographic and Financial Model (Dependent Var. = Excess Return on Bond)
0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
−0.025
−0.03
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
1961 1962 1963 1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000
End of Sample Period (from 1952)
1st principal component
2nd principal component
3rd principal component
one-lagged dependent variable(Right Axis)


Coefficient of Financial Model (Dependent Var. = Excess Return on Bond)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
End of Sample Period (from 1952)
one-lagged dependent variable

Post-War Period

Stock – Money

		PC1	PC2	PC3	Re – Rf
Demographc Model	coefficient	0.05	0.01	0.03	
	t-value	1.30	0.23	0.75	
Demographc and Financial Model	coefficient	0.06	0.01	0.03	−0.09
	t-value	1.42	0.23	0.73	−0.67
Financial Model	coefficient				−0.04
	t-value				−0.35

Bond – Money

		PC1	PC2	PC3	Rb – Rf
Demographc Model	coefficient	−0.02	0.00	0.00	
	t-value	−2.97	−0.47	−0.40	
Demographc and Financial Model	coefficient	−0.02	0.00	0.00	0.04
	t-value	−2.61	−0.44	−0.37	0.30
Financial Model	coefficient				0.20
	t-value				1.39

Table 2

One-Step Ahead Forecast for Excess Returns from Demographic, Demographic and Financial and Financial Models

1930-1939	$\overline{R}^2$	Root Mean Squared Error	Theil Ratio	RMSE no-change	correlation between forecast and actual excess return
BONDS					
Demographic model	0.629	0.00579	0.75106	0.0077	0.258
demographic and financial model	0.681	0.00550	0.71413	0.0077	0.337
financial model	0.646	0.00495	0.64251	0.0077	0.441
EQUITY					
Demographic model	-0.082	0.15976	1.25880	0.1269	-0.635
demographic and financial model	-0.153	0.18470	1.45535	0.1269	-0.543
financial model	-0.050	0.14650	1.15431	0.1269	-0.489

1962-2001	$\overline{R}^2$	Root Mean Squared Error	Theil Ratio	RMSE no-change	correlation between forecast and actual excess return
BONDS					
Demographic model	0.115	0.06724	0.99818	0.06736	0.119
demographic and financial model	0.096	0.06939	1.03017	0.06736	0.136
financial model	0.019	0.06840	1.01553	0.06736	0.116
EQUITY					
Demographic model	-0.014	0.29052	1.02135	0.28445	-0.073
demographic and financial model	-0.026	0.29861	1.04980	0.28445	-0.088
financial model	-0.019	0.28093	0.98762	0.28445	-0.252

Optimal Asset Allocation based on the forecasted excess return

Derive the optimal asset allocation among stock, bond and cash by considering the optimization problem of an investor with a mean-variance utility function

This optimization problem can be written as an optimization problem on the excess return

$$\max_{w_t^e, w_t^b} [\begin{pmatrix} w_t^e & w_t^b \end{pmatrix} \begin{pmatrix} E_t R_{t+1}^e - R_{t+1}^f \\ E_t R_{t+1}^b - R_{t+1}^f \end{pmatrix} - \frac{1}{2} k \begin{pmatrix} w_t^e & w_t^b \end{pmatrix} \begin{pmatrix} (\sigma_t^e)^2 & \sigma_t^{eb} \\ \sigma_t^{eb} & (\sigma_t^b)^2 \end{pmatrix} \begin{pmatrix} w_t^e \\ w_t^b \end{pmatrix} | \Omega_t]$$

Demographic model

$$E_t R_{t+1}^{e} - R_{t+1}^{f} \Leftarrow \widehat{(R_{t+1}^{e} - R_{t+1}^{f})} = \widehat{a}_0 + \widehat{a}_1 PC1_t + \widehat{a}_2 PC2_t + \widehat{a}_3 PC3_t$$

$$E_t R_{t+1}^{b} - R_{t+1}^{f} \Leftarrow \widehat{(R_{t+1}^{b} - R_{t+1}^{f})} = \widehat{a}_0 + \widehat{a}_1 PC1_t + \widehat{a}_2 PC2_t + \widehat{a}_3 PC3_t$$

Compare the optimal allocation using conditional excess returns estimated from demographic model with the allocation based on 3 simple buy-and-hold strategies (100% stock, 100% bond, and 50% stock and 50% bond)

Table 3: Mean Excess Returns and Sharpe Ratio from the optimal asset allocation and buy-and-hold allocation

Pre-War

1. Asset allocation from demographic model has higher mean excess return than all buy-and-hold allocations for k=1,2, and 5

2. Asset allocation from demographic model has higher Sharpe ratio than 100% equity for k=1,2,5, and 10

Post-War

1. Asset allocation from demographic model has higher mean excess return than all buy-and-hold allocations for k=1,2, and 5

2. Asset allocation from demographic model has higher Sharpe ratio than 100% equity AND 100% bond for k=1,2,5, and 10

Robustness Check on the Optimal Asset Allocation estimated from demographic model

