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Three Stage Least Squares with Inequality Constraints: Auto Theft and Police Expenditure[1])

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Abstract: Using behavorial models of utility maximization on the part of communities and criminals, a two equation model of police expenditure and auto theft is derived. On the basis of previous empirical work and economic theory, certain parameters should be estimated subject to inequality constraints. An appropriate simultaneous equations estimator is derived and applied to data for 230 communities in New Jersey.

1. Introduction

In the last decade the analysis of police expenditure has received considerable attention in the economics and public finance literature [*Beaton; Chapman/Hirsch/Sonenblum; Popp/Sebold; Walzer*]. A related question which has emerged following the pioneering work of *Becker* [1968] analyzes the rational behavior of criminals and its effects on the supply of offences. The interrelationship between the supply of crimes and the level of police expenditure or the deployment of police is the subject of an increasing number of studies [*Bahl/Gustely/Wasylenko; Carr-Hill/Stern; Furlong/Mehay; Greenwood/Wadycki; Hakim* et al., 1978; *McPheters/Stronge; Thaler*]. Our illustrative model concerns the factors which determine the level of police services. The decision on the level of policing is based upon variables that measure the demand for police protection (wealth of the community and level of property crimes) and variables that determine the production of police services (effectiveness of policing and resource availability). Community disposable income seems to be the primary argument from both supply and demand attributes that explains police expenditure. Our model of the supply of offenses is based on rational choices made by potential criminals weighing the opportunities confronting them.

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The cross section data on wealth, crime opportunity, police expenditure and property crime used to estimate the simultaneous equations model of expenditure and offenses can be divided into three groups of communities. Separation of the groups is according to distance from the central city, which reflects accessibility for potential criminals and level of economic development. Using a dummy variable to indicate accessibility and development results in multicollinearity between independent variables. Transforming the independent variables in a manner similar to that suggested by *Searle* [1971] for use in the analysis of covariance accomplishes three purposes: First, it reduces the linear dependence between independent variables. Second, it allows slope parameters, in addition to intercepts, for the three groups to be different. Finally, it allows us to use additional prior information about the effects of certain independent variables, in the form of inequality constraints, in our estimation procedure. By introducing sound prior information, we are able to improve our estimates of the remaining hypothesized coefficients.

The present paper proposes formal procedures for the incorporation of prior information, in the form of inequality constraints, in a classical simultaneous equations model. The proposed estimator is a variant of the well known three stage least squares estimator. We illustrate the use of such an estimator in a two equation model of burglaries and police expenditure in New Jersey.

The work presented here is innovative in that previous work on inequality constrained estimation has been confined to single equation models.[3]) *Hanson* [1965] proved the existence of inequality constrained maximum likelihood single equation estimators. A year later *Judge/Takayama* [1966] demonstrated the value of quadratic programming for obtaining estimates of the parameters in a model constrained by inequalities. More recently, *Liew* [1976a, b] and *Klemm/Sposito* [1976] have suggested closed form estimates. The work by Liew makes some reference to the properties of the inequality constrained least squares (ICLS) estimator. Other recent investigations of constrained estimation include *Wardle* [1974] and *Wedderburn* [1974]. A more formal effort to determine the properties of the ICLS estimator was made by *Lovell/Prescott* [1970] and most recently by *Buck/Hakim* [1981b].

In the second section we introduce the economic model which leads to the econometric specification. The third section presents the general statistical model and the assumptions to be used in estimation. Also included in this section is a discussion of the choice of the constraint generalization that is considered in detail. The empirical results are also reported.

In the last section the interpretations and conclusions of the ICLS estimators are cast in the light of their usefulness in applied modelling. The results demonstrate the usefulness of the consideration of constraints in modelling police outlays and criminal behavior, and suggests other areas of analysis where these techniques may be similarly useful.

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[3]) The referee has drawn our attention to Jöreskog's LISREL program, which permits interval restrictions in the maximum likelihood estimation of the parameters of multiequation models.

