

Temple University  
Department of Economics

**Econometrics I**  
**Hypothesis Testing**

1. In the recent crash of TWA 800 (summer 1996) the working hypothesis has been that the plane was brought down by an onboard explosion. In trying to prove this the authorities have been testing for chemical residue on the wreckage. There is a test lab on Long Island denoted R and a lab in D.C. denoted U. Some of the wreckage was initially tested at the L.I. site. Other pieces of wreckage were tested only at the DC lab. Whenever the result at the LI site was positive for residue the authorities in DC rejected the finding pending confirmation by the DC lab. This problem is designed to explore the statistical consequences of this behavior.

We wish to test the null hypothesis  $H_0: \theta=1$  versus the alternative hypothesis  $H_1: \theta=10$ . Two devices are available for measuring  $\theta$ : Device R produces a single observation  $y \sim N(\theta, 100)$  and device U produces a single observation  $y \sim N(\theta, 1)$ . Unfortunately, which device is available is determined by a random event J, with

$$P(J = R) = .05 - \varepsilon$$

$$P(J = U) = .95 + \varepsilon$$

$$\varepsilon = \frac{.95[1 - \Phi(4.5)]}{\Phi(4.5)}$$

Clearly the probability of selecting the more accurate testing device is quite high.

The decision rule for testing the hypothesis may be summarized as: When using device R the decision rule for the test of hypothesis is that  $H_0$  is always rejected regardless of the value of the test statistic  $y$ . For device U the null  $H_0$  is rejected if and only if  $y > 4.5$ .

- a. Compute  $P(\text{rejecting } H_0 \mid H_0, R)$ .
- b. Compute  $P(\text{rejecting } H_0 \mid H_0, U)$ .
- c. Compute  $P(\text{rejecting } H_0 \mid H_1)$ .
- d. Compute  $P(\text{rejecting } H_0 \mid H_2)$ .
- e. Suppose  $J=R$  and then  $y=1$  is observed. What do you conclude?
- f. Is your answer to part e counter-intuitive, given your answers to parts a-d?

2. Suppose  $Y \sim N(0, .25)$  and consider the hypotheses  $H_1: \mu=-1$  and  $H_2: \mu=1$ .

- a. Show that the critical region  $Y \geq 0$  yields error probabilities  $\alpha = \beta = .0228$ .
- b. Suppose  $y = 0$  is observed. Intuitively which hypothesis does this observation support?
- c. Suppose  $y = 1$  is observed. Which hypothesis does this result favor?
- d. For the realizations of  $y$  in both parts b and c perform a formal test of hypothesis of  $H_1$  versus  $H_2$  at the 5% level of significance. Do you obtain answers similar to your previous intuition?