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Nonclassical demand
A model-free examination of price–quantity relations in the Marseille fish market

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Abstract

We suggest a model for a market, the Marseille wholesale fish market, in which purchases do not correspond to standard competitive demand. We use nonparametric methods to detect the properties of price–quantity relations which reveal 'strategic demand'. Our data over three months include price quantity details of each transaction for each fish, and the identities of the buyers and sellers. The observed distributions of prices are stable over time, thus the market can be treated as a repeated game. Strategic demand curves are obtained by local fitting. They are downward-sloping at the aggregate level but not in general at the individual level. Thus regularities are generated by aggregation rather than derived from individual behaviour.

Key words: Aggregation; Demand; Markets; Nonparametric; Monotonicity
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1. Introduction

In economics, aggregate behaviour is often tested to see if it meets restrictions that can be derived from individual maximising behaviour. Thus it is common practice to treat data arising from aggregate purchases of some commodity over

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time as if these were the expression of the competitive demand of some representative individual. This approach involves a number of implicit assumptions, in particular that the underlying micro-data observed can be thought of as corresponding to individual Walrasian demand, and furthermore that aggregation considerations do not invalidate the use of restrictions derived from individual behaviour.

In this paper we find, empirically, that, for the particular market that we study, properties which we show to hold at the aggregate level and which might be thought of as 'downward-sloping demand curves' are not derived from similar properties at the individual level. Furthermore when this market is appropriately modelled we show that there is no theoretical reason to expect any such simple relation between individual and aggregate behaviour. In this we are following directly in the line of work by Becker (1962)[1] who showed that downward-sloping demand curves at the market level could be derived from random individual choice behaviour subject only to a budget constraint. Whereas he summarised his result as saying that 'households may be irrational and yet markets quite rational', a better summary of our results would be that 'sophisticated and complicated individual behaviour may lead to simple aggregate properties'.

In particular, we will be concerned with the properties of the purchases of particularly perishable goods, different types of fish, for which we have data at the individual level, from the Marseille fish market. Although fish markets have been widely used as an example in the economic literature, we will argue that, for several reasons, it is inappropriate to think of purchases on such markets as corresponding to Walrasian demand. Having briefly described the type of market involved, we then propose a simple theoretical model, which describes how individuals could be thought of as behaving when determining the quantities that they purchase and sell. This model shows that the quantities 'demanded' or 'supplied' by the individuals in question cannot be expected to correspond to those that would be 'demanded' or 'supplied' by the standard individual in a competitive market. Instead we develop a more appropriate theoretical notion, that of 'strategic demand and supply'.

Using this, it is clear that the appropriate equilibrium concept can be defined and corresponds to a nondegenerate price distribution (see Diamond, 1987; Butters, 1977). For the use of our model to be justified and if we are to be able to use data from the whole period that we study, it should be the case that the observed price distributions remain fairly constant over time. If this is the case, then one can think of the market as repeating itself from day to day, or at least from week to week. We test for the intertemporal stability of the price

[1] This idea has been developed recently by Gode and Sunder (1993).

distributions for some representative types of fish. In particular, it should be noted that we do this without imposing any a priori restrictions on the form of the distributions. Then, having established that these do indeed remain constant, we use nonparametric methods to fit two different aggregate price–quantity relations and find that these relations, in contrast to those at the individual level, exhibit monotonicity once a certain class of observations is identified and removed.

Since we are arguing that Walrasian demand is not the appropriate concept in our context, nor, indeed in many of the contexts in which it is used, it is worth looking at the way in which, historically, it has come to occupy such a central role in empirical studies. This is of particular interest since the market for fish, which we examine here, has been frequently used as an example.

In the nineteenth century there was a very active debate over the nature of demand and little concern about its estimation. An extensive debate took place between John Stuart Mill (1869, 1871, 1972) and Thornton (1869, 1870) over the meaning and nature of the equilibrium price in the fish market, and the interpretation that could be given to demand in such a market. More precisely, the particular structure considered in the examples they discussed, was that of an auction, and there was a suggestion that either the standard continuity property of demand was violated in the examples given or that transactions reflected disequilibrium behaviour.[2] Until Marshall, there was considerable discussion as to the correct definition of demand for a single commodity. However, in the more formal literature there was convergence on the rather abstract Walrasian notion that demand simply represented the quantity that an individual would purchase at given prices which he was unable to influence. The subsequent theoretical literature concentrated largely on the extension of the analysis to interdependent markets and the problem of demand systems rather than single demand equations still maintaining the abstract Walrasian approach. Until recently, the idea that demand should be treated in this way has not really been challenged, neither in the economic nor in the econometric literature.

Once the twentieth century literature had converged on this precise theoretical definition, econometricians concentrated on more sophisticated techniques for the estimation and identification of demand systems. The agreed definition, that of competitive demand, concerning the quantities of goods an individual would buy at given prices, were he only constrained by his income, was retained. In Working's (1927) paper the conceptual nature of demand and supply are not questioned. The only real problem for him was that of which of the two was

[2] This debate produced echoes recently, when Negishi (1985, 1986, 1989) (and Ekelund and Thommeson, 1989) discussed the precise nature of the difficulties involved in the Mill and Thornton examples.

fluctuating over time. However for many markets, and this is the subject of this paper, this conceptual framework is not satisfactory. For example, in our particular case, the wholesale fish market in Marseille, all transactions are bilateral and no prices are posted. When we look at the relation between the prices charged and the quantities purchased on this sort of market, a number of questions which were very present in the earlier debate as to the appropriate notion of 'demand' recur.

Let us, therefore, return to the implicit assumptions underlying the usual empirical analysis based on Walrasian demand theory and see whether they are appropriate in our context.

The first question that arises is whether the purchaser of a good is, in fact, the final consumer. If this is not the case, then one would have to show that properties of individual demand carry over to properties of quantities purchased by an intermediary at different prices. If one considers the simple case of a purchaser who is a retailer and has a monopoly locally of the product that he buys and resells, then it is easy to construct examples in which this will not be the case. This question was raised by Working (1927) and mentioned again in the classical studies by Schultz (1938), who although using individual properties of demand made his estimations using data for farm prices and not shop prices. More recently, in a specific study of the Belgian fish market, Barten and Bettendorf (1989) refer to this question.

The second problem arises even if one accepts that the final consumers are present on the market in question and that it does function 'competitively'. The problem is that of identification, in this case, separating out supply changes from demand changes. In a truly Walrasian, or Arrow–Debreu world such a distinction could, of course, not be made, since all transactions over time represent one supply and one demand decision taken in some initial period. However, this problem is usually circumvented in the empirical literature by making an implicit assumption of stationarity and separability, i.e., that the market is somehow repeated over time, and that decisions are taken in the same way at each point in time. This should, of course, be tested, but does mean that one can talk of successive observations. However, in this case the appropriate theory is that referred to as temporary general equilibrium theory. The problem with this is that, without full knowledge of future prices, expectations have to be taken into account. Without unreasonable assumptions about these, short-run demand loses many of the properties of its Walrasian counterpart. It does not satisfy homogeneity or the Weak Axiom of Revealed Preference, for example (see, e.g., Grandmont, 1983). Trying to fit a demand system based on the usual theoretical restrictions makes little sense therefore.

Nevertheless, if we are prepared to accept the idea that changes in the prices of fish do not result in a large amount of intertemporal substitution, then thinking of a sequence of equilibria in a market which repeats itself is more acceptable. This explains why, when considering particular markets, fish has been so widely used as an example (e.g., by Marshall, Pareto, Hicks) since with no stocks, successive markets can be thought of as independent. In our case, when fitting our price–quantity relations we are implicitly treating price changes as resulting from random shocks to the supply of fish, although the amount available is, at least in part, a result of strategic choice.