2. A Model of Police Expenditures and Auto Thefts

The model of police expenditure posits a community which both produces and consumes an abstract good which is called security, denoted S. The model developed here is adapted from *Lindahl* [1958] and is an economy with personalized prices for the public commodity. That is, prices which are unique to each consumer and which could be established following the procedures suggested by *Malinvaud* [1971]. One obvious assumption is that consumers correctly reveal their preferences.[4]) It seems to us that although security is a very abstract service, it is more encompassing and more accurately represents what the community believes it is buying from its police department than other possible services.

Unless otherwise noted, all the usual regularity, continuity and convexity assumptions for preferences and production are made in deriving the supply of and demand for protection.

As consumers of security, the members of the community wish to maximize their utility from the consumption of S as well as all other goods (Z), subject to a budget constraint. Thus, for each of n individuals in the community we have

$$\max_{s,z} U_i(S_i, Z_i) \qquad i = 1, \ldots, n \tag{2.1}$$

subject to the budget constraint

$$B_i = p_s^i S_i + P_z Z_i \tag{2.2}$$

where B is income and the P's are prices.

From the first order conditions from the maximization of (2.1) subject to (2.2) one can derive the individual's demand curve for security. Each individual is confronted with his personalized price of a unit of security and he announces truthfully his desired level of consumption. The security provided to the community is determined by the largest quantity demanded by any individual in the jurisdiction. Since security is a public good, the individual demand curves are being summed vertically. The market demand curve for security will be given by:

$$S^d = S^d\left(\sum_{i=1}^{n} P_s^i, P_z, B_i\right). \tag{2.3}$$

It is additionally known that the total amount spent on security must equal a given fraction of tax payments. That is,

$$S \sum_{i=1}^{n} P_s^i = k \sum_{i=1}^{n} (t_w W_i) \tag{2.4}$$

where t_w is the tax rate on private property (W), and k is the exogenously determined

[4]) One might argue that preferences are correctly revealed by one's choice of residence, both between communities and within a particular community [*Tiebout*]. The fact that security varies within the community and we do not assume identical utility curves necessitates the use of personalized prices.

fraction of the community budget designated for security.[5]) Thus, when W increases, expenditure on security also increases.

Now consider the community as a producer of security. Although it is a monopoly producer of security, the police department faces a zero profit constraint imposed by the residents. From particular choices of capital (K) and labor (L) inputs the police department allows a certain level of auto thefts (AUTO).[6]) These tangible results of the police function in combination with a set of demographic and economic characteristics that assigns the community to either one of three groups (A), produces a level of security, S. Thus, the maximization problem at hand is as follows:

$$\max_{K,L} \; S = f(\text{AUTO}, A) \tag{2.5}$$

subject to the profit and production constraints

$$S\,(\Sigma P_S^i) = P_K\,K + P_L L \tag{2.6}$$

$$\text{AUTO} = g\,(K, L). \tag{2.7}$$

From the maximization of (2.5) subject to (2.6) and (2.7) we can determine the quantity of security supplied. That is

$$S^s = S^s\,(P_K, P_L, \sum_{i=1}^{n} P_s^i)\,. \tag{2.8}$$

It should be noted that $S\Sigma P_s^i$ is equal to the right hand side of (2.4). In equilibrium we have equality between (2.3) and (2.8).[7])

Statistical modelling of the market for security presents several difficulties. The obvious problem is that one cannot measure the number of units of security consumed by a community, nor can one observe the unit price. Furthermore, there are considerable difficulties in observing P_K and P_L.

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[5]) The community continues to spend on police services up to the level at which the marginal benefit of the last dollar spent is equal for all local public services. Since this issue is only marginally related to our subject, the determination of k is not being treated explicitly.

[6]) Several researchers have examined the complexity of defining and measuring police output. Security is a joint product which is a function of both inputs and other socioeconomic and physical attributes of the community [*Hakim*, 41–44; *Hirsch*, p. 351; *Kakalik/Wilkhorn*, p. 8; *Ostrom*].

