Temple University
Department of Economics

Econometrics II
Linear Algebra

1. Show that the matrix

$$Q = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{30}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{30}} \\ \frac{1}{\sqrt{6}} & 0 & -\frac{5}{\sqrt{30}} \end{bmatrix}$$

is orthogonal, i.e., $Q' = Q^{-1}$.

2. Given $X' = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 3 \end{bmatrix}$ compute $M = X(X'X)^{-1}X'$ and $I - M$. Show that
$M^2 = M$ and $(I-M)^2 = I-M$ and therefore $M(I-M) = 0$.

3. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Compute the following:

a. the characteristic roots of A (from $|A-\lambda I|=0$),
b. the corresponding characteristic vectors.
c. Show that the characteristic vectors are orthogonal.
d. Find the matrix P such that (i) $P'P = I$ and (ii) $P'AP = D$, a diagonal matrix.

4. Let $A = \begin{bmatrix} 3 & \sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$

a. Find the characteristic roots of A.
b. Find a matrix P such that $P'P = I$ and $P'AP = D$, a diagonal matrix with the characteristic roots of A on the diagonal.
c. Find a matrix Q such that $Q'DQ = I$.
d. Plot the quadratic form $x'Ax = 4$.
e. Plot the quadratic form $y'Dy = 4$.
f. Plot the quadratic form $z'Iz = 4$.
g. Discuss the geometric interpretation of the transformations P and Q which you found. In particular, how does the transformation from x to y to z coordinates affect the objects plotted in d., e. and f.?