The next problem is that of aggregation. If we fit a demand system in the usual way, we are assuming that market behaviour corresponds to that of an individual. Examination of individual data reveals none of the properties that one would expect from standard individual demand. Thus, even if such properties are found at the aggregate level, they cannot be attributed to individual behaviour. This is one side of the problem of aggregation. The other is that, even if individuals satisfy certain properties, it is by no means necessary that these properties carry over to the aggregate level (see, e.g., Sonnenschein, 1972; Debreu, 1974). The two taken together mean that there is no direct connection between micro and macro behaviour. This basic difficulty in the testing of aggregate models has recently been insisted upon (see Kirman, 1992; Summers, 1991; Lewbel, 1989) when discussing representative individual macro models, but as Lewbel observes, this has not stopped, and is unlikely to stop, the profession from testing individually derived hypotheses at the aggregate level. Hence when we establish some empirical properties of the aggregate relationships between prices charged and quantities purchased, we suggest that these should be viewed as independent of standard maximising individual behaviour.

The next point is that the organisation of the market for the product in question may not be competitive. In this case, it is not possible to talk of a single market price. If different lots of the same good are auctioned off successively, for example, the average price will not necessarily correspond to the price which would have solved the Walrasian problem for that market. The problem here is that techniques for the econometric analysis of data arising from differently organized markets such as auctions, for example, have been little developed and there is always a temptation to return to standard and sophisticated techniques, even if these should not really be applied to the type of market in question. Barten and Bettendorf (1989) are well aware of this difficulty, and suggest that the aggregate behaviour in the fish market can be reduced to that of a Walrasian mechanism by looking at an inverse demand system. They reason as follows:

> 'Price taking producers and price taking consumers are linked by traders who select a price which they expect clears the market. In practice, this means that at the auction the wholesale traders offer prices for the fixed quantities which, after being augmented with a suitable margin, are suitably low to induce consumers to buy the available quantities. The traders set the prices as a function of the quantities. The causality goes from quantity to price.'

Although the authors are only making explicit what is commonly done, it is clear that one should *prove* that, even if the auction price is well defined, it is indeed related to prices charged to consumers through a simple mark-up. Necessarily, if different purchasers pay different prices and the mark-up principle does apply, then a distribution of prices will be observed on the retail market.

This bring us to a further point. Since our market does not function as a standard auction, and individual traders strike bargains amongst themselves and are well aware of each others' identities, different prices can be, and are, charged to different purchasers for the same product. This discrimination is an important feature of the market, and there are significant variations in the average prices paid by different buyers (see Kirman and McCarthy, 1990). This means that reducing prices to averages may well lose a significant feature of the data. Furthermore, it means that the average price cannot be regarded as a reasonable sufficient statistic and that other properties of the price distribution must be taken into account. This reduces the plausibility of the argument advanced by Barten and Bettendorf.

Lastly we emphasise that, when fitting the price–quantity relations, we use nonparametric estimation techniques, since these are less likely to lead to mistakenly accepting a monotone relation, and furthermore reveal interesting features of the data that standard techniques, using predefined functional forms, would have been unlikely to detect.

Now we turn to the development of our formal model of the market.

2. A simple strategic model

As we have already suggested and as has been emphasised by many authors, the structure and organisation of the market are of particular importance in determining the nature of the equilibrium realised. We therefore give a simple model of our type of market, restricting ourselves, for simplicity, to the case of one type of fish.

We thus consider the market for one perishable product with m sellers and n buyers.[3] The market evolves in a fixed number T of rounds. Each seller i has strategies which at each round t specify a vector $x_{it} \in R^n_+$ of the prices which he will charge to each of the buyers. A strategy for each buyer j specifies at each round t a demand function $q_{jt}(p)$: $R_+ \to R_+$. In both cases the choice of the prices set and the demand functions will depend on two things: firstly, the strategies of the other players and, secondly, on who has met whom in the

[3] In Kirman and Vignes (1991) we considered a continuum of buyers and sellers, but this was to facilitate the solution of the technical problem of establishing the continuity of strategies.

market. The model is then completed by specifying a matching process which, in keeping with the literature, will be assumed to be random. Thus a matching at time t, a realisation of the random variable, will be a mapping g from the integers $J = \{1, \ldots, n\}$ to the integers $I = \{1, \ldots, m\}$. A probability distribution must then be specified over the outcomes of the matching process for every time t. One might think, as an example, of each buyer as choosing a seller with uniform probability $1/m$, independently at each time t. However, many other matching processes could be considered, including those in which some particular buyers and sellers are always matched together. A best strategy for a buyer i then will consist for each realisation of the matching process and for the associated price vectors of each seller i and demand functions of the other buyers $h \neq \dot{c}$ of a demand function for each period t. Similarly, for a seller it will consist of specifying the best price vectors for each matching and each period.

Thus to sum up, the model consists of:

(1) *A basic strategy set* B for buyers which is a subset of the product space of Q where Q is the set of functions q: $R_+ \to R_+$ satisfying
   (i) every q is continuous and monotone decreasing,
   (ii) there exists a $\bar{p} > 0$ such that for every q in Q, $q(p) = 0$ for all $p > \bar{p}$.
   Thus $B \subset \prod_T Q$, i.e., $Q \times Q \times \ldots \times Q$. Furthermore, we assume B is compact and convex.
(2) *A basic strategy set* S for sellers which is a compact convex subset of R_+^{nT}.
(3) *A matching process* M, i.e.,
   (i) the mappings $f = (f_1, f_2, \ldots, f_t)$ where f_i: $I \to J$,
   (ii) a probability distribution over the finite set F of f.
(4) *A full strategy* $\tilde{q}$ for a buyer then associates with each f an element of Q, i.e., q: $F \to B$, and $\tilde{s}$ for a seller associates with each element of F, an element of S, i.e., s: $F \to S$. The set of full strategies for buyers is denoted $\tilde{Q}$ and for sellers $\tilde{S}$.
(5) There is *a continuous payoff function* for each player i. $\prod_i(q_1 \ldots q_n, s_{n+1} \ldots s_{n+m})$, $i = 1, \ldots, n+m$.
(6) *The response function* Γ for each player is given for buyers by Γ: $\tilde{Q}^n \times \tilde{S}^m \to \tilde{Q}$ where $\Gamma_i = \max_q \prod_i(q_1 \ldots q_{i-1}, q, q_{i+1} \ldots q_n, s_{n+1}, s_{n+m})$ and for sellers by Γ: $\tilde{Q}^n \times \tilde{S}^m \to \tilde{S}$ where $\Gamma_j = \max_s \prod_i(q_1 \ldots q_n, s_{n+1} \ldots s_{j-1}, s, s_{j+1} \ldots s_{n+m})$. We assume that for every i, $i = 1, \ldots, n$, and j, $j = n+1, \ldots, n+m$, Γ_i is continuous.

Denoting by $\tilde{Q}^n$ the n product of $\tilde{Q}$ and by $\tilde{S}^m$ the m product of $\tilde{S}$, then each Γ $(\Gamma_1 \ldots \Gamma_{n+m})$ defines a mapping from $\tilde{Q}^n \times \tilde{S}^m$ into itself. Given our assumptions, a standard fixed point argument can be used to show the existence of an equilibrium.

The market can be envisaged as follows:

Period 0: Initial stocks become available.

Period 1: Sellers specify prices, buyers specify demands. Matching takes place. Transactions occur.

Period 2: Given their information about what happened in period 1, sellers respecify prices, buyers respecify demands. Matching occurs. Exchanges follow.

Period T: Last specifications by sellers and buyers, last matching and exchanges.

As it stands, we have done no more than give a formal framework which enables us to define the concept of an equilibrium. To characterise precisely the nature of an equilibrium requires that the Γ_i be derived from maximising behaviour. For example, Γ_i for a buyer would maximise his expected utility at each round t given the known strategies of the other players and the matching up until t. The real difficulty here is *proving* the continuity of Γ_i for the players. This difficulty is illustrated by Kormendi (1979) and Benabou (1988).

