



Demographics and Asset Returns in Japan

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Motivation

The causal relation between the aging of Japanese society and the realized equity premium

Demographic variables predict expected excess returns in theory, is there any quantitatively large predictable variation in practice?

Analysis

Test the forecasting power of regression based on principal components from demographic variables by

- 1) obtaining the \bar{R}^2 , Theil ratio and the correlation between the forecast and actual returns
- 2) deriving the optimal asset allocation based on principal components from demographic variables and comparing the mean excess returns and Sharpe ratios from this asset allocation with those from simple buy and hold strategy.

Conclusion

Principal components from demographic variables do predict future excess stock and bond returns in both pre-war and post-war period, especially for bond excess returns in that

1) demographic model which forecast excess stock and bond returns from principal components of demographic variables (log population between 0 and 9, 10 and 19, 20 and 29, e.t.c.) has a Theil ratio of less than 1 and

2) the mean excess return and the Sharpe ratio of active trading strategy which forecasts excess stock and bond returns from principal components of the demographic variables is higher than those from a simple buy and hold strategy of 100% stock or 100% bond.

Literature Survey

1. Age and Asset Returns

Theoretical Evidence

Bakshi and Chen (1994): "life-cycle risk aversion hypothesis"

Risk aversion is linear in average age

>pricing kernel = f (aggregate consumption, average age)

expected excess return = f (aggregate consumption, average age)

Empirical Evidence

Poterba (2001): risk aversion = f (age)

>average age is not a good test statistic to explain returns

2. Demography and Asset Returns

Theoretical Evidence

Partial Equilibrium

Jagannathan and Kocherlakota (1996): Investors whose labor income is uncorrelated with stock return should buy stocks.

Campbell and Viceira (2002): lifecycle asset allocation with variable labor supply

General Equilibrium

Donaldson and Maddaloni (2002): risk premium is inversely related to population growth rate

Storresletten et al. (2001): hump shaped risky asset demand with variable labor supply

Empirical Evidence

Ang and Maddaloni (2003): long term predictability between excess returns and demographic variables (fraction of people over 65 years old, percentage of people in the age class 20-64, average adult age) for U.S., Japan, U.K., Germany & France

Asset return data: Global Financial Data

Population data: Japan Statistical Yearbook 1996

There was no significant relation between excess returns from correct total return data on financial assets estimated for pre-war and post-war periods using the same RHS variables.

We could not find any demographic variables such as fertility and average age that produce significant coefficient or high R^2

Ang and Maddaloni (2003): Japanese Data 1920 - 2001 (GFD)

| | log change of average age of 20+ | log change of fraction of 65+ | log change of fraction of | long - short bond yield | Adj R ² |
|----------------------------------|----------------------------------|-------------------------------|---------------------------|-------------------------|--------------------|
| log (stock return - bond return) | -13.12 | | | 0.005 | 0.04 |
| | | -2.56* | | 0.004 | 0.04 |
| | | | 9.59 | -0.009 | 0.07 |

Goyal (2002): U.S. Data 1926-1998 (S&P, Ibbotson Associates)

| | | average age of 25+ | fraction of 65+ | fraction of 25-44 | fraction of 45-64 | Adj.R ² |
|----------------------------------|----------------|--------------------|-----------------|-------------------|-------------------|--------------------|
| Log (stock return - T-bill rate) | percent change | 29.5 | | | | 0.15 |
| | | | -2.87* | 4.8* | -11.38* | 0.19 |
| | level | -0.01 | | | | 0.09 |
| | | | -0.37 | 1.31* | 1.14 | 0.14 |

Poterba (2001): U.S. Data 1926 - 1999 (Ibbotson Associates)

| | median age | average age of 20+ | fraction of 40-64/65+ | fraction of 40-64/20+ | fraction of 40-64 |
|---------------|------------|--------------------|-----------------------|-----------------------|-------------------|
| Treasury Bill | -0.001 | 0 | -0.002 | -0.39* | -1.30* |
| Government | 0.004 | -0.006 | 0 | -1.16* | -1.73* |
| Common Stock | 0.014 | 0.003 | -0.001 | -0.07 | 1.46 |

Data (Annual)

Pre-war (1920-1940)

Population: Population Estimate Series

Stock Returns: Rate of Yields on Equity Shares of All Industries

Bond Returns: Corporate Bond Yields

Risk Free Returns: Interest Rate on Postal Ordinary Savings

All asset returns are from Fujino and Akiyama (1977)

Post-war (1952-2001)

Population: Population Estimate Series & Japan Statistical Yearbook

Stock Returns: VW total return on the TSE (1st section)

Bond Returns: Gov Bond Returns (10 year maturity)

Risk Free Returns: overnight call rates

All asset returns data are from Ibbotson Associates Japan

Figure 1
Excess Returns in Pre WWII Period

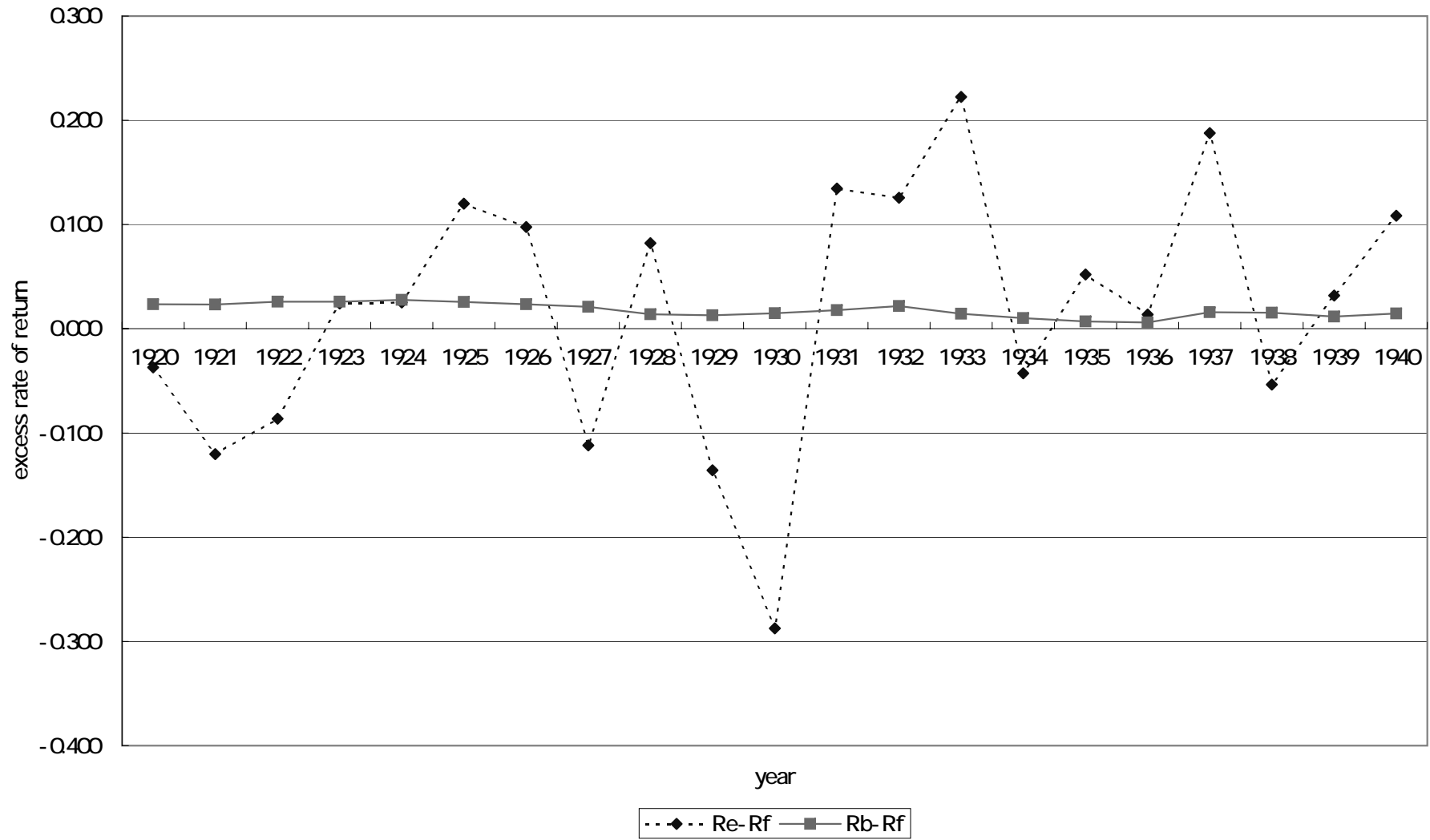
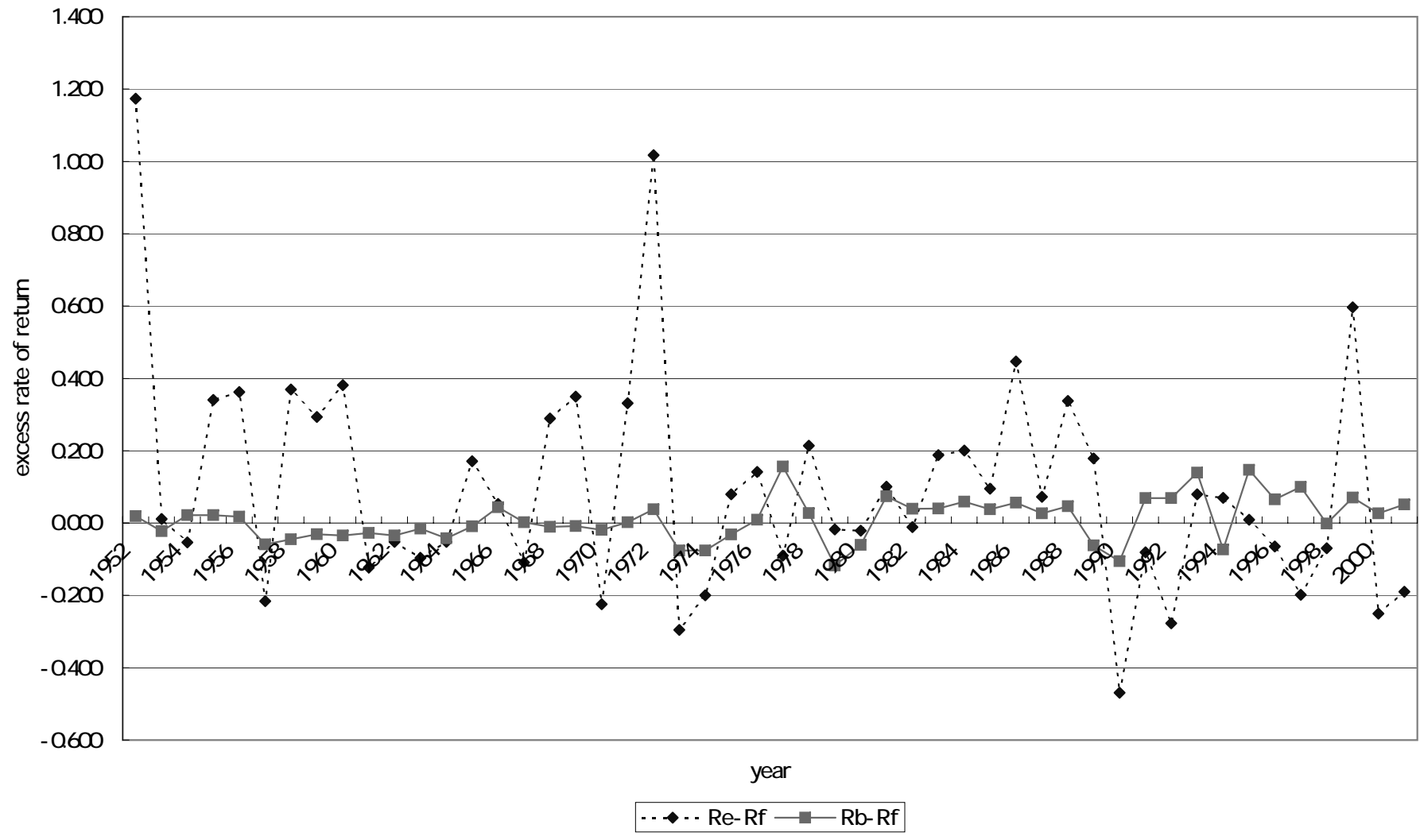
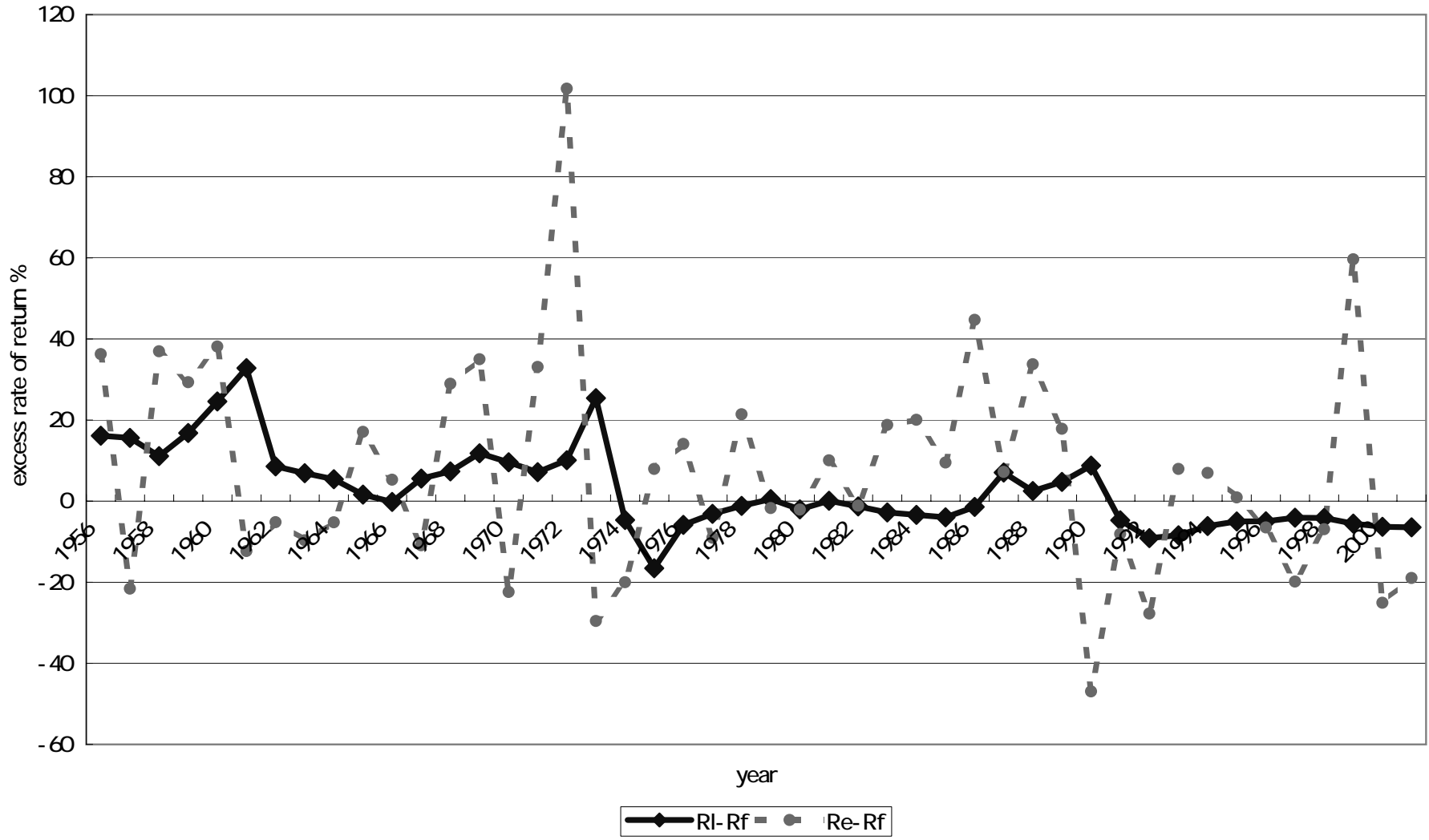