[7]) We also find that

$$\sum_{i=1}^{n} P_s^i/P_z = \sum_{i=1}^{n} MRS_{s,z}^i = MRT_{s,z}$$

These problems may be resolved when one recognizes that while P_s and S are unobservable one can observe the total expenditure on security, $(S\Sigma P_s^i)$. Further, while one cannot easily observe P_K or P_L the variable AUTO is a regularly reported statistic. This intermediate output of police departments depends on the quantities of K and L employed, and in turn on P_K and P_L by the usual duality theorems.

Thus, a reduced form equation that characterizes equilibrium in the market for security may be specified as:

$$S \sum_{i=1}^{n} P_s^i = F(\text{AUTO}, W, A). \tag{2.9}$$

One might interpret equation (2.9) as the demand for auto thefts, in which case it remains to model the supply side of the market.

The supply of offences may also be modelled as a utility maximization problem. The *Ehrlich* [1973] model is an obvious choice for adaptation. If the j-th criminal successfully avoids capture in any of the $i = 1, \ldots, n$ communities he victimizes then he earns

$$W_b^j = \sum_{i=1}^{n} br_{ij} \text{ AUTO}_{ij} + w_j \left(T - \sum_i t_{ij1} - t_{j2}\right) \tag{2.10}$$

where br_{ij} is the average return to an auto theft by the j-th criminal in the i-th community and depends on the opportunities in the community. AUTO_{ij} is the number of thefts he commits and depends on the amount of time he devotes to the activity, denoted t_{ij1}. The time spent in legitimate work is denoted by t_{j2} and the wage rate is w_j. If the criminal is apprehended and convicted, then his earnings are

$$W_a^j = \sum_i br_{ij} \text{ AUTO}_{ij} + w_j \left(T - \sum_i t_{ij1} - t_{i2}\right) - C\left(t_{ij1}\right) \tag{2.11}$$

where C is the cost of being apprehended and convicted, and depends on the time spent in illegal activity.

Let us denote the probability of apprehending and convicting the j-th criminal by $\text{Pr}_j = \sum_i \text{Pr}_{ij} (S \sum_i P_s^i)$. That is, the probability of incarceration depends on the level of police expenditure in the i communities. In this expression it is assumed that the outcomes of arrest in the i-th and j-th communities are non-intersecting. The expected utility of the criminal is thus

$$E\, U^j (W) = \text{Pr}_j\, U^j (W_a^j) + (1 - \text{Pr}_j)\, U^j (W_b^j). \tag{2.12}$$

The decision variables of the criminal are the periods of time spent in both legal and illegal work. Given the set of t_{ij1} one could solve for the number of auto thefts in the i-th community committed by the j-th criminal as a function of the criminal's opportunity cost and the illicit opportunities offered by the communities. Aggregating across the pool of potential criminals then gives the expected number of burglaries in the i-th community. [For an alternative approach see *Hakim* et al., 1979.]

3. The Empirical Model of Expenditure and Auto Thefts

3.1 Model Specification

On the basis of the model in Section 2 an empirical specification should be such that crime rates and expenditure on police protection are jointly dependent. Furthermore, it is often suspected that urban, suburban and rural communities form three distinct behavioral groups. Thus, our three equation model is as follows:

$$\text{AUTO} = \alpha_0 + \alpha_1 \text{EU} + \alpha_2 \text{ES} + \alpha_3 \text{ER} + \alpha_4 \text{VSA} + \alpha_5 \text{POV} + \alpha_6 \text{UNEMP} + \alpha_7 \text{MALE} + \alpha_8 \text{DURB} + \alpha_9 \text{DSUB} + \epsilon \quad (3.1)$$

$$\text{EXP} = \beta_0 + \beta_1 \text{AUTO} + \beta_2 \text{DENU} + \beta_3 \text{DENS} + \beta_4 \text{DENR} + \beta_5 \text{POPC} + \beta_6 \text{OCC} + \beta_7 \text{DURB} + \beta_8 \text{DSUB} + \beta_9 \text{REC} + \xi \quad (3.2)$$

$$\text{EXP} = \text{EU} + \text{ES} + \text{ER}. \quad (3.3)$$

The variable definitions and data sources are given in Table 1.

