Workouts in Hypothesis Testing: Part I

1. Hypothesis Testing and Intuition:

The random variable y follows a normal distribution with unknown mean (θ) and σ^2=.25. The null and alternate are:

H_o: $\theta = -1$
H_1: $\theta = 1$

a. You observe y=0. Which hypothesis does this support? Intuitively, it does not support either.

b. You observe y=1. Which hypothesis does this support? Most would respond that they would be inclined to reject the null hypothesis.

c. Choose c* = 0 as the critical value of y for the purpose of hypothesis testing. What is the probability of a Type I error?

$P(y>0 \mid \theta=-1) = P(z>\frac{(0+1)}{.5}) = P(z>2) = .0228$

What is the probability of a Type II error?

$P(y<0|\theta=1) = P(z < (0-1)/.5) = P(z<-2) = .0228$

Comment on your intuition in light of these results.

d. The 5% critical value is z = 1.645, or y = -.178, for a one tail test.
i. When we observe y=0, we reject the null hypothesis.
ii. When we observe y=1, we reject the null.
For the 'conventional' level of a test our conclusion differs from our original intuition.

2. A Test for the Variance

H_1: σ^2=40 ($\sigma := \sqrt{40}$)

H_2: σ^2<>40

We have the following sample information:

$n := 9 \qquad s := \sqrt{32}$

Now under the null $\frac{(n-1)\cdot s^2}{\sigma^2}$ has a chi-square distribution. At the 1% level the critical values are 1.35 and 21.96.

$$\frac{(n-1)\cdot s^2}{\sigma^2} = 6.4$$

Do not reject the null hypothesis.

3. Equality of means in two samples.

From our sample data

$s_1 := \sqrt{32}$ $\quad xbar_1 := 4$ $\quad n_1 := 9$

$s_2 := \sqrt{42}$ $\quad xbar_2 := 8$ $\quad n_2 := 16$

The samples are independent. The null and alternate , to be tested at the 5% level, are

H_o: $\mu_1 = \mu_2$
H_1: μ_1<>μ_2

Define a new random variable

$xdiff := xbar_1 - xbar_2$ $\quad xdiff = -4$

If the two samples are drawn from the same population then a pooled estimate of the population standard deviation is

$$s := \sqrt{\frac{(n_1 - 1)\cdot(s_1)^2 + (n_2 - 1)\cdot(s_2)^2}{n_1 + n_2 - 2}}$$

$$s = 6.207$$

Since the samples are independent of one another, Var(xdiff) is just the sum of the variances of the sample means. You should be able to derive the denominator in the following test statistic.

The test statistic is

$$t := \frac{xdiff}{\sqrt{s^2 \cdot \frac{(n_1 + n_2)}{n_1 \cdot n_2}}}$$

$$t = -1.547$$

With 23 degrees of freedom, the critical values are -2.069 and +2.069. Do not reject the null.

4. Testing for equality of Variances

At the 2% level test the hypothesis

H_o: $\sigma_1^2 = \sigma_2^2$

H_1: $\sigma_1^2 <> \sigma_2^2$

We have observed the following sample information

$n_1 := 9$ $\quad s_2 := \sqrt{42}$ $\quad xbar_2 := 8$

$n_2 := 16$ $\quad s_1 := \sqrt{32}$ $\quad xbar_1 := 4$

The test statistic is

$$F := \frac{\frac{(s_1)^2}{\sigma^2}}{\frac{(s_2)^2}{\sigma^2}} \qquad F = 0.762$$

There are 8 degrees of freedom in the numerator and 15 degrees of freedom in the denominator. The critical values are 1/4.00 and 4.00. Since the test statistic lies between these values, do not reject the null.

5. A Joint Test

At the 5% level test the hypothesis

H_o: μ_1=0 and μ_2=0
H_1: one or the other is not zero

using the two-sample data of the previous examples.

$$F := \begin{bmatrix} xbar_1 & xbar_2 \end{bmatrix} \cdot \begin{bmatrix} \frac{(s_1)^2}{n_1} & 0 \\ 0 & \frac{(s_2)^2}{n_2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} xbar_1 \\ xbar_2 \end{bmatrix}$$

$$F = 28.881$$

There are 2 degrees of freedom in the numerator (from the two restrictions in the null hypothesis) and n_1+n_2-2 = 23 degrees of freedom in the denominator. The 5% critical value is 3.42, so we reject the null.