Figure 2
Excess Returns in Post WWII Period



Excess Returns in Post WWII Period
Stock - Money vs. Land - Money



Amounts of Savings and Liabilities Held per Household (All Households)

(Value in thousands of yen)

| | No. of tabulated households | Yearly income | Total Savings | Financial institutions | Stocks & shares | Public & corporate bonds | Total Liabilities |
|------|-----------------------------------|------------------|------------------|---------------------------|--------------------|--------------------------------|----------------------|
| 1959 | ... | 432 | 302 | 302 | 78 | 5 | 40 |
| 1960 | ... | 453 | 359 | 339 | 84 | 6 | 69 |
| 1961 | ... | 573 | 462 | 440 | 118 | 7 | 109 |
| 1962 | ... | 583 | 441 | 419 | 89 | 8 | 68 |
| 1963 | ... | 626 | 647 | 609 | 129 | 13 | 88 |
| 1964 | ... | 706 | 689 | 642 | 115 | 13 | 116 |
| 1965 | ... | 784 | 764 | 721 | 128 | 16 | 131 |
| 1966 | ... | 882 | 910 | 852 | 120 | 19 | 127 |
| 1967 | ... | 976 | 995 | 934 | 113 | 29 | 159 |
| 1968 | 4,305 | 1078 | 1126 | 1069 | 130 | 37 | 190 |
| 1969 | 4,847 | 1235 | 1395 | 1326 | 191 | 34 | 225 |
| 1970 | 5,091 | 1394 | 1603 | 1524 | 188 | 36 | 284 |
| 1971 | 4,313 | 1621 | 1829 | 1755 | 218 | 41 | 339 |
| 1972 | 4,630 | 1816 | 2150 | 2057 | 312 | 54 | 462 |
| 1973 | 5,173 | 2124 | 2426 | 2327 | 271 | 55 | 623 |
| 1974 | 5,180 | 2598 | 2704 | 2576 | 232 | 65 | 756 |
| 1975 | 5,185 | 2990 | 3168 | 3027 | 244 | 72 | 850 |
| 1976 | 5,172 | 3428 | 3768 | 3618 | 281 | 103 | 874 |
| 1977 | 5,231 | 3769 | 4271 | 4091 | 334 | 133 | 1078 |
| 1978 | 5,676 | 3932 | 4511 | 4328 | 342 | 118 | 1404 |
| 1979 | 6,065 | 4314 | 5212 | 5027 | 445 | 146 | 1708 |
| 1980 | 6,045 | 4643 | 5794 | 5597 | 429 | 196 | 1772 |
| 1981 | 6,051 | 5017 | 6500 | 6291 | 515 | 273 | 1758 |
| 1982 | 5,909 | 5051 | 6972 | 6772 | 557 | 319 | 1858 |
| 1983 | 5,964 | 5235 | 7263 | 7038 | 495 | 354 | 2077 |
| 1984 | 5,981 | 5297 | 7697 | 7458 | 602 | 315 | 2404 |
| 1985 | 5,974 | 5557 | 8528 | 8265 | 708 | 382 | 2721 |
| 1986 | 5,893 | 5710 | 9095 | 8822 | 902 | 351 | 2843 |
| 1987 | 5,695 | 5923 | 10452 | 10170 | 1441 | 377 | 3113 |
| 1988 | 5,732 | 6075 | 11198 | 10880 | 1434 | 366 | 3096 |
| 1989 | 5,734 | 6413 | 13110 | 12760 | 2335 | 380 | 3742 |
| 1990 | 5,627 | 6773 | 13530 | 13212 | 1829 | 360 | 3592 |
| 1991 | 5,701 | 7189 | 14654 | 14289 | 1594 | 346 | 3753 |
| 1992 | 5,395 | 7505 | 15368 | 14914 | 1331 | 359 | 3926 |
| 1993 | 5,449 | 7510 | 14982 | 14539 | 1140 | 316 | 3998 |
| 1994 | 5,548 | 7552 | 15921 | 15455 | 1145 | 377 | 4391 |
| 1995 | 5,481 | 7618 | 16035 | 15620 | 991 | 320 | 4599 |
| 1996 | 5,496 | 7545 | 16553 | 16127 | 917 | 340 | 5100 |
| 1997 | 5,350 | 7548 | 16345 | 15952 | 898 | 331 | 4985 |
| 1998 | 5,419 | 7584 | 16607 | 16134 | 853 | 268 | 5347 |
| 1999 | 5,458 | 7550 | 17377 | 16947 | 1071 | 339 | 5773 |
| 2000 | 5,466 | 7213 | 17812 | 17414 | 915 | 309 | 5382 |

Percentages of Household Holding Savings and Liabilities (All Households)

(Holding percentage in percent)