The data used in the case study includes 230 communities in New Jersey for 1970, with population greater than 2500.

Police expenditure is standardized by the number of dwelling units and not by the population size on the presumption that there is a high correspondence between the number of dwelling units and autos in the community. Our approach allows us to model the comparative intensity of protection (e.g. patrolling) provided by local police departments with respect to the object of the criminal's attention, while an expenditure per capita model focuses on the factors which explain the per capita fiscal burden communities choose to undertake. The expenditure variables are proxies for the intensity of policing in that part of the municipality in which most of the policing takes place.[8])

AUTO is the crime variable. The residents of a community recognize AUTO as an intermediate output of their police department. If, in a given class of community, the number of auto thefts is increasing, the residents will try to buy more security by spending a greater amount on police protection.

We have omitted violent crime from our behavioral model and the reduced form equations (3.1) – (3.3) on the basis of previous empirical work [*Chapman; Greenwood/ Wadycki*]. It would appear that crimes of passion do not respond to explicit economic incentives. Thus, it is assumed that consumers of security recognize this and make their budgetary decisions on the basis of the prevailing rate of property crime.

[8]) While it may be more appropriate to standardize by the number of autos, this information was not readily available. Furthermore, it is quite likely that there is a very high correspondence between the number of autos and the number of dwelling units in any given community.

No.	Notation	Description of Variable	Source
1.	AUTO	Incidence of Auto Theft	New Jersey Division of State Police, 1970 *Uniform Crime Report* of the State of New Jersey.
2.	DENU, DENS, DENR	Population density (in urban, suburban and rural places, respectively)	New Jersey Division of State Police, 1970, *Uniform Crime Report* of the State of New Jersey.
3.	DURB, DSUB	Dummy variable indicating the character and locations of the community with respect to central cities	Determined by the researchers. DURB = 1 are urban places, DSUB = 1 suburban places. When DURB = DSUB = 0 then rural places.
4.	EU, ES, ER	Police expenditure per housing unit (in urban, suburban and rural places, respectively) EXP = ES + EU +ER	Division of Local Government Services: *1970 Statements of Financial Conditions of Countries and Municipalities,* N.J. Dept. of Community Affairs.
5.	MALE	% of population between 15 and 24 years of age and male	U.S. Bureau of the Census 1970 Census of Population and Housing.
6.	OCC	% of owner occupied homes	U.S. Bureau of the Census, *1970, Census of Population and Housing,* New Jersey, Volume.
7.	POPC	% population change between 1960 and 1970	U.S. Bureau of the Census, *1970 Census of Population and Housing,* New Jersey Volume.
8.	POV	% of population below the poverty line (in urban, suburban, rural places, respectively)	U.S. Bureau of the Census, *1970 Census of Population and Housing,* New Jersey Volume.
9.	REC	% of all stolen property recovered	New Jersey Division of State Police, 1970 *Uniform Crime Report.*
10.	UNEMP	Unemployment rate	U.S. Bureau of the Census, 1970 [Census of Population and Housing.]
11.	VSA	Value of stolen autos	New Jersey Division of State Police, 1970 *Uniform Crime Report.*

Tab. 1: Definition of Variables and Sources of Data

In the equation explaining the incidence of auto thefts (AUTO), the variables EU, ES, and ER are urban, suburban and rural police expenditure per year-around dwelling unit expressed as deviations from group means, or zero if the observation is not a member of the particular group. On the basis of the models of police expenditures and thefts we posit the following: We expect that as more is spent on policing, the greater is the deterrent effect, and fewer car thefts are expected (∂ (AUTO)/∂ $(E_i) < 0$, where $i = U, S, R$). However, prior studies have consistently shown the opposite sign [e.g. *Allison; Carr-Hill/Stern; Greenwood/Wadycki; McPheters/Stronge; Thaler; Zipin* et al.].