Whether or not we give a complete specification of the maximising behaviour of the individuals, our model would allow for extensive price dispersion (particularly since we have assumed that price discrimination is possible as each seller knows the buyers' characteristics), there will be no necessary tendency for prices to decline during the day as is commonly supposed and, as we have mentioned, there is no a priori reason to assume that individual buyers will or will not search.

One important point to emphasise is that any strategy must be such that if the information set up to time τ is the same in two realisations, the next component of the strategy at time $t + 1$ should be the same. Thus it is important to specify what is known at each time. If, for instance, the individuals know only their own initial stocks and only observe their own transaction outcomes, they will be much more limited than if they observe everything that has occurred. Furthermore, it may well be the case that individuals actually choose to condition strategies on a limited part of the information they have available.

Although it is difficult to prove the continuity of strategies in a fully optimising context, it is possible that agents develop simple rules which are continuous. An interesting problem is how such rules are developed.

Having given an outline of the structure of the sort of process we examine, it is not surprising that the outcomes do not necessarily satisfy standard demand properties at the individual level, since observed transactions are the results of the interaction between buyers' and sellers' strategies. We shall refer to the observed purchases as reflecting '*strategic demand*' since they reflect the outcome of the process which clears the market at the end of the day. The difference from day to day on the market is the amount of fish available. This is due in part to exogenous factors such as weather, but is also due to the choice of sellers when anticipating demand changes. This latter factor should be incorporated into a complete model.

Table 1
Characteristics of the data for the Marseille fish market

Period	July–September 1988
Organization	Pairwise trading, no posted prices
Number of buyers	574
Number of sellers	37
Number of types of fish	129
Level of disaggregation	Every transaction recorded
Data for each transaction	a) Name of seller b) Name of buyer c) Type of fish d) Quantity sold (weight) e) Price per kilo
Time	Transactions listed in chronological order during the day for each seller
Salient features of the market	
Concentration	
Sellers	One seller accounts for 15.5% of all transactions; the six biggest sellers account for 50% of all transactions
Buyers	One buyer accounts for 14% of all transactions; all others account for less than 1%
Diversity	Average number of fish types traded by a seller during a day varies from 1 to 32; 50% of traders trade in less than 10 fish types
Fish types selected	Sole, sardine, whiting, and trout

In Table 1 a description of the data set and some of its characteristics are given, and we now turn to an analysis of some of its features.

3. Price–quantity relations

What we have learned from our theoretical analysis is that there is no a priori reason to expect any particular structure of the relationship between prices (or average prices) and quantities sold. Testing standard properties to verify the theory underlying demand functions or demand systems would make little sense in this context for the reasons we have indicated. What we are observing does not reflect consumer demand, discriminatory pricing is taking place and prices evolve strategically over the day.

However, determining whether or not our data do satisfy certain properties is of interest. The one feature that we do observe is that over the day markets do more or less clear in the sense that the surplus left unsold never exceeds 4%. Since sellers become aware, from the reactions of buyers to their offers, of the amount available on the market and vice versa, it would not be unreasonable to expect average strategic equilibrium prices to be lower on those days where the quantity is higher, but some buyers transact early, before such information becomes available, and others only make one transaction for a given fish on a given day. Thus to deduce such a property formally would require much stronger assumptions than we have made. If we can establish such a property, i.e., of 'downward-sloping demand', it certainly could not be attributed to the normal utility-maximising model as is frequently done, but is rather a property that emerges from a rather complicated noncooperative game.

To look at this we now proceed to an empirical examination of the behaviour of the market. We might like to find out whether, for example, when we consider the four fish that we have taken as examples, the quantities purchased at each price $D(p)$ for those fish display the monotonicity property, i.e., for $p \neq p'$ and $p > 0$, $p' > 0$, p in R^4_+.

$$(D(p) - D(p')) \cdot (p - p') \leqslant 0.$$

Such a property, when $D(p)$ is interpreted as a standard demand system, is described as the 'Law of Demand' by Hildenbrand (1983) following Hicks. In particular, it implies that each partial 'own demand curve' for the fish is downward-sloping.[4] One approach would be to estimate in a standard parametric way the whole 'demand system', but since we have no a priori reason to impose any sort of functional form on the system, we have chosen to look at the weaker property, negatively sloped price quantity relations for each individual fish. In doing so we are open to the criticism that we are not taking into account substitution effects between fish. Thus, it could be argued that what we gain in using more flexible estimation methods is offset by what we lose in overlooking these effects. There are three responses to this. Firstly, many buyers such as restaurant owners have a pre-determined vector of fish quantities which they do not vary much in response to relative price changes. Secondly, there are other buyers who only buy one type of fish and therefore do not substitute. Lastly, some of the exogenous factors influencing the amount of fish available, such as weather, are common to many fish, thus limiting the amount of substitution possible. For all of these reasons we have analysed each of our four fish separately.

In undertaking our analysis of the 'demand' for each fish, we do an exercise designed to elicit some of the basic characteristics of the data. Basically, we take the data for a given fish and aggregate it by taking the quantity of that fish sold on a particular day and the weighted average price for that day. There is

[4] Of course to take observed quantities purchased as representing a marginal curve is not correct since the ceteris paribus condition is violated. However, this makes the resultant monotonicity more rather than less convincing.

a problem of separation of strategies here. There are not only variations in the supply of fish due to weather, etc., but more fish is landed on active market days by choice. The variations over the week are due in part to obvious institutional factors (fish-shops are closed on Sundays), but also to more indirect ones. As Robbins (1935) observed before his discussion of the market for herring in England:

> 'The influence of the Reformation made no change in the forces of gravity. But it certainly must have changed the demand for fish on Fridays.'

We then fit the resulting data by nonparametric smoothing methods. Sufficient details to give a basic understanding of the techniques used are given in Appendix A (for a full account see Härdle, 1990). We use nonparametric methods since they enable us to pick up any lack of monotonicity of the fitted curve over some particular price range. Nevertheless in all four cases the fitted curves are indeed monotone decreasing and two examples are given in Figs. 1a and 1b.

Simple inspection of the graphs is, of course, not sufficient, and since we have no explicit functional form for the fitted curves, we have actually to *show* that