| | Yearly income | Savings | Demand deposits | Time deposits | Stocks & shares | Public & corporate bonds | Liabilities 4) |
|------|------------------|---------|--------------------|------------------|--------------------|--------------------------------|-------------------|
| 1959 | ... | 92.2 | 69.6 | 56.5 | 15.9 | 5.1 | 20.2 |
| 1960 | ... | 93.5 | 68.4 | 58.6 | 18.0 | 5.6 | 27.7 |
| 1961 | ... | 94.8 | 69.9 | 56.8 | 19.8 | 5.6 | 31.0 |
| 1962 | 598.0 | 94.3 | 70.9 | 57.6 | 19.8 | 5.6 | 32.5 |
| 1963 | 638.4 | 98.8 | 80.1 | 62.3 | 20.5 | 8.2 | 39.2 |
| 1964 | 719.9 | 99.6 | 80.8 | 65.3 | 22.0 | 8.5 | 34.7 |
| 1965 | 799.3 | 97.5 | 79.5 | 65.9 | 19.9 | 8.8 | 35.6 |
| 1966 | 891.8 | 97.5 | 83.7 | 69.4 | 20.7 | 11.4 | 34.8 |
| 1967 | ... | 97.5 | 84.6 | 70.7 | 18.5 | 11.7 | 37.6 |
| 1968 | 1,095.2 | 99.4 | 87.6 | 74.1 | 18.7 | 12.7 | 38.4 |
| 1969 | 1,237.2 | 99.7 | 90.0 | 76.3 | 20.2 | 12.7 | 38.9 |
| 1970 | 1,397.7 | 98.9 | 89.5 | 77.4 | 18.7 | 12.1 | 40.6 |
| 1971 | 1,625.2 | 98.9 | 90.1 | 79.1 | 17.3 | 12.7 | 41.3 |
| 1972 | 1,818.7 | 98.8 | 90.7 | 79.7 | 16.2 | 11.9 | 43.8 |
| 1973 | 2,123.4 | 98.8 | 90.8 | 81.3 | 15.3 | 10.8 | 44.8 |
| 1974 | 2,597.7 | 99.0 | 91.7 | 82.1 | 15.3 | 10.5 | 44.4 |
| 1975 | 2,991 | 99.3 | 91.5 | 83.4 | 15.8 | 10.8 | 44.3 |
| 1976 | 3,438 | 99.3 | 92.3 | 85.8 | 16.3 | 9.9 | 42.2 |
| 1977 | 3,776 | 99.7 | 94.1 | 87.6 | 16.4 | 9.9 | 46.0 |
| 1978 | 3,932 | 99.7 | 93.9 | 88.5 | 15.6 | 9.1 | 47.4 |
| 1979 | 4,314 | 99.3 | 92.9 | 88.1 | 15.3 | 9.0 | 49.1 |
| 1980 | 4,643 | 99.5 | 92.6 | 89.8 | 16.3 | 9.4 | 49.9 |
| 1981 | 5,017 | 99.5 | 91.3 | 89.5 | 16.9 | 10.2 | 49.4 |
| 1982 | 5,051 | 99.5 | 90.8 | 89.8 | 15.6 | 10.3 | 49.0 |
| 1983 | 5,234 | 99.2 | 90.4 | 89.5 | 15.4 | 11.0 | 49.9 |
| 1984 | 5,297 | 99.0 | 89.9 | 89.7 | 15.7 | 10.5 | 50.3 |
| 1985 | 5,557 | 99.3 | 90.3 | 89.0 | 15.7 | 11.2 | 51.9 |
| 1986 | 5,710 | 99.2 | 89.2 | 89.7 | 15.5 | 10.7 | 49.0 |
| 1987 | 5,923 | 99.1 | 88.0 | 90.2 | 18.4 | 10.3 | 50.0 |
| 1988 | 6,075 | 99.2 | 89.3 | 90.3 | 18.5 | 10.1 | 48.6 |
| 1989 | 6,413 | 99.2 | 89.2 | 89.9 | 19.2 | 9.6 | 49.7 |
| 1990 | 6,773 | 99.5 | 90.8 | 90.6 | 20.5 | 10.2 | 48.2 |
| 1991 | 7,189 | 99.4 | 91.0 | 91.0 | 20.1 | 9.0 | 47.4 |
| 1992 | 7,505 | 99.4 | 90.6 | 89.2 | 19.5 | 8.4 | 46.4 |
| 1993 | 7,510 | 99.5 | 90.0 | 88.0 | 19.9 | 7.7 | 48.0 |
| 1994 | 7,552 | 99.3 | 90.5 | 88.3 | 19.3 | 7.8 | 46.7 |
| 1995 | 7,618 | 99.2 | 90.4 | 87.8 | 19.0 | 7.0 | 46.9 |
| 1996 | 7,545 | 99.2 | 90.3 | 87.9 | 17.5 | 6.2 | 45.3 |
| 1997 | 7,548 | 98.9 | 89.9 | 86.4 | 17.5 | 5.5 | 45.4 |
| 1998 | 7,584 | 98.9 | 90.6 | 86.8 | 18.5 | 5.9 | 45.7 |
| 1999 | 7,550 | 99.1 | 91.4 | 87.0 | 19.6 | 6.5 | 45.2 |
| 2000 | 7,213 | 98.7 | 91.5 | 86.0 | 18.8 | 6.6 | 43.0 |

Construction of 5 principal Components

$$\begin{bmatrix} \log(pop09_1) & \cdots & \log(pop80p_1) \\ \vdots & \ddots & \vdots \\ \log(pop09_{10}) & \cdots & \log(pop80p_{10}) \end{bmatrix} \Rightarrow \begin{bmatrix} PC1_1 & \cdots & PC5_1 \\ \vdots & \ddots & \vdots \\ PC1_{10} & \cdots & PC5_{10} \end{bmatrix}$$

10×9 10×5

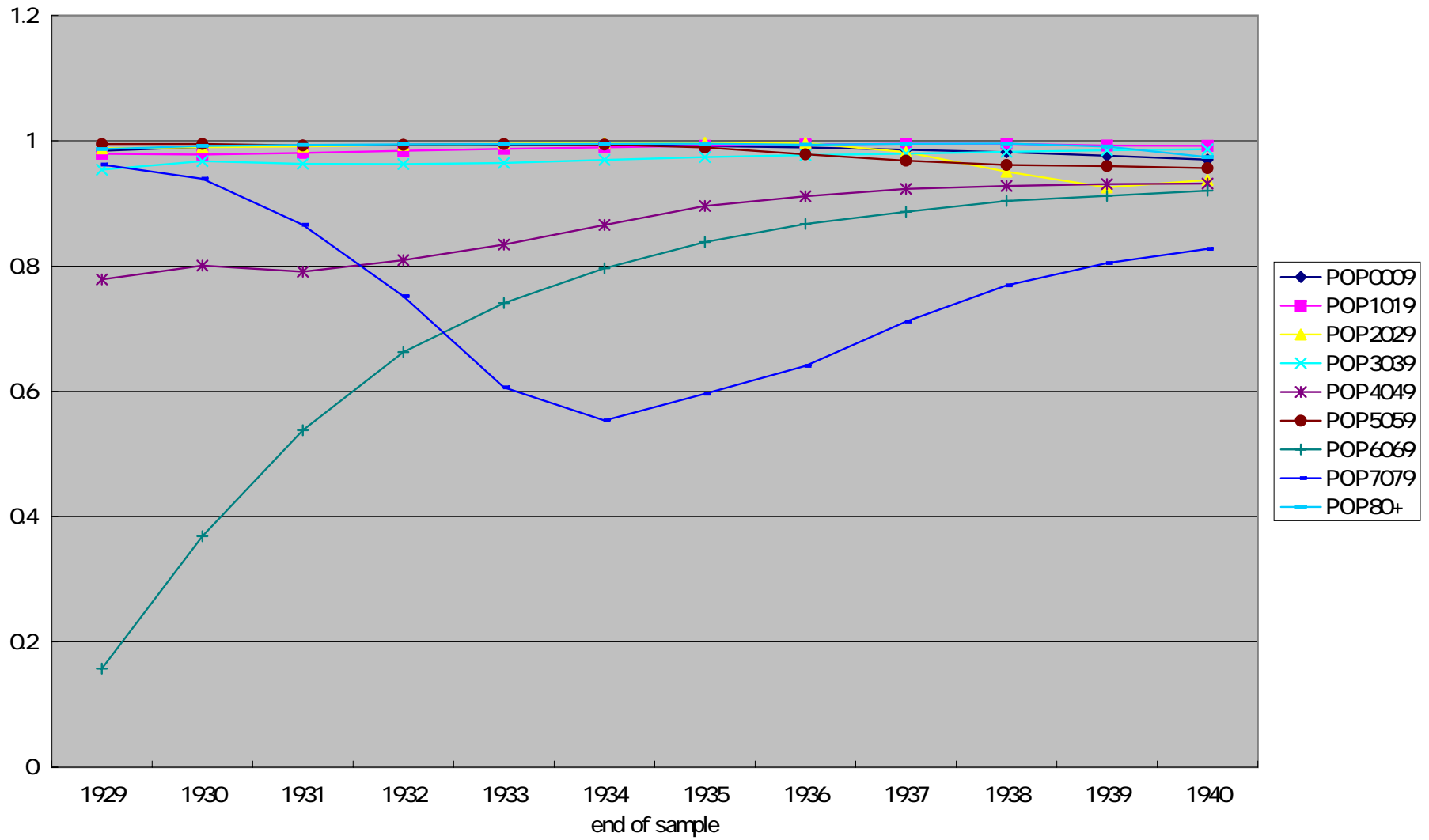
$$\begin{bmatrix} \log(pop09_1) & \cdots & \log(pop80p_1) \\ \vdots & \ddots & \vdots \\ \log(pop09_{11}) & \cdots & \log(pop80p_{11}) \end{bmatrix} \Rightarrow \begin{bmatrix} PC1_1 & \cdots & PC5_1 \\ \vdots & \ddots & \vdots \\ PC1_{11} & \cdots & PC5_{11} \end{bmatrix}$$

11×9 11×5

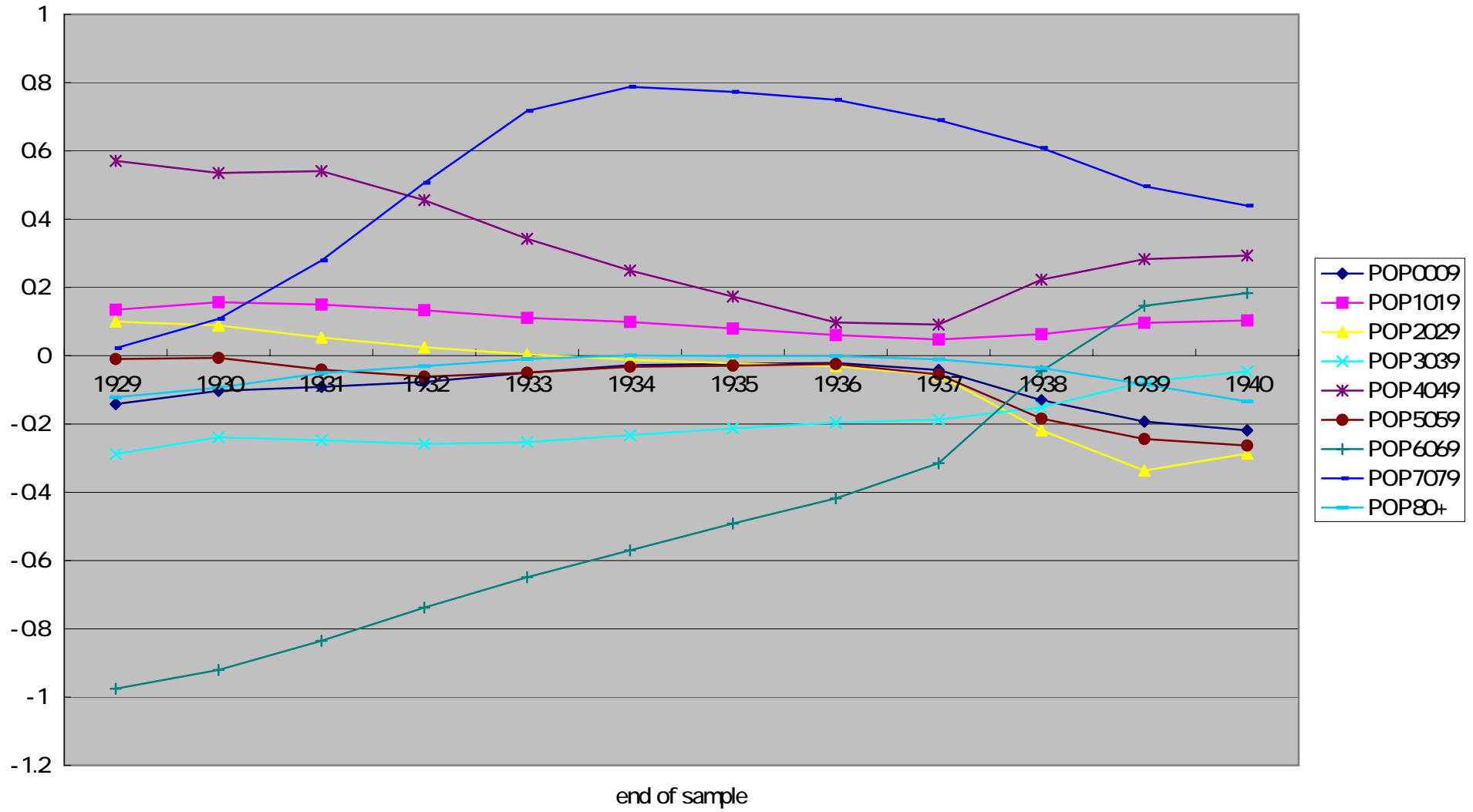
$$\begin{bmatrix} \log(pop09_1) & \cdots & \log(pop80p_1) \\ \vdots & \ddots & \vdots \\ \log(pop09_T) & \cdots & \log(pop80p_T) \end{bmatrix} \Rightarrow \begin{bmatrix} PC1_1 & \cdots & PC5_1 \\ \vdots & \ddots & \vdots \\ PC1_T & \cdots & PC5_T \end{bmatrix}$$