The poverty variable (POV) expresses the portion of the population with low opportunity cost for legal income and which is more inclined to steal autos (∂ (AUTO)/∂ (POV) > 0). As measures of crime opportunity for thieves we have introduced the average value of stolen autos (VSA) which is assumed to be positively related to the value of autos in the community.

The variables MALE and UNEMP measure the pool of potential criminals. Most automobiles are stolen by males between the age of 15 and 24 and/or unemployed individuals (∂ (AUTO)/∂ (MALE) > 0, ∂ (AUTO)/∂ (UNEMP) > 0).

DURB, DSUB are dummy variables used to express the nature of the locale and its accessibility from central cities. The first ring suburbs experience more vehicular traffic and hence are more familiar to potential thieves than rural places which are farther away. We expect the level of thefts to be higher in urban than in suburban locales and the lowest in rural places (∂ (AUTO)/∂ (DURB) $>$ ∂ (AUTO)/∂ (DSUB) > 0).

Turning to the police expenditure equation, DURB, and DSUB can also be thought of as measures of the existence of economies of scale in the provision of police services. We assume efficient and spatially equal provision of police services by all locales, and equal effects of density, wealth and population change in all three types of communities. In such a case, DURB and DSUB implicitly express production of service attributes. Further, since the population size in urban areas is larger than in suburban places, and the latter is larger than in rural places, we might assume that the dummy variables express structural shifts due to population differences. Hence, our measure of returns to scale parallels that of other studies.

If ∂ (E EXP)/∂ (DURB) $>$ ∂ (E EXP)/∂ (DSUB) > 0, then one might interpret it as indicating the presence of diseconomies of scale. If, however, we observe that ∂ (E)/∂ (DURB) $<$ ∂ (E EXP)/∂ (DSUB) < 0, then one might infer the presence of economies of scale [*Allison; Hirsch; McPheters/Stronge; Popp/Sebold*].

In the equation explaining police expenditure, the lower the population density (DENU, DENS, DENR), the more expensive it becomes to provide the same level of security (∂ (E EXP)/∂ (DEN_i) < 0, [President's Commission on Law Enforcement and Administration of Justice]). Lastly, percent population change between 1960 and 1970 (POPC) is indicative of the stability of the whole population and commitment to the community. The higher the instability rate the smaller are the resources which the community is willing to allocate for security (∂ (E EXP)/∂ (POPC) < 0, [*McPheters/ Stronge*]).

The percent of owner occupied housing is included as a measure of the ability and willingness to pay. More wealthy communities are expected to be more eager to protect their property (∂ (E EXP)/∂ (OCC) > 0). The variable REC, % of all stolen property recovered, is an indication of the success of the policing effort. Its sign is ambiguous in that expenditure might increase as a reward for performance, or decrease in response to too much policing.

High multicollinearity is built into the model between the type of community (urban, suburban, rural) and two other independent variables in the theft equation – police expenditure and poverty. We wish to reveal whether there exists a significantly different level of theft for the three groups, and to measure the separate statistical effects of expenditure and poverty on thefts in the three groups. The method used here was developed by *Searle* [1971, 355–358] and requires a linear transformation of the original values of all cases in each group.

The transformation involves calculating the means of particular variables for each of the groups and measuring each observation as a deviation from the appropriate group mean. The variable expressed in deviation form is then split into three new ones; each one corresponding to a given group. Thus, for example, observations on expenditure in urban places are expressed as either a deviation for an urban area, or zero if it is not an urban area. This transformation preserves the explanatory power of the original variables while reducing collinearity. [For the use and interpretation of this transformation see *Buck/Hakim*, 1981a.]