Table 3

Mean Excess Retuns and Sharpe Ratios for Demographic Model and Passive Model

Sample Period: 1930-1939	Mean Excess Return	Sharpe Ratio
100% equity	3.84%	0.26
100% bond	1.35%	1.70
3 asset demographic model		
k=1	5.47%	0.60
k=2	5.43%	0.59
k=5	4.05%	0.59
k=10	3.20%	0.67
k=5000	0.13%	0.17
2 asset demographic model, equity		
k=1	3.95%	0.43
k=2	3.91%	0.43
k=5	3.88%	0.47
k=10	2.25%	0.48
k=5000	0.00%	0.01
2 asset demographic model, bonds		
k=1	1.35%	1.70
k=2	1.35%	1.70
k=5	1.35%	1.70
k=10	1.35%	1.70
k=5000	0.01%	0.15

Sample Period: 1962-2001	Mean Excess Return	Sharpe Ratio
100% equity	6.26%	0.24
100% bond	1.55%	0.27
3 asset demographic model		
k=1	8.54%	0.37
k=2	8.00%	0.39
k=5	5.60%	0.37
k=10	3.83%	0.48
k=5000	0.01%	0.00
2 asset demographic model, equity		
k=1	6.35%	0.28
k=2	5.77%	0.28
k=5	3.50%	0.23
k=10	1.75%	0.22
k=5000	0.00%	0.00
2 asset demographic model, bonds		
k=1	2.50%	0.53
k=2	2.54%	0.54
k=5	2.43%	0.52
k=10	2.34%	0.55
k=5000	0.01%	0.00

Demographic and financial model

$$E_t R_{t+1}^{e} - R_{t+1}^{f} \Leftarrow \widehat{(R_{t+1}^{e} - R_{t+1}^{f})} = \widehat{a}_0 + \widehat{a}_1 PC1_t + \widehat{a}_2 PC2_t + \widehat{a}_3 PC3_t + \widehat{a}_4 (R_t^{e} - R_t^{f})$$

$$E_t R_{t+1}^{b} - R_{t+1}^{f} \Leftarrow \widehat{(R_{t+1}^{b} - R_{t+1}^{f})} = \widehat{a}_0 + \widehat{a}_1 PC1_t + \widehat{a}_2 PC2_t + \widehat{a}_3 PC3_t + \widehat{a}_4 (R_t^{b} - R_t^{f})$$

Financial model

$$E_t R_{t+1}^{e} - R_{t+1}^{f} \Leftarrow \widehat{(R_{t+1}^{e} - R_{t+1}^{f})} = \widehat{a}_0 + \widehat{a}_1 (R_t^{e} - R_t^{f})$$

$$E_t R_{t+1}^{b} - R_{t+1}^{f} \Leftarrow \widehat{(R_{t+1}^{b} - R_{t+1}^{f})} = \widehat{a}_0 + \widehat{a}_1 (R_t^{b} - R_t^{f})$$

Table 4: Mean Excess Returns and Sharpe Ratios for demographic, financial and demographic and financial models

Pre-War

K=1, allocations from all three models are the same

K=2, allocation from demographic model has highest return

K=5,10,5000, allocation from demographic and financial model has higher return than that from demographic model which has higher return than that from financial model

Post-War

K=1, allocation from financial model has higher return but lower Sharpe ratio

K=2,5 allocation from demographic model has the highest return

K=10, allocation from demographic and financial model is the best

demographic components contribute to returns even when financial component is in the model

Table 4

Mean Excess Retuns and Sharpe Ratios for 3 asset optimal portfolios that trade on Demographic and Financial Indicators or Financial Indicators only

Sample Period: 1930-1939	Mean Excess Return	Sharpe Ratio
demographic and financial model		
k=1	5.47%	0.60
k=2	5.41%	0.59
k=5	4.43%	0.58
k=10	3.46%	0.66
k=5000	0.13%	0.18
financial model		
k=1	5.47%	0.60
k=2	5.12%	0.58
k=5	3.48%	0.66
k=10	2.42%	0.91
k=5000	0.11%	0.15

Sample Period: 1962-2001	Mean Excess Return	Sharpe Ratio
demographic and financial model		
k=1	8.45%	0.37
k=2	7.02%	0.33
k=5	5.13%	0.36
k=10	3.99%	0.53
k=5000	0.03%	0.01
financial model		
k=1	8.75%	0.34
k=2	5.11%	0.30
k=5	3.61%	0.45
k=10	2.83%	0.55
k=5000	0.02%	0.01

Conclusion and Extension

The demographic variables affect excess asset returns!

We need to distinguish between an observed change in demography and an unobserved change in or a shock to demography

> We can model and forecast demographic changes and relate the shock to demographic change to future excess returns

We can expand the asset class and include real estate returns. The problem with real estate data is that it does not include dividends and we need to adjust the real estate index. Ad-hoc way is to assume that the dividend is 2%.

Results on Real Estate Index Return

Data: Urban Land Price Index of Nationwide (average of commercial, residential and industrial) 1956-2001

Results: Active (land, bond and money) with 2% dividend outperforms 100% land with or 100% bond

	Land	Bond	Active
Excess Return	7.1%	7.5%	150%, 78%, 34%, 20% (k=1),(k=2),(k=5),(k=10)
Sharpe Ratio	0.169	0.335	0.589,0.588,0.585,0.579 (k=1),(k=2),(k=5),(k=10)

Problem with Real Estate Index

1. Does not include "dividends". Need to assume dividend. In the U.S., 10% is assumed in Nakajima (2003).

2. Strong trend in the index return. Ex-post return was higher than money in the 50s, 60s and 70s, but lower in the 80s and 90s.

3. The index is not transactions based. Condominium transaction price index might be a better alternative.