$T \times 9$ $T \times 5$

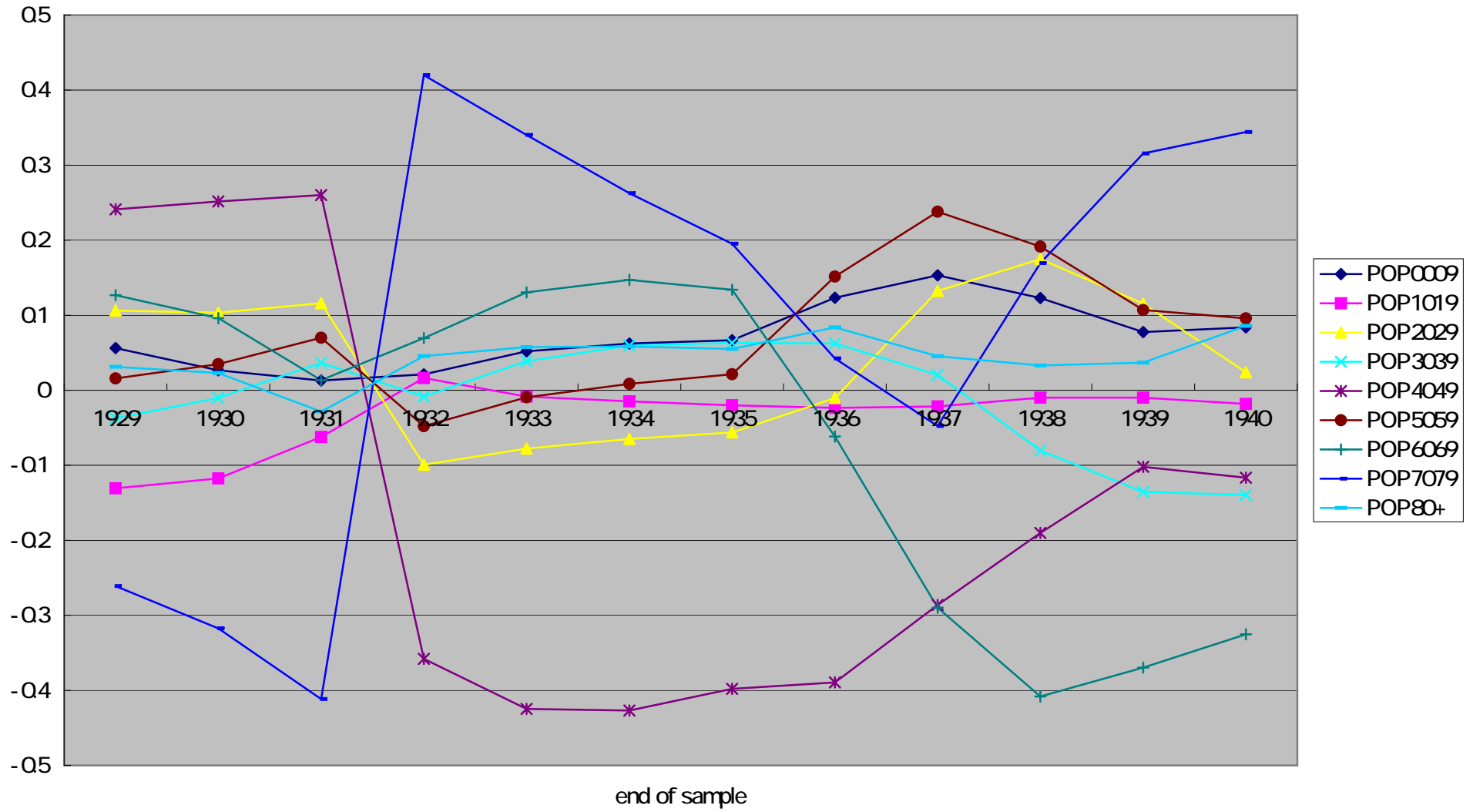
First eigenvalue: Pre War



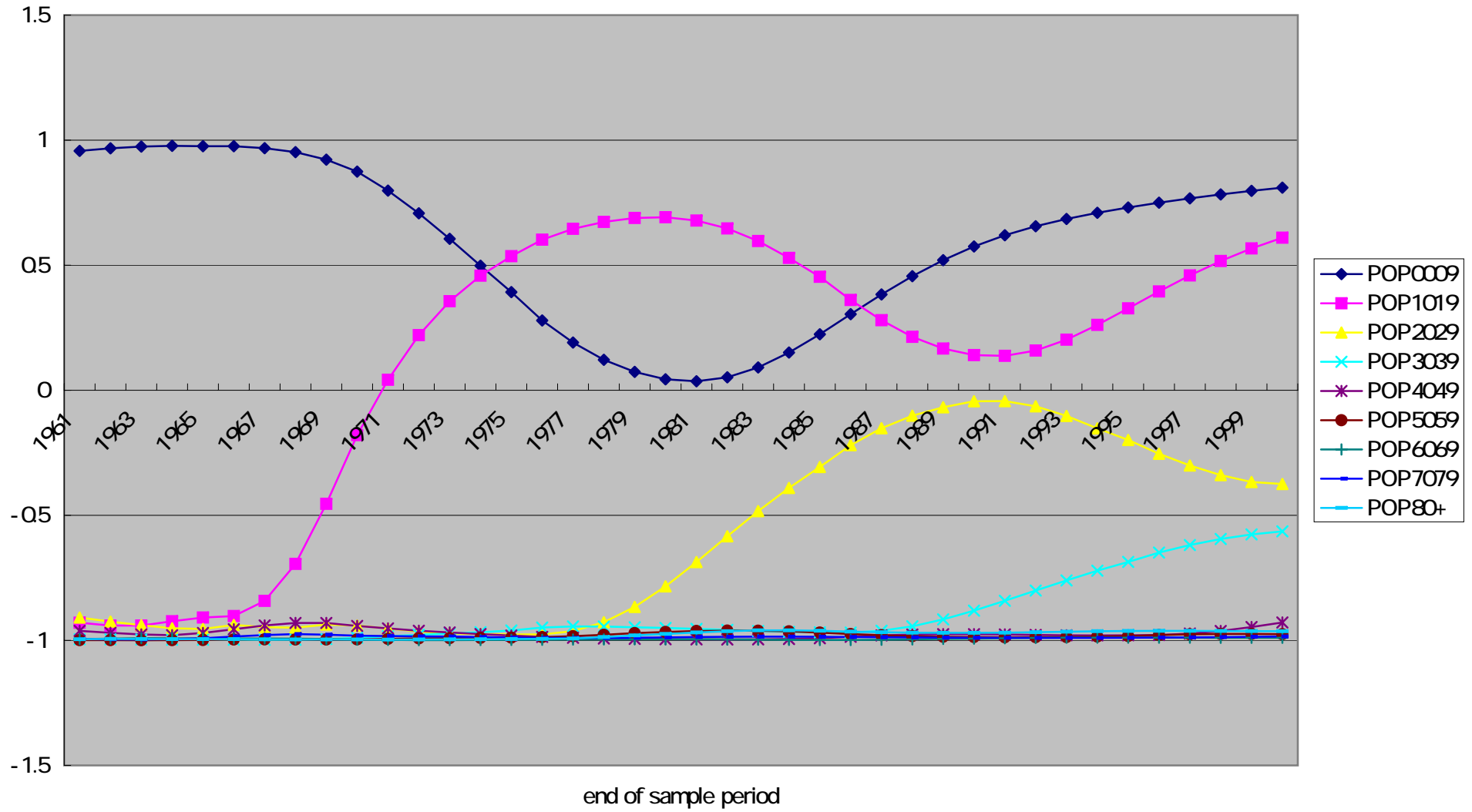
Second Eigenvalue: Pre War



Third eigenvalue: Pre War



First eigenvalue: Post war



Second Eigenvalue: Post War

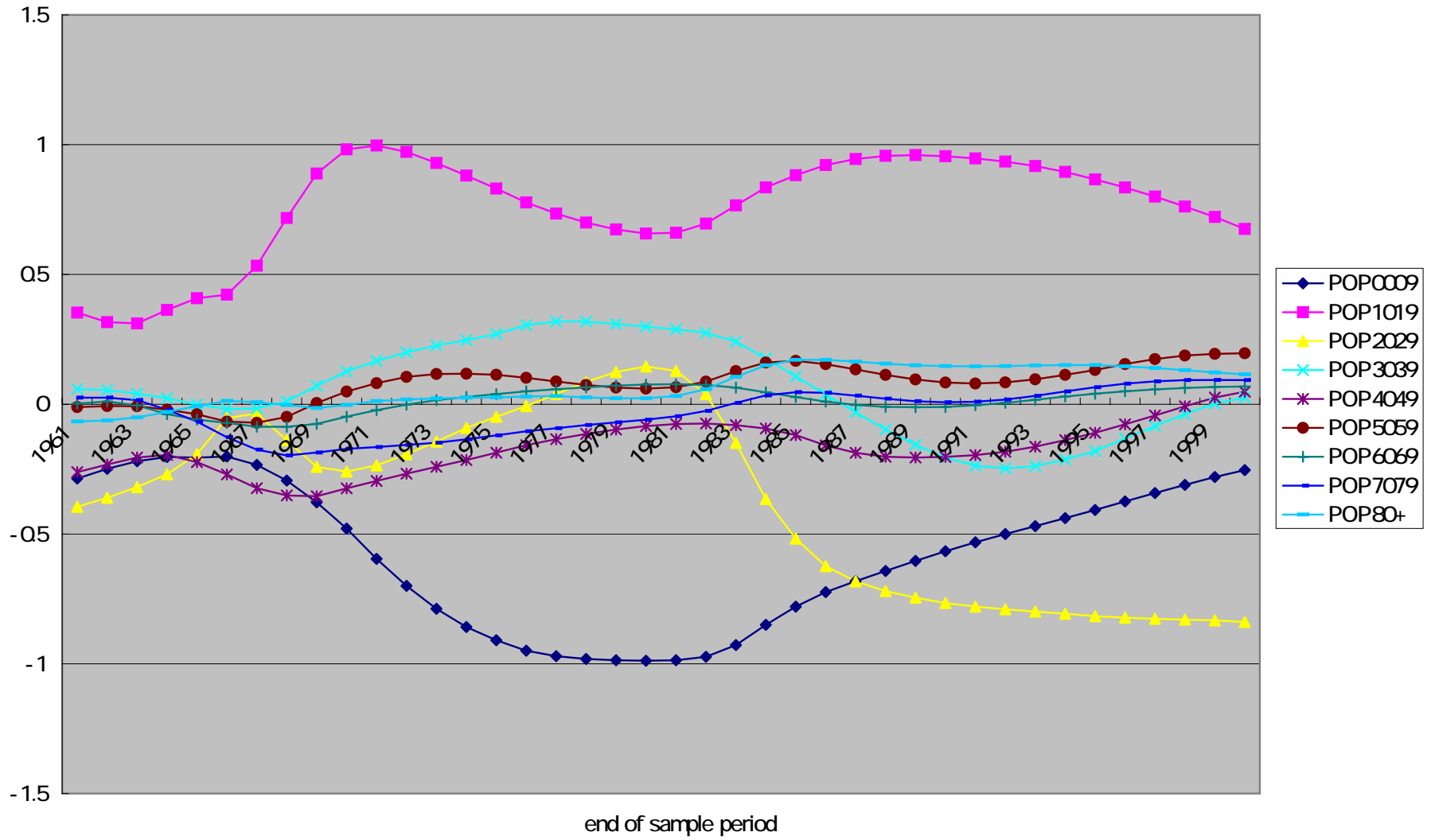


Table 1
Principal components estimation of demographic variables for population levels