Two criteria are used in order to select the restated continuous variables: (1) variables which theoretically seem to be highly correlated with the (0, 1) dummy, and (2) variables which the data suggests will improve the interpretation of the model when they are restated. This method applies only to the case in which at least one continuous variable is not multiplied by the dummy variable or by its complement. When all continuous independent variables are restated, then we actually observe a separate equation for each group, as if the cases in each group were derived from different populations.

One of the conclusions of the economic theory of criminal behavior presented in Section 2 and the works cited above is that expenditure should have an inverse relationship to the theft rate in the auto theft equation. Also, we impose prior restrictions on most of the coefficients of the independent variables. Thus, we impose on the model the interval restrictions:

$$\begin{aligned}
&\alpha_1, \alpha_2, \alpha_3 < 0 \\
&\alpha_4, \alpha_5, \alpha_6, \alpha_7 > 0 \\
&\beta_1, \beta_6 > 0 \\
&\beta_2, \beta_3, \beta_4, \beta_5 < 0
\end{aligned} \tag{3.4}$$

3.2 Estimation of Model Parameters

Let us estimate the following structural equation:

$$y_j = -Y_j \beta_j - Z_j \gamma_j + U_j \qquad j = 1, \ldots, G \tag{3.5}$$

where y_j is the vector of n observations on the j-th dependent variable, Y_j is the matrix of n observations on the remaining $G - 1$ jointly dependent variables, Z_j is the matrix of n observations on the included exogenous variables of which there are, say, k_j in the j-th equation and U_j is the disturbance vector. The whole system may be rewritten as:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_G \end{bmatrix} = \begin{bmatrix} -Y_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & & -Y_G \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_G \end{bmatrix} - \begin{bmatrix} Z_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & & Z_G \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_G \end{bmatrix} + \begin{bmatrix} U_1 \\ \vdots \\ U_G \end{bmatrix} \tag{3.6}$$

or

$$Y = X\delta + U \tag{3.7}$$

where Y corresponds to the nG vector of observations on the included endogenous variables, X corresponds to the $nG \times G(G-1) + Gk$ matrix of observations on the other dependent variables and exogenous variables. In determining the dimension of X note that there are G equations with n observations each, hence nG rows. Also, there are $\Sigma k_j = k$ exogenous variables for each equation although some of the coefficients must have zero restrictions, thus there are Gk columns in the exogenous variable matrix plus $G(G-1)$ columns in the endogenous variable matrix. Define $\overline{Z}$ as the $n \times k$ matrix of all predetermined variables. Using the G dimensional identity matrix construct

$$Z = I \otimes \overline{Z}$$

where $\otimes$ is the Kroneker product [see *Theil*, p. 509], and premultiply the system (3.7) by Z to obtain

$$Z'y = Z'X\delta + Z'U. \tag{3.8}$$

We note that

$$\text{Var}(Z'U) = \Sigma \otimes (Z'Z). \tag{3.9}$$

The three stage least squares estimator follows as:

$$\hat{\hat{\delta}} = (X'Z(\hat{\Sigma} \otimes (Z'Z))^{-1}Z'X)^{-1}X'Z(\hat{\Sigma} \otimes (Z'Z))^{-1}Z'Y \tag{3.10}$$

where

$$\hat{\Sigma}_{ij} = [n^{-1}(y_i - X_i\hat{\delta}_i)'(y_j - X_j\hat{\delta}_j)]$$

and $\hat{\delta}_i$ are the two stage estimates of the parameter vector for each equation. [For the details of the derivation of equation (3.10) the reader is referred to *Theil*, 508–514.]

In many applied statistical problems we may have prior information in the form of an inequality constraint. In general this type of constraint may be written as

$$r_1 \leqslant H\delta \leqslant r_2 \tag{3.11}$$

where r_1 and r_2 are vectors and H is a matrix of dimension $h \times [G(G-1) + Gk]$. It was noted above that δ is a vector of coefficients of dimension $[G(G-1) + Gk] \times 1$.