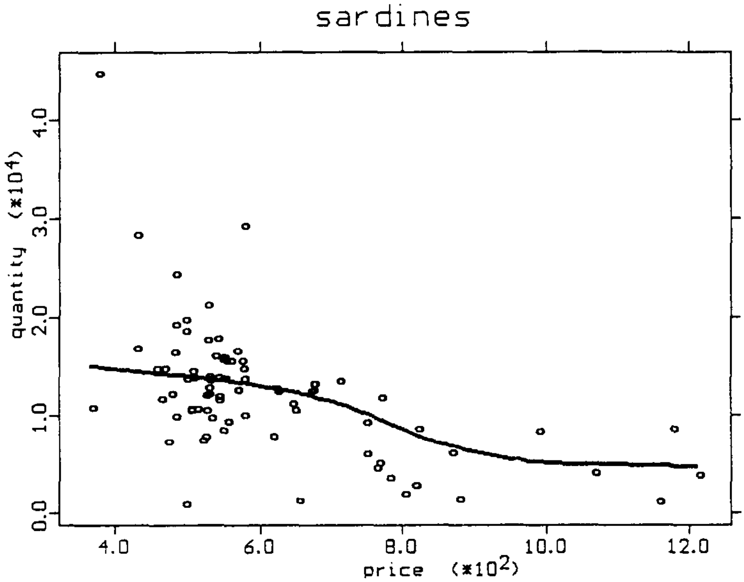

Fig. 1a

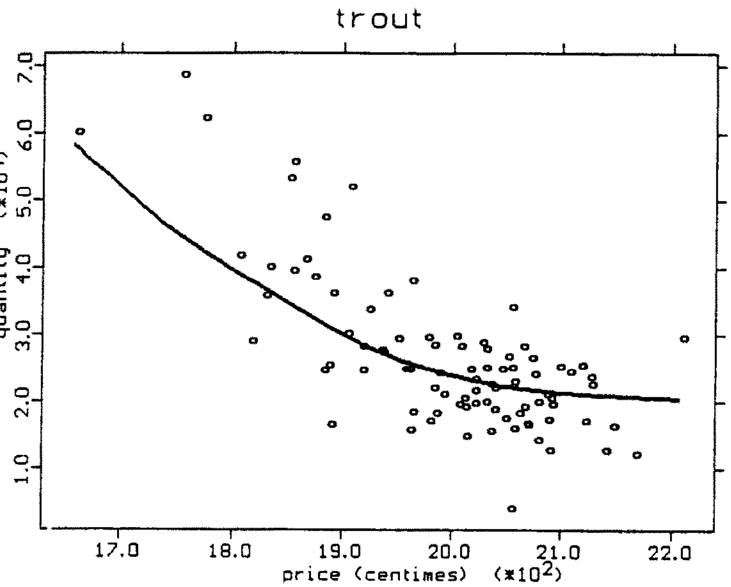

Fig. 1b

they are monotonic. This is easily done since we can check successive differences in the y values for each of the x values corresponding to the grid imposed by the original observations. If the maximum of these is negative, then it can easily be shown that the continuous fitted curve is monotone decreasing. This was the case for all of our curves. As explained in Appendix A, the local smoothing procedure used is 'optimal', and this monotonicity property is not vulnerable to changes about the optimum. Although we have established the monotonicity of the fitted relation, what we are really interested in is the monotonicity of the 'true' relation. This requires establishing an upper confidence band on the *derivative* of the function and showing that this is negative. In our case, it is enough to establish that the upper confidence bound on the maximum of the derivative is negative. However, since no theoretical results are available for this, we had to make this calculation at each discrete point on the x axis corresponding to one of the bin means. In every case the negativity property was satisfied. In the light of this evidence, that the monotonicity property is apparently robust, an economist might naively have suggested that these curves represented aggregate demand for the fish in question and that their monotonicity was derived from the underlying classical individual demands.

The important thing to re-emphasise here is that the 'nice' monotonicity property of the aggregate price quantity curves does *not* reflect and is *not* derived

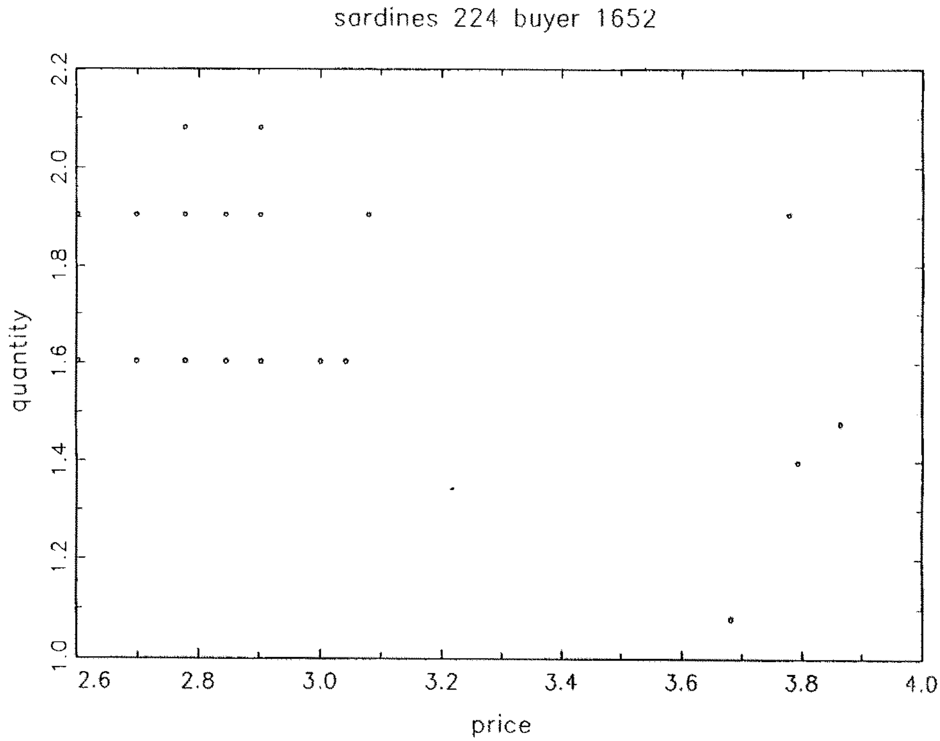

Fig. 2a

from the corresponding characteristics of individual behaviour. Nor indeed, given our discussion, should we expect it to be.

Transactions at the individual level

To illustrate the lack of 'good behaviour' at the micro level, it is therefore worth looking at the plots for the quantities of sardines purchased at different prices by three individuals. These are illustrated in Figs. 2a, 2b, and 2c. The observed price–quantity pairs of the first two buyers are far from corresponding to what classical demand theory might lead us to expect, whereas the third might conceivably meet those conditions. The sardine was chosen to illustrate this, not because it has any particular significance but since the price–quality relations for all four fish are, as we have seen, well-behaved on the aggregate level. Once again it is important to recall the nature and organization of the transactions on this market, both over the day and as it varies between matching

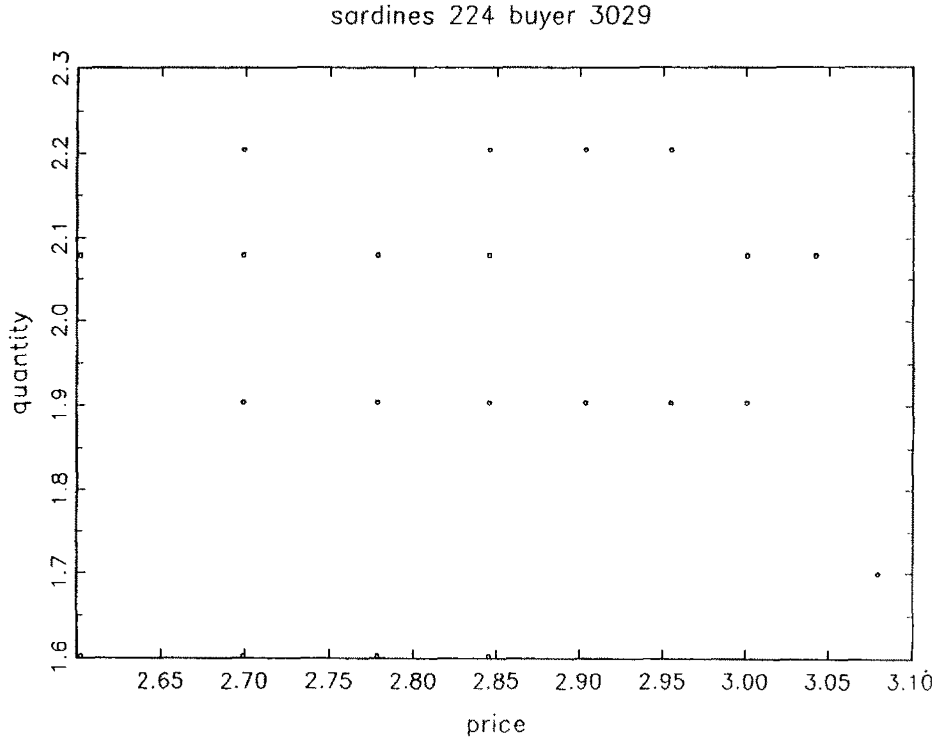

Fig. 2b

of different buyers and sellers, to understand the apparent eccentricity of the individual demands.