| Factor sensitivities | | | | |
|----------------------|-----------------------|----------------------|-----------------------|----------------------|
| first eigenvector | second eigenvector | third eigenvector | fourth eigenvector | fifth eigenvector |
| 0.973 | 0.210 | 0.073 | 0.052 | 0.012 |
| 0.993 | -0.090 | -0.006 | -0.030 | 0.001 |
| 0.944 | 0.271 | 0.036 | -0.157 | -0.075 |
| 0.985 | 0.051 | -0.145 | 0.048 | -0.041 |
| 0.928 | -0.300 | -0.111 | -0.175 | 0.073 |
| 0.960 | 0.253 | 0.095 | 0.009 | 0.016 |
| 0.913 | -0.178 | -0.349 | 0.108 | -0.032 |
| 0.829 | -0.433 | 0.346 | 0.051 | -0.047 |
| 0.976 | 0.140 | 0.089 | 0.095 | 0.086 |
| 0.895 | 0.953 | 0.986 | 0.995 | 0.998 |

| Factor sensitivities | | | | |
|----------------------|-----------------------|----------------------|-----------------------|----------------------|
| first eigenvector | second eigenvector | third eigenvector | fourth eigenvector | fifth eigenvector |
| 0.792 | 0.131 | 0.438 | 0.389 | 0.111 |
| 0.622 | -0.636 | 0.207 | -0.389 | 0.114 |
| -0.418 | 0.822 | 0.183 | -0.334 | 0.069 |
| -0.686 | -0.126 | 0.690 | -0.051 | -0.183 |
| -0.944 | -0.084 | 0.218 | 0.072 | 0.215 |
| -0.977 | -0.195 | -0.025 | 0.069 | 0.002 |
| -0.993 | -0.064 | -0.086 | -0.007 | 0.043 |
| -0.989 | -0.093 | -0.061 | 0.091 | -0.006 |
| -0.984 | -0.126 | -0.112 | 0.023 | 0.014 |
| 0.716 | 0.847 | 0.938 | 0.987 | 0.999 |

Demographic model for equity excess returns

$$\begin{bmatrix} Re_2 - Rf_2 \\ \vdots \\ Re_{11} - Rf_{11} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} PC1_1 \\ \vdots \\ PC1_{10} \end{bmatrix} + a_2 \begin{bmatrix} PC2_1 \\ \vdots \\ PC2_{10} \end{bmatrix} + a_3 \begin{bmatrix} PC3_1 \\ \vdots \\ PC3_{10} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{10} \end{bmatrix}$$

$$\Rightarrow (\widehat{a}_0, \dots, \widehat{a}_3) \Rightarrow \widehat{Re}_{12} - \widehat{Rf}_{12} = \widehat{a}_0 + \widehat{a}_1 PC1_{11} + \widehat{a}_2 PC2_{11} + \widehat{a}_3 PC3_{11}$$

$$\begin{bmatrix} Re_2 - Rf_2 \\ \vdots \\ Re_{12} - Rf_{12} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} PC1_1 \\ \vdots \\ PC1_{11} \end{bmatrix} + a_2 \begin{bmatrix} PC2_1 \\ \vdots \\ PC2_{11} \end{bmatrix} + a_3 \begin{bmatrix} PC3_1 \\ \vdots \\ PC3_{11} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{11} \end{bmatrix}$$

$$\Rightarrow (\widehat{a}_0, \dots, \widehat{a}_3) \Rightarrow \widehat{Re}_{13} - \widehat{Rf}_{13} = \widehat{a}_0 + \widehat{a}_1 PC1_{12} + \widehat{a}_2 PC2_{12} + \widehat{a}_3 PC3_{12}$$

The same procedure for bond excess returns

Financial model for excess stock returns

$$\begin{bmatrix} R^e_2 - R^f_2 \\ \vdots \\ R^e_{11} - R^f_{11} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} R^e_1 - R^f_1 \\ \vdots \\ R^e_{10} - R^f_{10} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{10} \end{bmatrix}$$

$$\Rightarrow (\widehat{a}_0, \widehat{a}_1) \Rightarrow \widehat{R^e_{12} - R^f_{12}} = \widehat{a}_0 + \widehat{a}_1 (R^e_{11} - R^f_{11})$$

$$\begin{bmatrix} R^e_2 - R^f_2 \\ \vdots \\ R^e_{12} - R^f_{12} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} R^e_1 - R^f_1 \\ \vdots \\ R^e_{11} - R^f_{11} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{11} \end{bmatrix}$$

$$\Rightarrow (\widehat{a}_0, \widehat{a}_1) \Rightarrow \widehat{R^e_{13} - R^f_{13}} = \widehat{a}_0 + \widehat{a}_1 (R^e_{12} - R^f_{12})$$

Demographic and financial model for excess stock returns:

$$\begin{bmatrix} Re_2 - Rf_2 \\ \vdots \\ Re_{11} - Rf_{11} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} PC1_1 \\ \vdots \\ PC1_{10} \end{bmatrix} + a_2 \begin{bmatrix} PC2_1 \\ \vdots \\ PC2_{10} \end{bmatrix} + a_3 \begin{bmatrix} PC3_1 \\ \vdots \\ PC3_{10} \end{bmatrix} + a_4 \begin{bmatrix} Re_1 - Rf_1 \\ \vdots \\ Re_{10} - Rf_{10} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{10} \end{bmatrix}$$

$$\Rightarrow (\widehat{a}_0, \dots, \widehat{a}_4) \Rightarrow \widehat{Re}_{12} - \widehat{Rf}_{12} = \widehat{a}_0 + \widehat{a}_1 PC1_{11} + \widehat{a}_2 PC2_{11} + \widehat{a}_3 PC3_{11} + \widehat{a}_4 (Re_{11} - Rf_{11})$$

$$\begin{bmatrix} Re_2 - Rf_2 \\ \vdots \\ Re_{12} - Rf_{12} \end{bmatrix} = a_0 + a_1 \begin{bmatrix} PC1_1 \\ \vdots \\ PC1_{11} \end{bmatrix} + a_2 \begin{bmatrix} PC2_1 \\ \vdots \\ PC2_{11} \end{bmatrix} + a_3 \begin{bmatrix} PC3_1 \\ \vdots \\ PC3_{11} \end{bmatrix} + a_4 \begin{bmatrix} Re_1 - Rf_1 \\ \vdots \\ Re_{11} - Rf_{11} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{11} \end{bmatrix}$$

$$\Rightarrow (\widehat{a}_0, \dots, \widehat{a}_4) \Rightarrow \widehat{Re}_{13} - \widehat{Rf}_{13} = \widehat{a}_0 + \widehat{a}_1 PC1_{12} + \widehat{a}_2 PC2_{12} + \widehat{a}_3 PC3_{12} + \widehat{a}_4 (Re_{12} - Rf_{12})$$

Theil Ratio

Index to compare root mean squared error of the model in question with that of the no change model

$$RMSE = \sqrt{\frac{\sum_{i=12}^T \{R_i^e - R_i^f - (R_i^e - R_i^f)\}^2}{T-12+1}}$$

$$RMSE_{no\ change} = \sqrt{\frac{\sum_{i=12}^T [\{1/i-1 \sum_{k=1}^{i-1} (R_k^e - R_k^f)\} - (R_i^e - R_i^f)]^2}{T-12+1}}$$

$$\text{Theil Ratio} = \frac{RMSE}{RMSE_{no\ change}}$$

If Theil Ratio is less than 1, the model in question performs better than the no change model

Table 2: Demographic, financial and demographic and financial model performance

Pre-War Bonds:

- 1. \bar{R}^2 is more than 60% for all the models**
- 2. Theil ratios for all the models are less than one**
- 3. The correlations between the forecast and actual excess returns are more than 25% for all the models**
- 4. Financial model performs best**

Pre-War Equity:

- 1. \bar{R}^2 is negative and small for all the models**
- 2. Theil ratios for all the models are more than one**
- 3. The correlations between the forecast and actual excess returns are negative for all the models**
- 4. Financial model performs best**

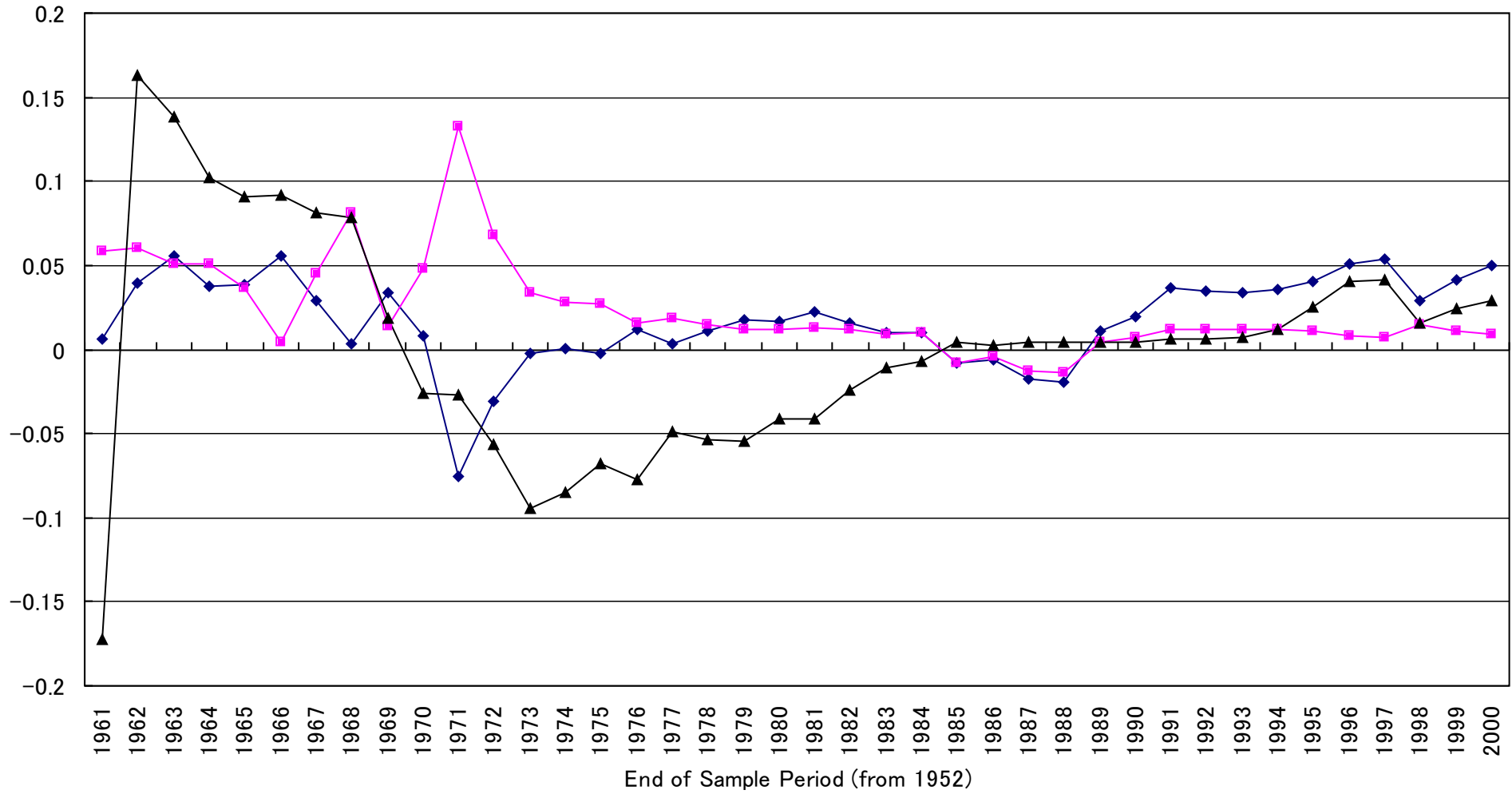
Post-War Bonds

- 1. \bar{R}^2 is positive but small for all the models**
- 2. Theil ratios for all the models are close to one**
- 3. The correlations between the forecast and actual excess returns are more about 12% for all the models**
- 4. Demographic model performs best**