The estimator we propose to use is

$$\begin{aligned}\delta^* = {} & \hat{\hat{\delta}} + S^{-1}H\,(HS^{-1}H)^{-1}\,(r_1 - H)\,I_{(-\infty, r_1)}\,(H\hat{\hat{\delta}}) \\ & + S^{-1}H\,(HS^{-1}H)^{-1}\,(r_2 - H\hat{\hat{\delta}})\,I_{(r_2, \infty)}\,(H\hat{\hat{\delta}})\end{aligned} \tag{3.12}$$

where

$$S = X'Z\,(\hat{\hat{\Sigma}} \otimes Z'Z)^{-1}Z'X$$

and $I_{(\cdot)}(\cdot)$ is a vector indicator function whose elements take the value of one when the corresponding element of $H\hat{\hat{\delta}}$ falls in the subscripted interval and zero otherwise. If $H\hat{\delta}$ falls between r_1 and r_2 then the estimate is given by $\hat{\hat{\delta}}$. If $H\hat{\hat{\delta}}$ falls below r_1 then the estimate is given by $\hat{\hat{\delta}}$ plus the second term. Similarly, if $H\hat{\hat{\delta}}$ falls above r_2 then the estimate is given by $\hat{\hat{\delta}}$ plus the third term. This particular estimator is a direct application of the Kuhn-Tucker Theorem and is implemented by first estimating S from ordinary least squares then using quadratic programming to minimize $(Y - X\delta)'\,S^{-1}\,(Y - X\delta)$ subject to $H\delta \leqslant r_2$ and $-H\delta \leqslant r_1$. More intuitively, consider two dimensions of the parameter space in Figure 1. The unconstrained estimates are

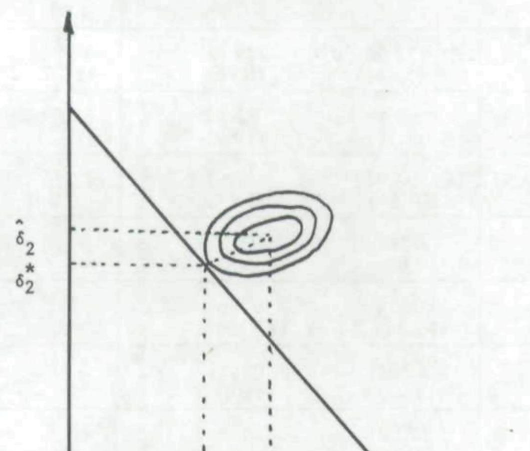

Fig. 1

denoted by $\hat{\delta}_1$ and $\hat{\delta}_2$, the ellipses are empirical likelihood contours, the diagonal is the constraint $\hat{\delta}_1 + \hat{\delta}_2 < r$. The quadratic programming estimator projects the unconstrained estimator obliquely onto the constraint, that is, δ_1^* and δ_2^*. [For an exposition of the process in single equation models see *Judge/Takayama*.]

Estimating the parameters of the model defined by equations (3.1) through (3.3) according to the procedure just outlined reveals that for our New Jersey data set, auto thefts are directly related to expenditure (see Table 2) and that not all of the inequality constraints of (3.4) are binding. The important point is that the results of Table 2

		Expenditure Equation		Auto Theft Equation	
		Unconstrained	Constrained	Unconstrained	Constrained
Jointly Dependent Variables	AUTO	6.03×10^3 (3.15)	5.73×10^3 (3.05)		
	EU			$-.73 \times 10^{-6}$ (0.00)	$-.74 \times 10^{-6}$ (0.00)
	ES			$.23 \times 10^{-6}$ (0.00)	0.0
	ER			$.47 \times 10^{-5}$ (0.00)	0.0
	VSA			$.75 \times 10^{-5}$ (4.39)	$.74 \times 10^{-5}$ (4.47)
	POV			.4356 (2.85)	.4314 (2.88)
	UNEMP			.1184 (.39)	.1140 (.38)
	MALE			−2.792 (−1.09)	0.0
	DURB	232.06 (5.88)	239.41 (6.16)	3.496 (2.76)	3.508 (2.78)
	DSUB	58.43 (1.73)	62.95 (1.89)	1.309 (1.15)	1.284 (1.14)
	CONSTANT	−104.92 (−.64)	44.67 (1.44)	−1.256 (−.87)	−1.200 (−.85)
	DENU	.0273 (10.85)	.0272 (10.83)		
	DENS	.0098 (10.25)	.0098 (10.19)		
	DENR	.0284 (3.92)	.0281 (3.88)		
	POPC	172.95 (1.08)	0.0		
	OCC	2.916 (1.32)	2.747 (1.25)		
	REC	−.0594 (−.91)	0.0		