4. Stability of price distributions

If, after this initial examination of the data, we are to consider deriving something corresponding to short-term 'strategic demand' from observations over time in our particular market, then we have to be sure that market conditions remained essentially the same over the whole period. Since, as we have already observed, we would not expect an equilibrium price, but rather an equilibrium price distribution for the game that we have described, we should therefore check that the distributions observed in successive periods remain the same. To do this we test the hypothesis that for each individual fish the daily observed price distribution is stable over time. It is important to understand

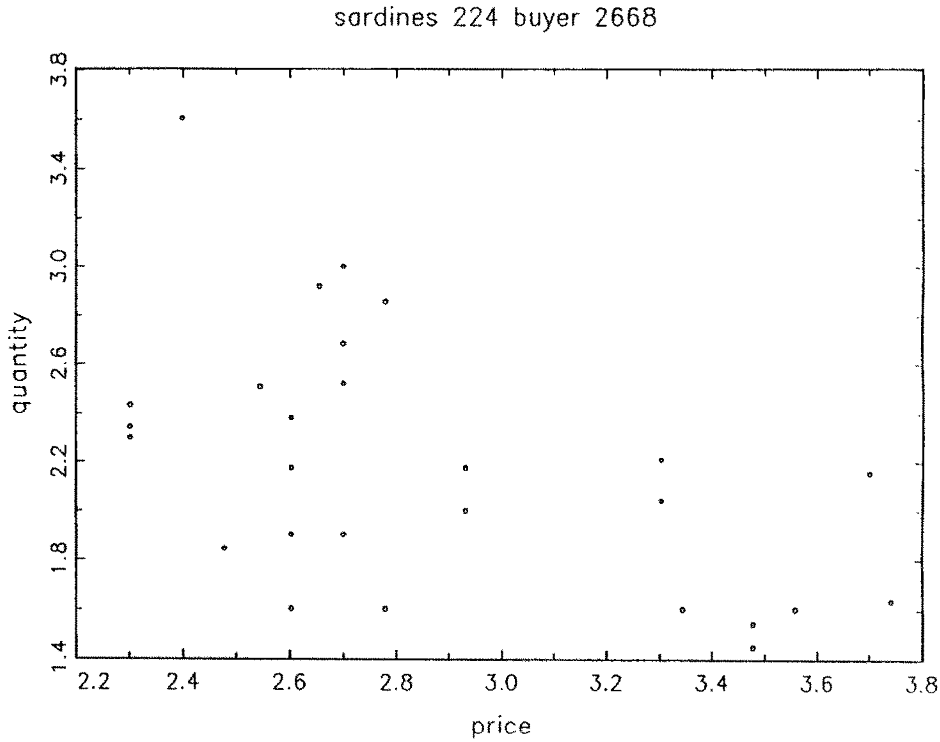

Fig. 2c

what we mean here by distribution. We count the total number of kilos transacted in each price interval. The alternative would be to count the *number of transactions* at each price level, but, in effect, we consider each kilo as a separate transaction. This distinction is usually avoided in the literature on price dispersion where individuals demand one unit of an invisible good (see Rothschild, 1973, and Diamond, 1987, for example). Thus the distribution h of prices is given by

$$h(p_j) = \frac{\sum \textit{quantities sold at prices in the jth interval}}{\textit{Total quantities sold}}.$$

We proceed by fitting a function to each of the distributions and then seeing by how much the distance of each of these functions from the others varies. In Figs. 3a and 3b the results for the three months for sardines on a general and on a focused scale are shown. Fig. 3c shows the same analysis of trout on a focused scale.

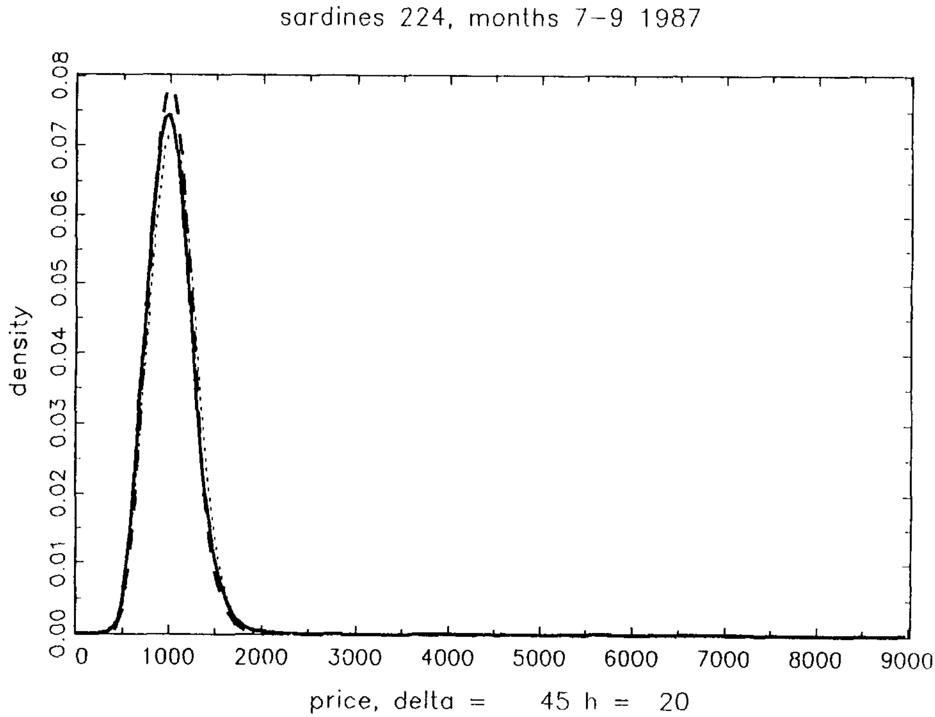

Fig. 3a

Full details of the tests for the stability of the distribution are given in Appendix B. We could not reject the hypothesis that the distributions were constant over time. That is, when we considered the following hypotheses:

$$H_0:\ f_i = f_j, \qquad i \neq j,$$

$$H_1:\ f_i \neq f_j, \qquad i \neq j,$$

for each of our four fish over the three months in question, in none of the cases could we reject H_0.

Since the numerical values of the confidence bounds constitute curves we have not reproduced the graphs here, but similar illustrations may be found in Härdle (1990). However, the relative stability of the smoothed fits over the three months in question can be seen in Figs. 3a to 3c.[5] The importance of the discretisation

[5] In each case, July is given by the solid line, August by the dashed line, and September by the dotted line.

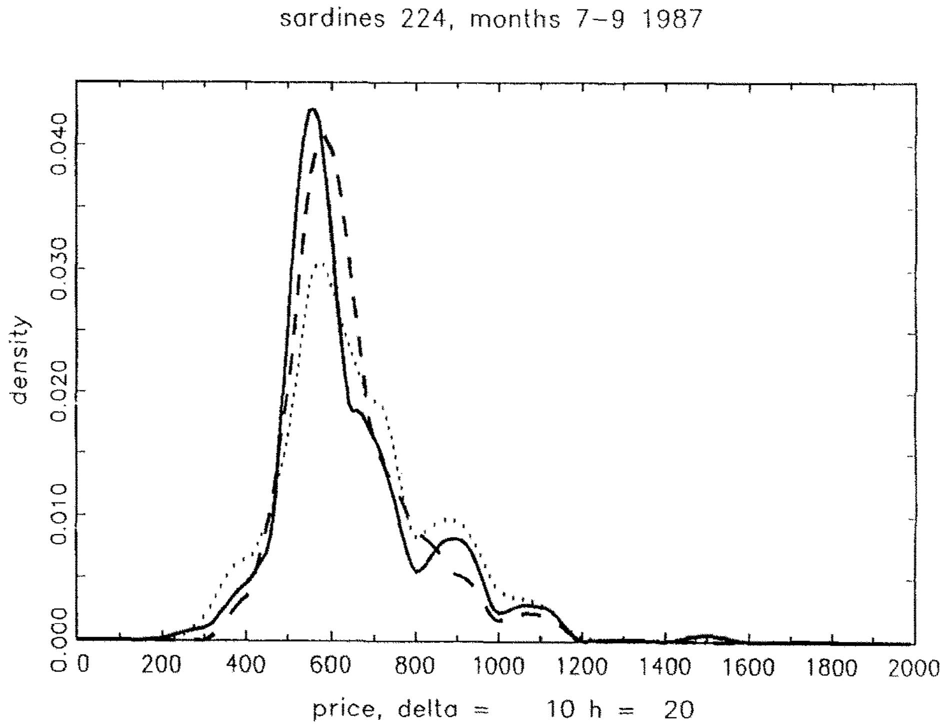

Fig. 3b

parameter d is seen by comparing the two figures for sardines, Figs. 3a and 3b. The smoothing parameter is h.