Post-War Equity

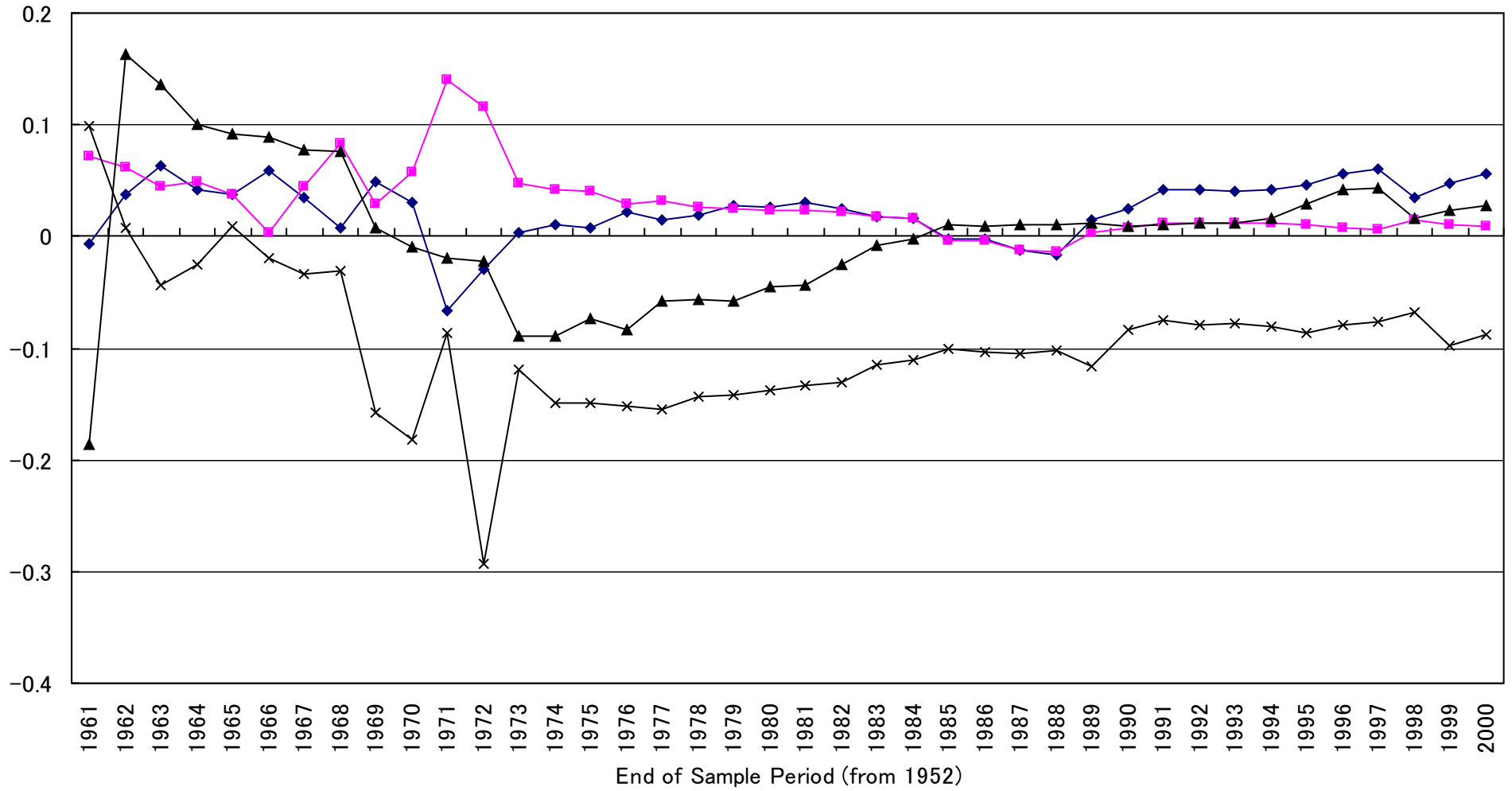
- 1. \bar{R}^2 is negative and small for all the models**
- 2. Theil ratios for all the models are close to one**
- 3. The correlations between the forecast and actual excess returns are negative for all the models**
- 4. Financial model performs best**

Coefficients of Demographic Model (Dependent Var.=Excess Return on Equity)



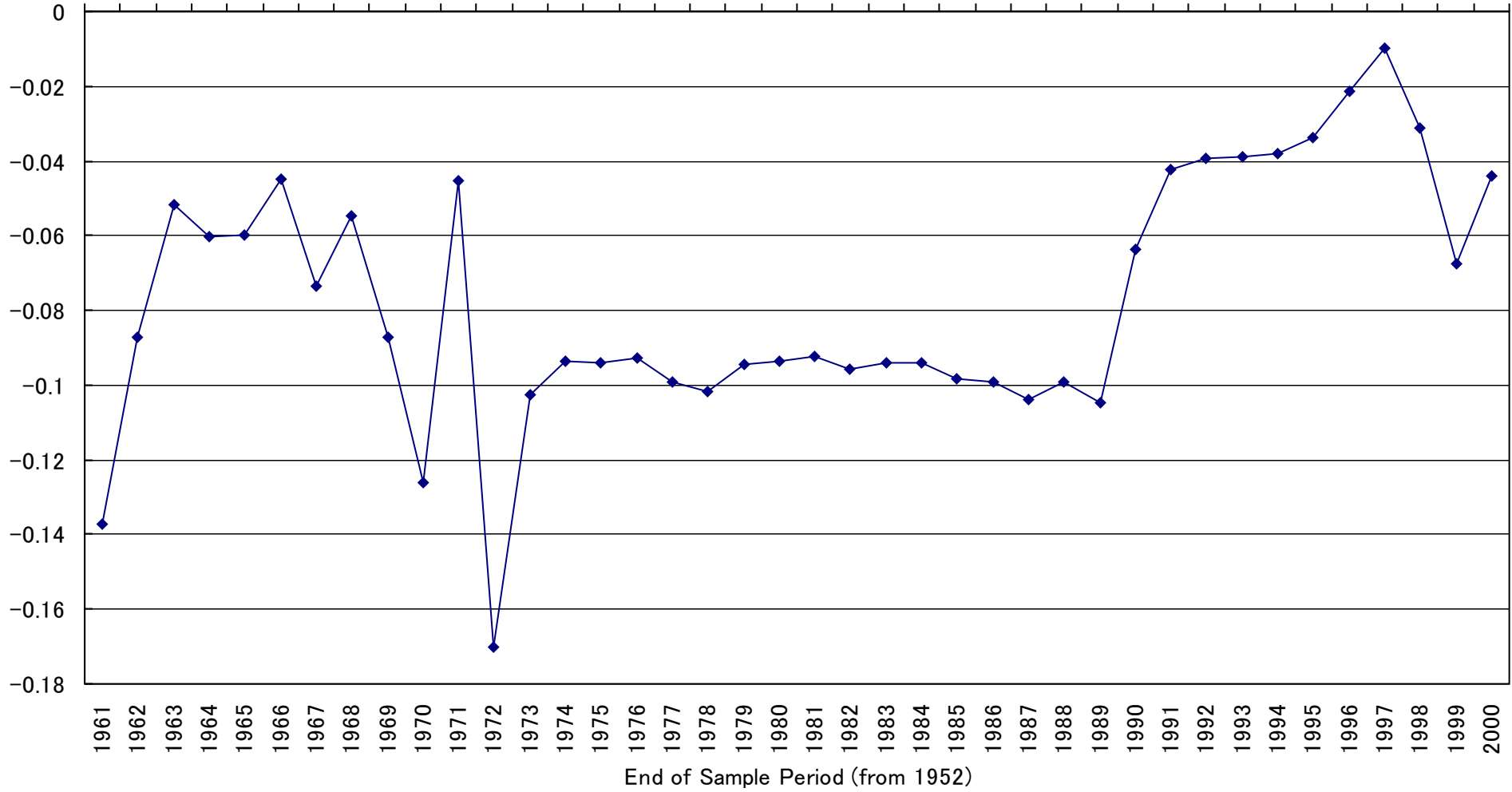
◆ 1st principal component ■ 2nd principal component ▲ 3rd principal component

Coefficients of Demographic and Financial Model (Dependent Var.=Excess Return on Equity)



