

Temple University  
Department of Economics

**Econometrics I**  
**Functions of Random Variables and Sampling Distributions**

1. Let  $X_1$  and  $X_2$  be independent random variables each with probability density function  $f(x) = 2x$  for  $0 < x < 1$ , and zero otherwise. Define  $Y = X_1/X_2$ . Find  $P(Y > 1/2)$ . (Hint: The probability density for  $Y$  defined as the ratio of two r.v.'s is in

general given by

$$g(Y) = \int_{-\infty}^{\infty} |x_2| f(x_2 y, x_2) dx_2$$

2. Consider the sample values  $Y_t$  ( $t=1, 2, \dots, T$ ). Show:

$$\max_{1 \leq t \leq T} |y_t - \bar{y}| \geq \left( \frac{T-1}{T} \right)^{\frac{1}{2}} s$$

where  $\bar{y}$  and  $s$  denote the realized sample mean and sample standard deviation.

3. Let  $X$  denote the hours of study required for a passing grade in an econometrics course. Assume that the distribution of  $X$  is  $N(147.8, 12.3^2)$ .

a. Find  $P(X < 163.3)$ .

b. Suppose you sample 25 econometrics students at random.

What is  $P(\bar{X} \leq 159.9)$ ?

c. Find constants  $a$  and  $b$  such that  $P(a < S^2 < b) = .90$ .