As our statistical analysis shows, once we accept the idea that we are dealing with a distribution of prices which reflects the equilibrium strategies of the different players, we cannot reject the hypothesis that these distributions are stable over time.

An important point which merits further discussion is to what extent is it legitimate to use stability tests developed for independently drawn observations on the sort of data we examine here? Two remarks can be made. Firstly, it is by no means infrequent to apply a stochastic model to data which is not derived from such a model. Stochastic models of deterministic processes are often very useful. (See Erdös and Spencer, 1974, for example.)

Thus, even if buyers met sellers in a predetermined way, without the appropriate information the modeller may well have to treat the data as if generated by some stochastic matching process.

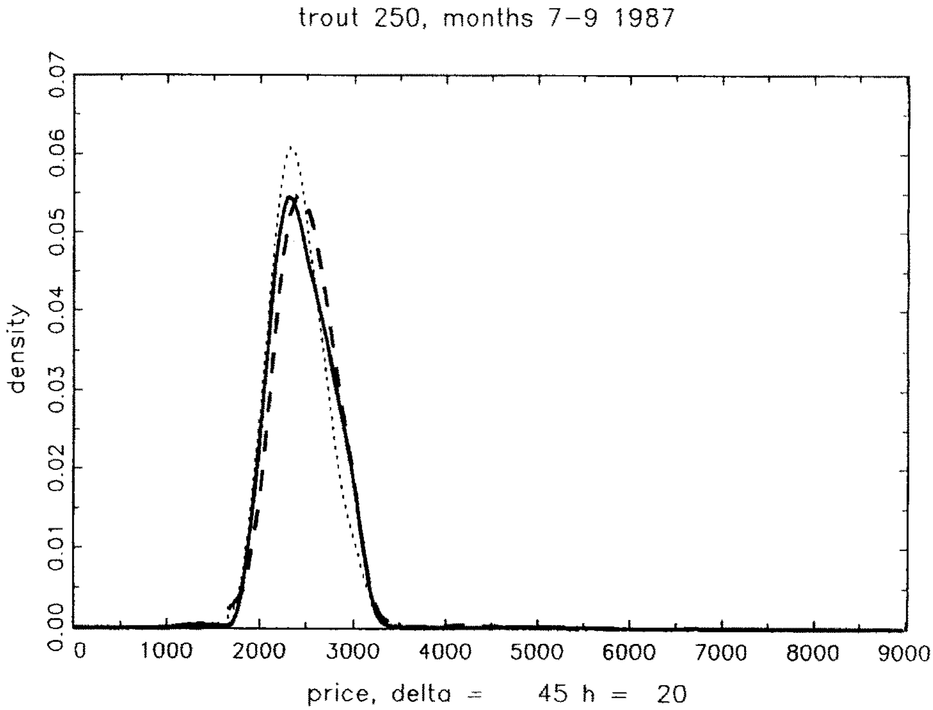

Fig. 3c

In fact, there is a certain amount of stochastic behaviour in the market in that searching for low prices does take place. The problem is that, although the evidence from the fitted densities seems to be clear, for the *statistical* tests for stability to be valid, the observations should be independently identically distributed. This cannot be strictly true, since certain buyers pay higher prices for example. Although these buyers are probably of particular types, restaurants, etc., they are only identified by code. We therefore do not have prior information on which to condition and cannot treat them as different.[6]

[6] In treating our observations as drawn from the same population in this way we are following Theil (1971), for example, who in his 'convergence' approach thought of N consumers as independent elements of an infinite consumer population and the parameters of their utility functions as identically distributed.

5. Price–quantity relations: A second analysis

When we estimated our smoothed 'demand' curves in the previous section we did so by averaging prices. However, this loses a lot of the information contained in our data. We might therefore ask a simpler question for which generally data are not available. Are more kilos of fish transacted at lower prices? By this we mean taking as one observation the total quantity transacted over the whole period at a specific price. Fitting these observations with a smoothed curve corresponds to finding the best fit for the average quantity sold for prices in some appropriately chosen price interval.

We thus obtained smoothed curves by summing over quantities at each given price and by 'binning' data in price intervals. The results are shown in Figs. 4a to 4c. Aggregation over transactions produces nicely behaved, that is, essentially monotone demand curves for three out of the four fish. Sardines are illustrated in Figs. 4a and 4b. Whiting in Fig. 4c, however, displays a monotone increasing