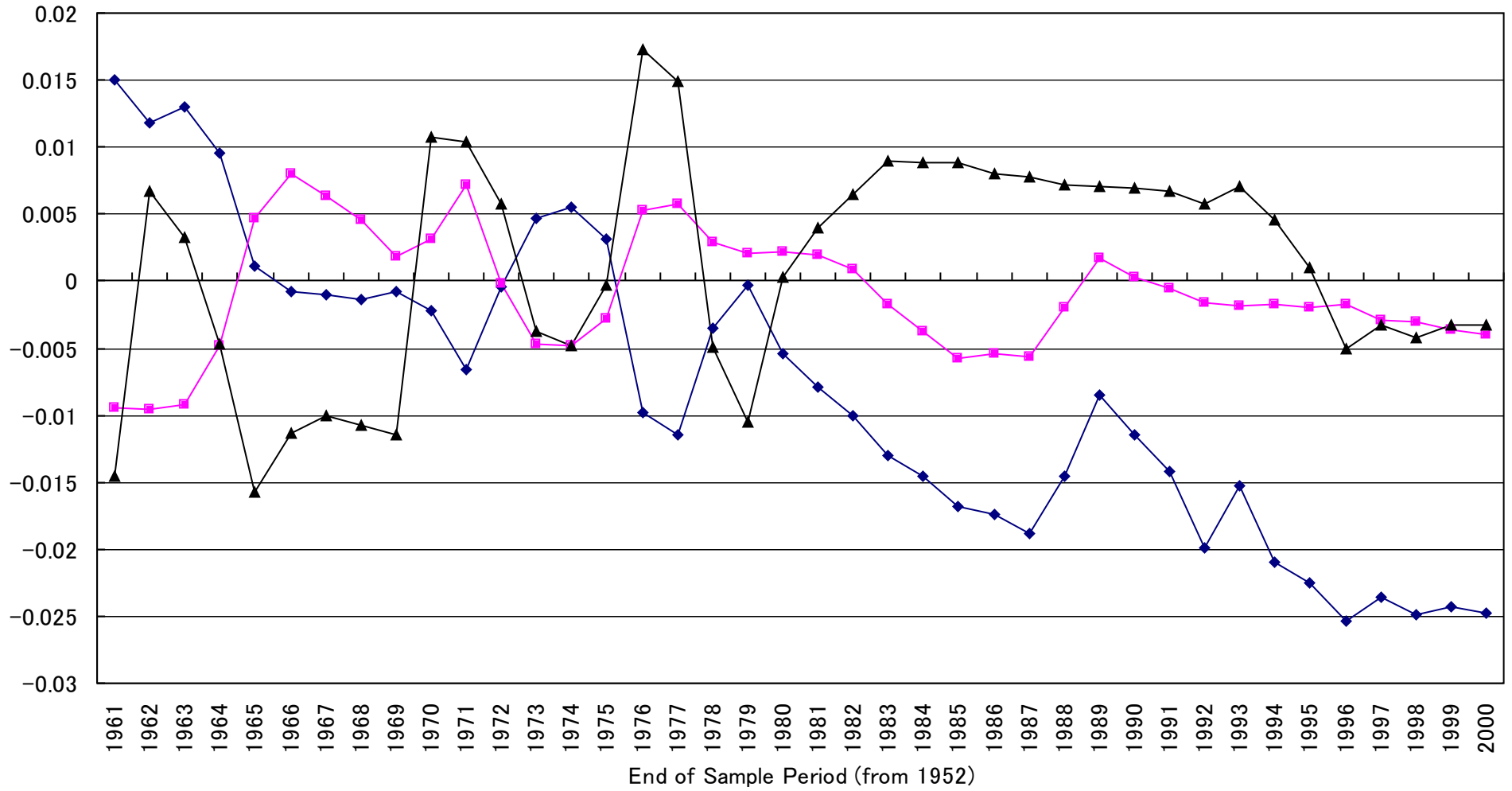
◆ 1st principal component ■ 2nd principal component ▲ 3rd principal component × one-lagged dependent variable

Coefficients of Financial Model (Dependent Var.=Excess Return on Equity)



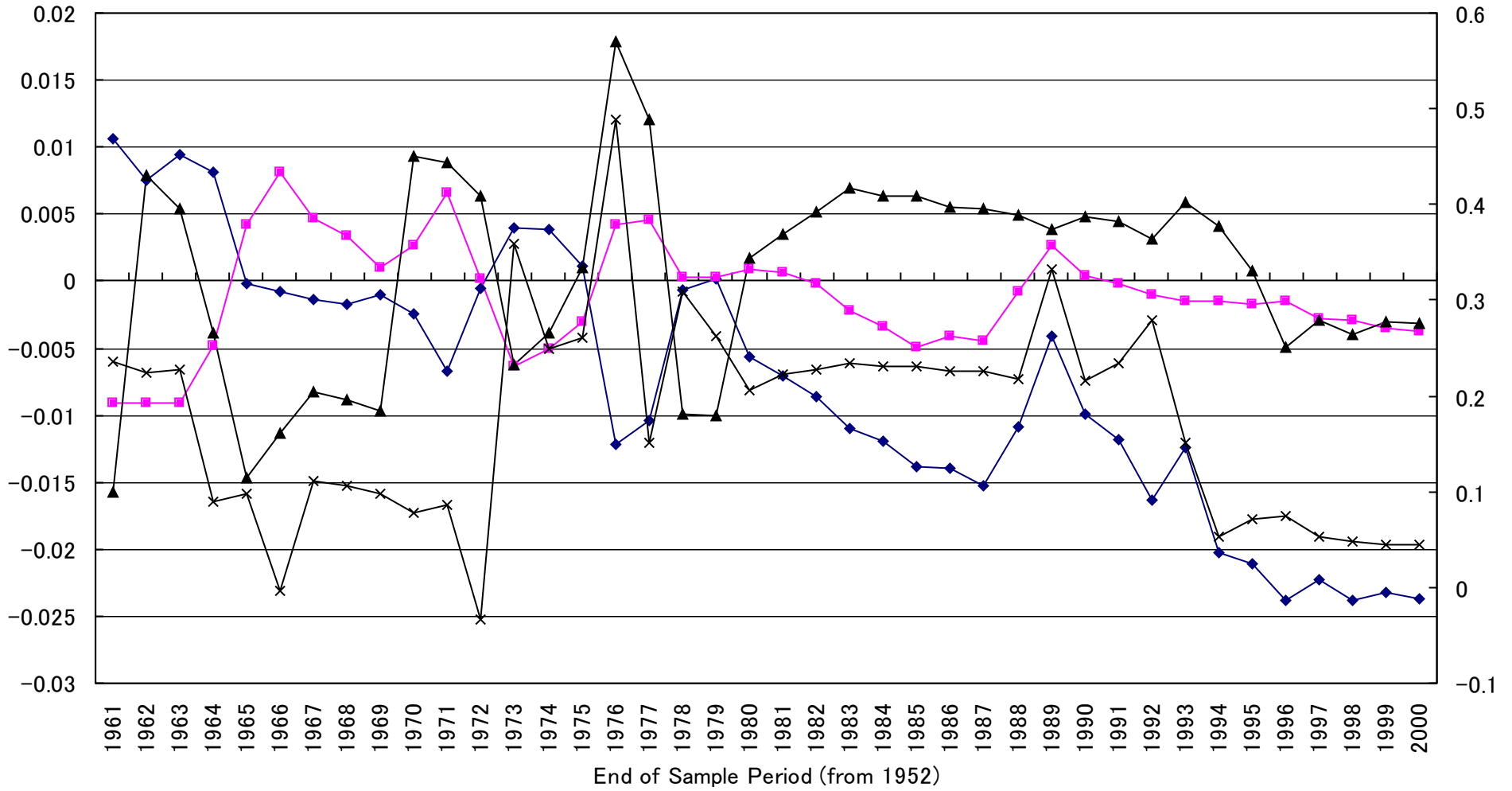
◆ one-lagged dependent variable

Coefficients of Demographic Model (Dependent Var.= Excess Return on Bond)



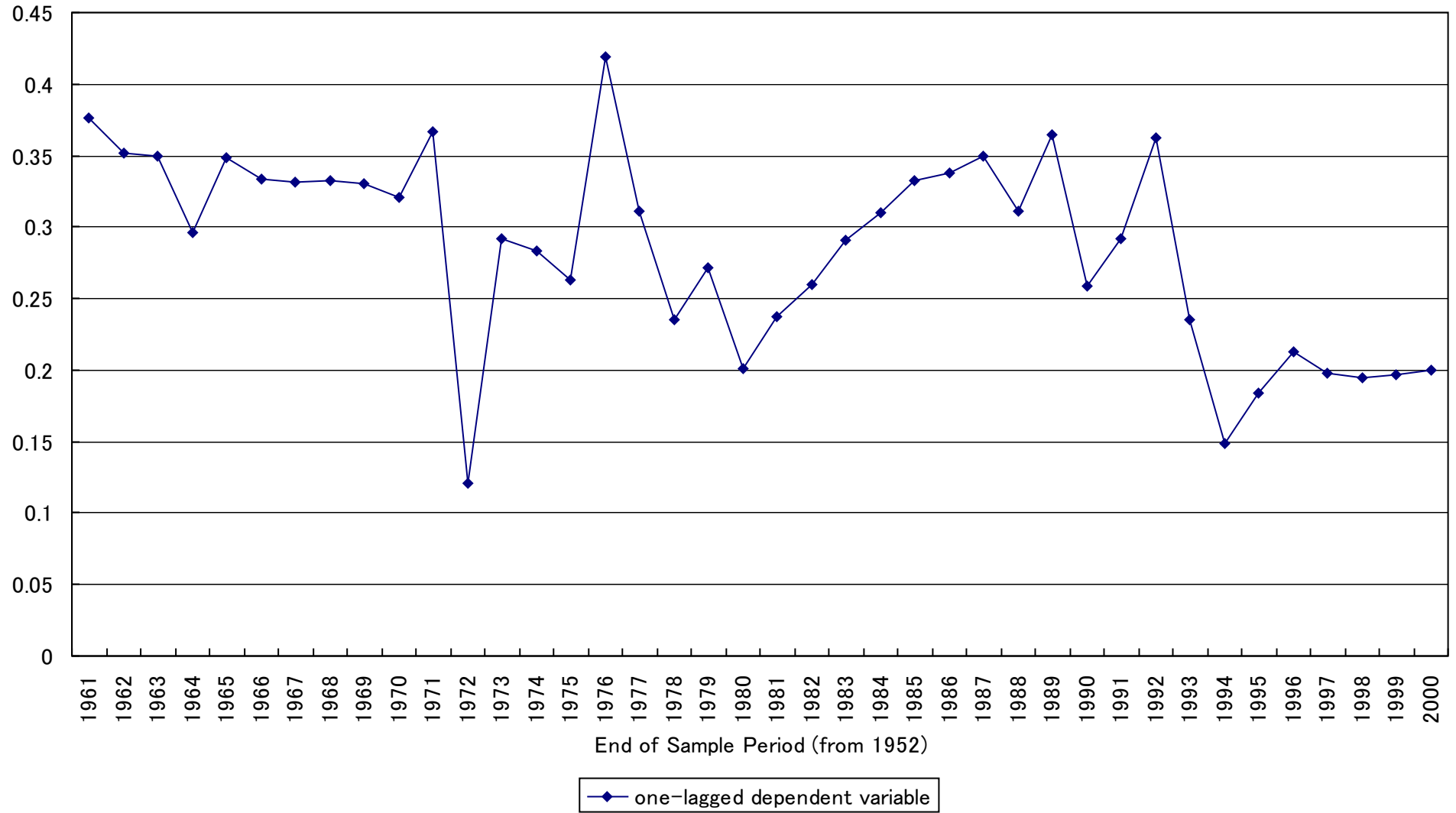
—◆— 1st principal component —■— 2nd principal component —▲— 3rd principal component

Coefficients of Demographic and Financial Model (Dependent Var. = Excess Return on Bond)



◆ 1st principal component ■ 2nd principal component ▲ 3rd principal component × one-lagged dependent variable (Right Axis)

Coefficient of Financial Model (Dependent Var. = Excess Return on Bond)



Post- War Period

Stock - Mbney

| | PC1 | PC2 | PC3 | Re - Rf |
|---------------------------------|------|------|------|---------|
| Demographic Model | | | | |
| coefficient | 0.05 | 0.01 | 0.03 | |
| t-value | 1.30 | 0.23 | 0.75 | |
| Demographic and Financial Model | | | | |
| coefficient | 0.06 | 0.01 | 0.03 | -0.09 |
| t-value | 1.42 | 0.23 | 0.73 | -0.67 |
| Financial Model | | | | |
| coefficient | | | | -0.04 |
| t-value | | | | -0.35 |

Bond - Mbney

| | PC1 | PC2 | PC3 | Rb - Rf |
|---------------------------------|-------|-------|-------|---------|
| Demographic Model | | | | |
| coefficient | -0.02 | 0.00 | 0.00 | |
| t-value | -2.97 | -0.47 | -0.40 | |
| Demographic and Financial Model | | | | |
| coefficient | -0.02 | 0.00 | 0.00 | 0.04 |
| t-value | -2.61 | -0.44 | -0.37 | 0.30 |
| Financial Model | | | | |
| coefficient | | | | 0.20 |
| t-value | | | | 1.39 |