Tab. 2: Unconstrained and Constrained Parameter Estimates[1])

[1]) Numbers in parentheses are asymptotic t-statistics. Due to the biases introduced by incorrect constraints, tests of hypothesis are not directly applicable in the second and fourth rows.

represent both the initial and final tableaus of the quadratic programming estimator derived above. Estimating the parameters of the constrained model, equations (3.1) through (3.4), yields the result presented in the 2nd and 4th columns of Table 2.

The results of the unconstrained equations generally confirm our earlier hypothesized relationships. However, in the auto theft equation, two police expenditure variables appear with positive signs contradicting our expectations, although this unexpected relationship appears frequently in other studies [*Allison; Carr-Hill/Stern; Greenwood/Wadycki; Hakim* et al., 1978; *McPheters/Stronge; Thaler; Zipin* et al.]. Poverty appears to be positively related to car thefts in urban and suburban places. The variable MALE also had an unexpected sign. All other signs are as earlier hypothesized.

In the expenditure equation OCC appears to explain the variation in the dependent variable as hypothesized. Police services are characterized as a normal good, the consumption of which increases with the increase in wealth. The signs and magnitudes of the dummy variables (DURB, DSUB) might suggest the presence of diseconomies of scale [same as *Walzer*]. It is important to note that this result confirms the findings of most other studies [e.g. *Allison; Hirsch,* p. 360; *Popp/Sebold*].

Table 2 reveals that, with the exception of the intercept coefficient, little has changed other than the constrained coefficients. The small increases in the magnitudes of the coefficients can be attributed to a non-spherical error term and any remaining multicollinearity.

4. Conclusions

This paper has developed an inequality constrained least squares estimator for simultaneous equation models. The estimator we have developed was applied to a study of simultaneously determined auto thefts and police expenditure. The implementation of the estimator by quadratic programming involves oblique projections of the ordinary three stage estimates onto the constraints. Our example reveals that if the estimated error covariance matrix is nearly diagonal and the columns of the design matrix are independent, then the unconstrained parameter estimates will not change appreciably.

The results of the restricted burglary equation show a marked increase in the marginal effect of the opportunity set for the criminal (VSA) and the pool of criminals (POV). In the expenditure function the marginal effect of wealth (OCC) decreased. There are two possible explanations for these changes: first, the quadratic programming projections are oblique rather than orthogonal; second, in the initial tableau some explanatory power was incorrectly attributed to the wealth variable while under the binding constraint the other independent variables become more important, i.e. their coefficients become larger.

Because of the lack of knowledge about the ordinary three stage least squares (3SLS) estimator, there is little we can say about the comparison between ordinary 3SLS and inequality constrained 3SLS in terms of risk or mean square error. However, the constrained estimator will have smaller variance at the expense of increased bias.[9])

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[9]) While an incorrect constraint results in some bias, the variance of a constrained estimator is always smaller than that for an unconstrained estimator. The bias however may be large enough to swamp the smaller variance so that risk and *MSE* are larger in the constrained case than in the unconstrained case. See *Buck/Hakim* [1981b].

Our estimator is of considerable value since it permits us to incorporate prior information about a particular statistical model into classical sampling theoretic estimation. The principal advantage over a Bayesian approach is the reduction in quantity and quality of prior information.

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