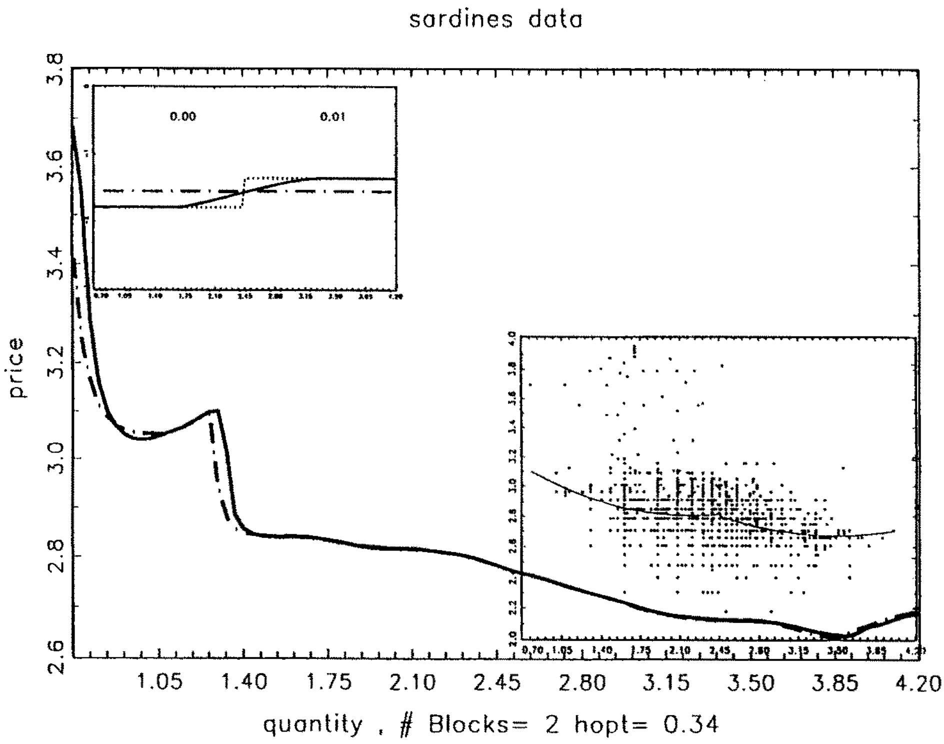

Fig. 4a

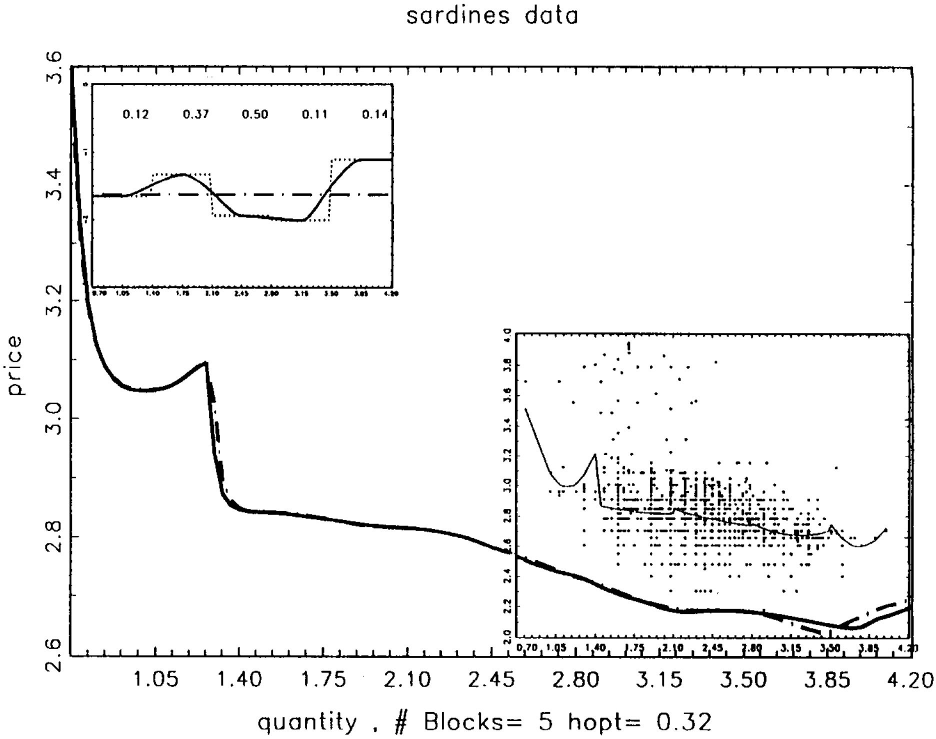

Fig. 4b

characteristic over higher prices. The reason for this is the cluster of points near the origin. One explanation that could be advanced for this is the existence of many transactions at low prices corresponding to the clearing of stocks at the end of the day. To check for this we isolated data for late transactions, and as Table 2 indicates, we found examples in which the prices of late transactions fall sharply.

This result must be treated with some caution since we also find examples where prices fall, only to rise again at the end of the day – a situation corresponding to that when a buyer with inelastic demand wished to add to his purchases before the end of the market.

Nevertheless the important thing to realise here is that the use of nonparametric methods enabled us to identify this particular feature. Trying various nonlinear but *parametric* forms gave excellent fits with monotone decreasing functions. Thus we would not have detected the presence of this group of 'less well-behaved' observations. Once the offending observations were removed we obtained a monotone decreasing nonparametric curve.

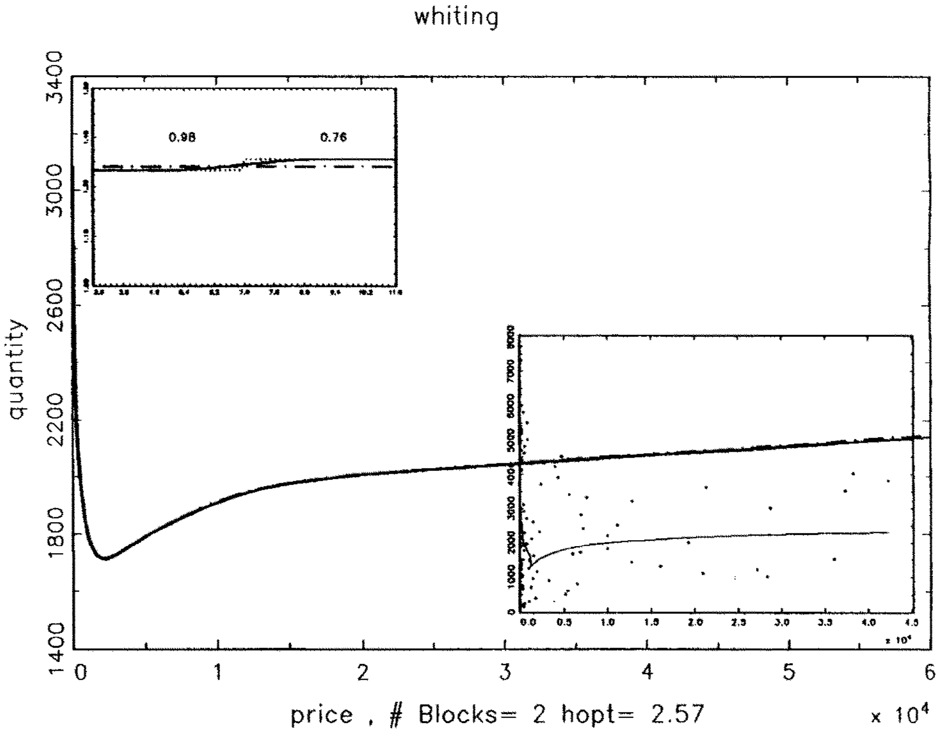

Fig. 4c

Table 2
Late transactions, in chronological order (price dropped if transaction is in last 10% of those made by seller in the day)

Date	Seller no.	Price	Quantity
87/07/04	40		
Normal prices		82	60
		85	60
		75	100
		75	50
		85	60
		75	50
		85	60
		85	70
Dropped price		25	85
87/07/08			
Normal prices	28	82	50
		82	20
Dropped price		20	64

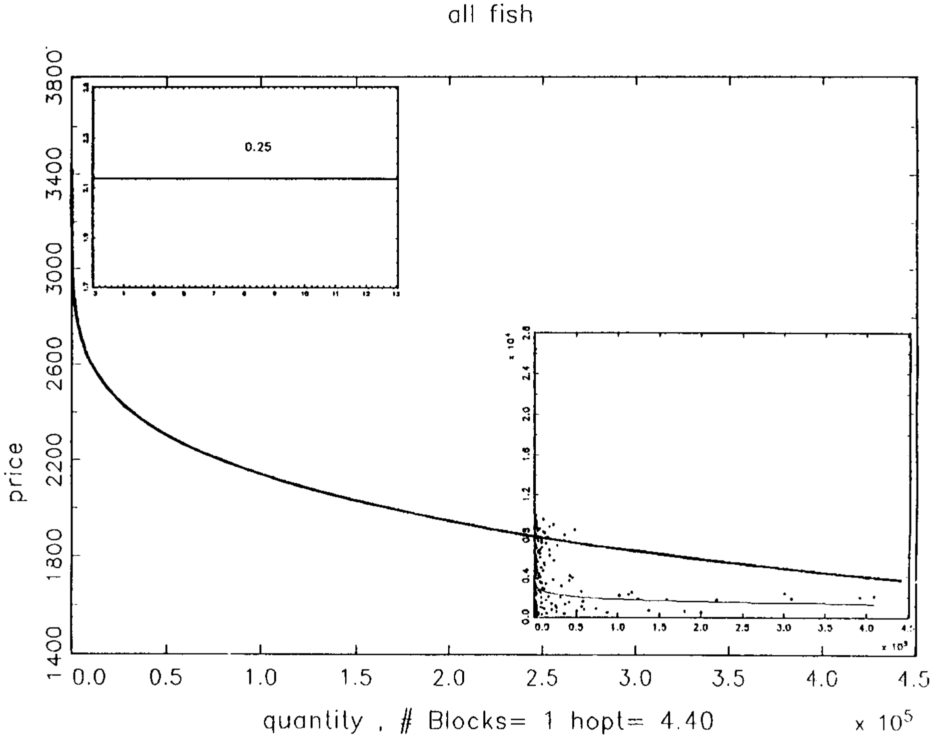

Fig. 5a

Finally we look at all fish with quantities added as if they were one commodity, and the data and resultant curves are shown in Figs. 5a and 5b. Although, of course, this simple addition is extremely primitive as a procedure, the resultant smoothed demand curves are monotone. Weighting the quantities more appropriately would not have modified this. Thus aggregating, even in an arbitrary way, reinforces a characteristic which is weaker at a less aggregated level and even absent for many observations at the micro level. The market is, therefore, 'well-behaved', even though many of the individuals participating in it are not.