Table 2

One-Step Ahead Forecast for Excess Returns from Demographic, Demographic and Financial and Financial Models

| 1930-1939 | \bar{R}^2 | Root Mean Squared Error | Theil Ratio | RMSE no-change | correlation between forecast and actual excess return |
|---------------------------------|-------------|-------------------------|-------------|----------------|---|
| BONDS | | | | | |
| Demographic model | 0.629 | 0.00579 | 0.75106 | 0.0077 | 0.258 |
| demographic and financial model | 0.681 | 0.00550 | 0.71413 | 0.0077 | 0.337 |
| financial model | 0.646 | 0.00495 | 0.64251 | 0.0077 | 0.441 |
| EQUITY | | | | | |
| Demographic model | -0.082 | 0.15976 | 1.25880 | 0.1269 | -0.635 |
| demographic and financial model | -0.153 | 0.18470 | 1.45535 | 0.1269 | -0.543 |
| financial model | -0.050 | 0.14650 | 1.15431 | 0.1269 | -0.489 |
| <hr/> | | | | | |
| 1962-2001 | \bar{R}^2 | Root Mean Squared Error | Theil Ratio | RMSE no-change | correlation between forecast and actual excess return |
| BONDS | | | | | |
| Demographic model | 0.115 | 0.06724 | 0.99818 | 0.06736 | 0.119 |
| demographic and financial model | 0.096 | 0.06939 | 1.03017 | 0.06736 | 0.136 |
| financial model | 0.019 | 0.06840 | 1.01553 | 0.06736 | 0.116 |
| EQUITY | | | | | |
| Demographic model | -0.014 | 0.29052 | 1.02135 | 0.28445 | -0.073 |
| demographic and financial model | -0.026 | 0.29861 | 1.04980 | 0.28445 | -0.088 |
| financial model | -0.019 | 0.28093 | 0.98762 | 0.28445 | -0.252 |

Optimal Asset Allocation based on the forecasted excess return

Derive the optimal asset allocation among stock, bond and cash by considering the optimization problem of an investor with a mean-variance utility function

This optimization problem can be written as an optimization problem on the excess return

$$\max_{w_t^e, w_t^b} \left[\begin{pmatrix} w_t^e & w_t^b \end{pmatrix} \begin{pmatrix} E R^e & -R^f \\ t & t+1 & t+1 \end{pmatrix} - \frac{1}{2} k \begin{pmatrix} w_t^e & w_t^b \end{pmatrix} \begin{pmatrix} (\sigma^e)^2 & \sigma^{eb} \\ t & t \end{pmatrix} \begin{pmatrix} w_t^e \\ t \\ w_t^b \\ t \end{pmatrix} \Big|_{\Omega_t} \right]$$

Demographic model

$$E_t R_{t+1} e^{-R_{t+1}} f \leftarrow \overbrace{(R_{t+1} e^{-R_{t+1}} f)} = \hat{a}_0 + \hat{a}_1 PC1_t + \hat{a}_2 PC2_t + \hat{a}_3 PC3_t$$

$$E_t R_{t+1} b - R_{t+1} f \leftarrow \overbrace{(R_{t+1} b - R_{t+1} f)} = \hat{a}_0 + \hat{a}_1 PC1_t + \hat{a}_2 PC2_t + \hat{a}_3 PC3_t$$

Compare the optimal allocation using conditional excess returns estimated from demographic model with the allocation based on 3 simple buy-and-hold strategies (100% stock, 100% bond, and 50% stock and 50% bond)

Table 3: Mean Excess Returns and Sharpe Ratio from the optimal asset allocation and buy-and-hold allocation

Pre-War

- 1. Asset allocation from demographic model has higher mean excess return than all buy-and-hold allocations for $k=1,2$, and 5**
- 2. Asset allocation from demographic model has higher Sharpe ratio than 100% equity for $k=1,2,5$, and 10**

Post-War

- 1. Asset allocation from demographic model has higher mean excess return than all buy-and-hold allocations for $k=1,2$, and 5**
- 2. Asset allocation from demographic model has higher Sharpe ratio than 100% equity AND 100% bond for $k=1,2,5$, and 10**

Robustness Check on the Optimal Asset Allocation estimated from demographic model

Table 3

Mean Excess Returns and Sharpe Ratios for Demographic Model and Passive Model

| Sample Period: 1930-1939 | Mean Excess Return | Sharpe Ratio |
|-----------------------------------|--------------------|--------------|
| 100% equity | 3.84% | 0.26 |
| 100% bond | 1.35% | 1.70 |
| 3 asset demographic model | | |
| <i>k=1</i> | 5.47% | 0.60 |
| <i>k=2</i> | 5.43% | 0.59 |
| <i>k=5</i> | 4.05% | 0.59 |
| <i>k=10</i> | 3.20% | 0.67 |
| <i>k=5000</i> | 0.13% | 0.17 |
| 2 asset demographic model, equity | | |
| <i>k=1</i> | 3.95% | 0.43 |
| <i>k=2</i> | 3.91% | 0.43 |
| <i>k=5</i> | 3.88% | 0.47 |
| <i>k=10</i> | 2.25% | 0.48 |
| <i>k=5000</i> | 0.00% | 0.01 |
| 2 asset demographic model, bonds | | |
| <i>k=1</i> | 1.35% | 1.70 |
| <i>k=2</i> | 1.35% | 1.70 |
| <i>k=5</i> | 1.35% | 1.70 |
| <i>k=10</i> | 1.35% | 1.70 |
| <i>k=5000</i> | 0.01% | 0.15 |

| Sample Period: 1962-2001 | Mean Excess Return | Sharpe Ratio |
|-----------------------------------|--------------------|--------------|
| 100% equity | 6.26% | 0.24 |
| 100% bond | 1.55% | 0.27 |
| 3 asset demographic model | | |
| <i>k=1</i> | 8.54% | 0.37 |
| <i>k=2</i> | 8.00% | 0.39 |
| <i>k=5</i> | 5.60% | 0.37 |
| <i>k=10</i> | 3.83% | 0.48 |
| <i>k=5000</i> | 0.01% | 0.00 |
| 2 asset demographic model, equity | | |
| <i>k=1</i> | 6.35% | 0.28 |
| <i>k=2</i> | 5.77% | 0.28 |
| <i>k=5</i> | 3.50% | 0.23 |
| <i>k=10</i> | 1.75% | 0.22 |
| <i>k=5000</i> | 0.00% | 0.00 |
| 2 asset demographic model, bonds | | |
| <i>k=1</i> | 2.50% | 0.53 |
| <i>k=2</i> | 2.54% | 0.54 |
| <i>k=5</i> | 2.43% | 0.52 |
| <i>k=10</i> | 2.34% | 0.55 |
| <i>k=5000</i> | 0.01% | 0.00 |

Demographic and financial model

$$E_t R_{t+1} e^{-R_{t+1}} f \leftarrow \overbrace{(R_{t+1} e^{-R_{t+1}} f)} = \widehat{a}_0 + \widehat{a}_1 PC1_t + \widehat{a}_2 PC2_t + \widehat{a}_3 PC3_t + \widehat{a}_4 (R_t e^{-R_t} f)$$

$$E_t R_{t+1} b^{-R_{t+1}} f \leftarrow \overbrace{(R_{t+1} b^{-R_{t+1}} f)} = \widehat{a}_0 + \widehat{a}_1 PC1_t + \widehat{a}_2 PC2_t + \widehat{a}_3 PC3_t + \widehat{a}_4 (R_t b^{-R_t} f)$$

Financial model

$$E_t R_{t+1} e^{-R_{t+1}} f \leftarrow \overbrace{(R_{t+1} e^{-R_{t+1}} f)} = \widehat{a}_0 + \widehat{a}_1 (R_t e^{-R_t} f)$$

$$E_t R_{t+1} b^{-R_{t+1}} f \leftarrow \overbrace{(R_{t+1} b^{-R_{t+1}} f)} = \widehat{a}_0 + \widehat{a}_1 (R_t b^{-R_t} f)$$

Table 4: Mean Excess Returns and Sharpe Ratios for demographic, financial and demographic and financial models

Pre-War

K=1, allocations from all three models are the same

K=2, allocation from demographic model has highest return

K=5,10,5000, allocation from demographic and financial model has higher return than that from demographic model which has higher return than that from financial model

Post-War

K=1, allocation from financial model has higher return but lower Sharpe ratio

K=2,5 allocation from demographic model has the highest return

K=10, allocation from demographic and financial model is the best demographic components contribute to returns even when financial component is in the model

Table 4**Mean Excess Returns and Sharpe Ratios for 3
asset optimal portfolios that trade on
Demographic and Financial Indicators or
Financial Indicators only**

| Sample Period: 1930-1939 | Mean Excess Return | Sharpe Ratio |
|---------------------------------|--------------------------|-----------------|
| demographic and financial model | | |
| k=1 | 5.47% | 0.60 |
| k=2 | 5.41% | 0.59 |
| k=5 | 4.43% | 0.58 |
| k=10 | 3.46% | 0.66 |
| k=5000 | 0.13% | 0.18 |
| financial model | | |
| k=1 | 5.47% | 0.60 |
| k=2 | 5.12% | 0.58 |
| k=5 | 3.48% | 0.66 |
| k=10 | 2.42% | 0.91 |
| k=5000 | 0.11% | 0.15 |

| Sample Period: 1962-2001 | Mean Excess Return | Sharpe Ratio |
|---------------------------------|--------------------------|-----------------|
| demographic and financial model | | |
| k=1 | 8.45% | 0.37 |
| k=2 | 7.02% | 0.33 |
| k=5 | 5.13% | 0.36 |
| k=10 | 3.99% | 0.53 |
| k=5000 | 0.03% | 0.01 |
| financial model | | |
| k=1 | 8.75% | 0.34 |
| k=2 | 5.11% | 0.30 |
| k=5 | 3.61% | 0.45 |
| k=10 | 2.83% | 0.55 |
| k=5000 | 0.02% | 0.01 |

Conclusion and Extension

The demographic variables affect excess asset returns!

We need to distinguish between an observed change in demography and an unobserved change in or a shock to demography

> We can model and forecast demographic changes and relate the shock to demographic change to future excess returns

We can expand the asset class and include real estate returns. The problem with real estate data is that it does not include dividends and we need to adjust the real estate index. Ad-hoc way is to assume that the dividend is 2%.

Results on Real Estate Index Return

Data: Urban Land Price Index of Nationwide (average of commercial, residential and industrial) 1956-2001

Results: Active (land, bond and money) with 2% dividend outperforms 100% land with or 100% bond

| | Land | Bond | Active |
|---------------|-------|-------|---|
| Excess Return | 7.1% | 7.5% | 150%, 78%, 34%, 20% (k=1),(k=2),(k=5),(k=10) |
| Sharpe Ratio | 0.169 | 0.335 | 0.589,0.588,0.585,0.579 (k=1),(k=2),(k=5),(k=10) |

Problem with Real Estate Index

- 1. Does not include “dividends”. Need to assume dividend. In the U.S., 10% is assumed in Nakajima (2003).**
- 2. Strong trend in the index return. Ex-post return was higher than money in the 50s, 60s and 70s, but lower in the 80s and 90s.**
- 3. The index is not transactions based. Condominium transaction price index might be a better alternative.**