Temple University
Department of Economics

Econometrics I
Estimation

1. Consider two random variables X and Y and the scalar parameter θ related by $f(x|\theta) = \theta e^{-\theta x}$ for x > 0 and $\theta > 0$, and $f(y|x,\theta) = f(y|x) = x e^{-xy}$ for y > 0 and x > 0. Suppose the researcher wishes to estimate θ, observes Y, but does not observe X. Obtain analytical expressions for each of the following three likelihood functions:

    1. $\Lambda_1(\theta; y) = f(y|\theta) = \int_0^\infty f(y|x)f(x|\theta)dx$

    2. $\Lambda_2(\theta; x, y) = f(y, x|\theta)$

    3. $\Lambda_3(\theta, y; x) = f(x|\theta, y)$

Which of the three is an appropriate likelihood function?

2. Suppose Y_1 and Y_2 are independently distributed with the same variance σ^2, but with different means: $E(Y_1)=2\theta$ and $E(Y_2)=4\theta$. Consider the estimator $\hat{\theta} = w_1 Y_1 + w_2 Y_2$, where w_1 and w_2 are unknown weights. Find w_1 and w_2 so that $\hat{\theta}$ has the smallest possible variance, and yet is unbiased.

3. Suppose Y_t (t=1,2,...,T) are i.i.d. Bernoulli random variables such that $P_t = P(Y_t=1) = \Phi(\alpha)$ and $1-P_t = P(Y_t=0) = 1 - \Phi(\alpha)$ where $\Phi(\cdot)$ is the standard normal cdf. The sample corresponds to a cross section of individuals. The first m individuals are homeowners and the last T-m are renters. Find the maximum likelihood estimator for α.

4. Suppose a random sample of size T=7 from a $N(\mu,\sigma^2)$ yields $\bar{y} = 5$ and $s^2 = 1.96$. Find a 95% confidence interval for μ when:

    a. $\sigma^2 = 2$ is known.
    b. σ^2 is unknown.