6. Conclusion

In this paper we have examined detailed data from the fish market for Marseille. We built a theoretical model for the behaviour in this market. The perishable nature of the product enables us to think of successive markets as

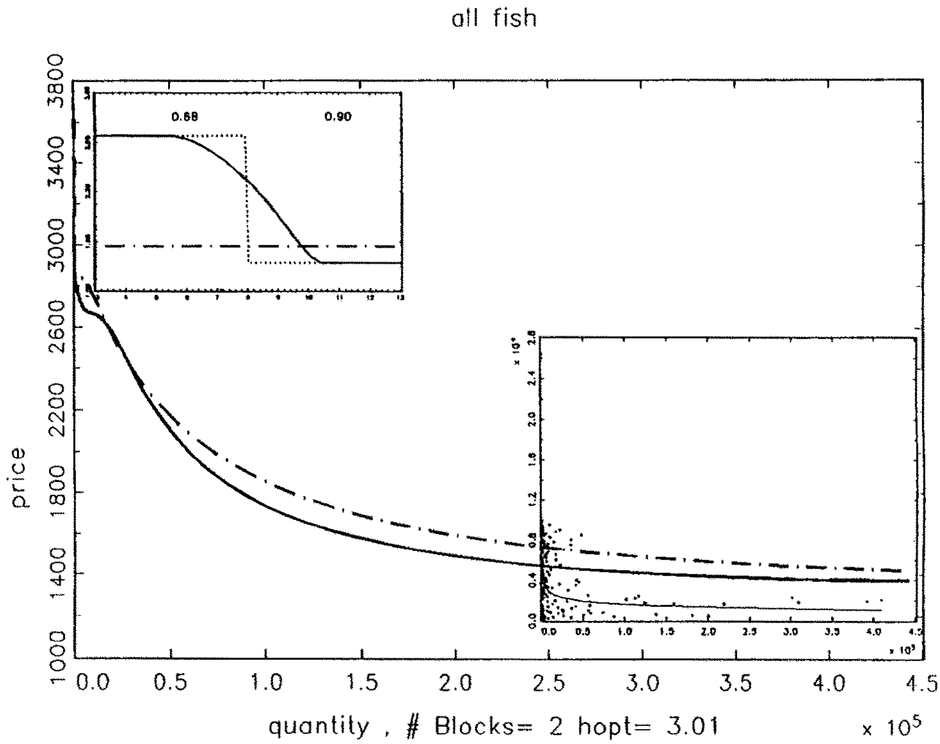

Fig. 5b

being separated and to analyse the short-run behaviour of the relation between prices and quantities in such a market. Although it is tempting to look at these data as resulting from the interaction between competitive supply and demand, the organization of the market and the identity of the participants makes this unreasonable. Individual transactions show none of the characteristics that standard demand analysis would lead one to expect. Fitting seasonalised average price over a day to quantities sold on that day did at the *aggregate level* give rise to a monotone decreasing relation. Thus, this property does not reflect individual behaviour but rather results from aggregation. The use of nonparametric methods makes this finding particularly striking.

We then turned to looking at the data in the light of our models. The price distributions, the appropriate equilibrium notion were stable over time, allowing us to think of the market game as being repeated over time. We then analysed nonparametrically the transactions at each price and were able to identify a particular feature of the data, many small transactions at low prices at the end of trading which destroyed the monotone character of the relation for one fish. However, once this problem was identified, trading for the earlier part of the day did have a monotone price–quantity relation.

Although this analysis now needs to be generalised to all types of fish over longer periods, two aspects are important. Firstly, the underlying behaviour should not just be taken to reflect a standard competitive market. Doing so may lead to false inferences on the aggregate level about individual behaviour. Aggregate phenomena in this sort of market are not simply the magnified reflection of micro phenomena. Testing aggregate data for properties derived from the theory of individual behaviour is not an appropriate procedure. Secondly, once this is taken into account, it is interesting to observe that using nonparametric methods to fit a different price–quantity relation did allow us to identify features of the special behavioural structure of this market, at the micro level.

Appendix A

A.1. The local smoothing method

Our smoothing method assumes that the response variables $\{Y_i\}_{i=1}^n$ are of the form:

$$Y_i = m(X_i) + \varepsilon_i, \qquad i = 1, \dots, n,$$

with explanatory variables X_i, independent errors $\{\varepsilon_i\}_{i=1}^n$, and the smooth regression function $m(x)$. We are interested in estimating the function m. The kernel smoother is defined by

$$\hat{m}_h(x) = n^{-1} \sum_{i=1}^{n} K_h(x - X_i) Y_i \Big/ n^{-1} \sum_{i=1}^{n} K_h(x - X_i), \tag{A.1}$$

where $K_h(\cdot) = h^{-1}K(\cdot/h)$ is the rescaled kernel function K with bandwidth h. Behaviour of this smoother is crucially dependent on the choice of h. A simple and useful quantification of the influence of h is the analysis of the asymptotic mean integrated squared error. The variance of the kernel smoother $\hat{m}_h(x)$ is approximated by

$$n^{-1}h^{-1}V(x) = n^{-1}h^{-1}\int K^2(u)\,du\,\frac{\sigma^2(x)}{f(x)}, \tag{A.2}$$

where $\sigma^2(x)$ denotes the variance function $E(Y^2|x) - m^2(x)$ and $f(x)$ is the marginal density of the X variables. The bias is approximated by

$$h^2B(x) = \frac{h^2}{2}\int u^2K(u)\left[m''(x) + 2\,\frac{m'(x)f'(x)}{f(x)}\right]. \tag{A.3}$$

This indicates that the best bandwidth at each location x is well represented by

$$h_0(x) = n^{-1/5}\left[\frac{V(x)}{4B^2(x)}\right]^{1/5}.$$

A good global bandwidth, i.e., one suitable in an average sense, is given by

$$h_l(x) = n^{-1/5}\left[\frac{\int V(x)}{4\int B^2(x)}\right]^{1/5}. \tag{A.4}$$

This bandwidth is obtained by minimising the approximate integrated mean squared error, $n^{-1}h^{-1}\int V(x)\mathrm{d}x + h^4\int B^2(x)\mathrm{d}x$.

A.2. A global smoothing parameter

Practical use of the above representation for h_1 in formula (A.4) requires estimates of $\int V(x)$ and $\int B^2(x)$, which in turn can be built up, using formulas (A.2) and (A.3), from estimates of $m(x)$ and $f(x)$. We shall use simple estimates like histograms for this.

Histograms are constructed by first partitioning the design interval $[a, b]$ into blocks B_j, $j = 1, \ldots, N$. For simplicity, we work explicitly here with equal length intervals

$$B_j = \left[a + \frac{(j-1)(b-a)}{N}, a + \frac{j(b-a)}{N}\right].$$

Let B denote a generic block B_j, and r and l denote right and left boundaries of this block. The proportion of X_i falling in each interval reflects the height of the density near the centre of the block (bin). Let $c = (r + 1)/2$ denote the block centre and $r_b = (r - 1)/2$ denote the block radius. The histogram density estimate is

$$\hat{f}(c) = \frac{1}{2nr_b}\sum_{i=1}^{n} I(|c - X_i| \leqslant r_b). \tag{A.5}$$

To estimate the derivative of f on B we use a simple differencing method. Define

$$n_l = \sum_{i=1}^{n} I(l \leqslant X_i < c), \qquad n_r = \sum_{i=1}^{n} I(c \leqslant X_i < r).$$

If these frequency estimates are combined we obtain the score function estimate

$$(\hat{f'/f})(c) = 2(n_r - n_l)/r_b(n_r + n_l). \tag{A.6}$$

The estimation of $V(x)$ in (A.4) is constructed from a sum of squared residuals (RSS) about an estimate $\hat{m}(x)$ of $m(x)$, normalised by an estimate of f. In particular, in the generic block B define $RSS = \sum_{X_i \in B}(Y_i - \hat{m}(X_i))^2$. An estimate of (σ^2/f) is then given by

$$\widehat{(\hat{\sigma}^2/f)}(c) = \frac{2nr_b}{n_l + n_r} RSS,$$

which leads to

$$\hat{V}(c) = \tfrac{5}{7}\,\widehat{(\sigma^2/f)}(c), \tag{A.7}$$

for the quartic kernel,

$$K(u) = \tfrac{15}{16}(1-u^2)^2 I(|u| \leqslant 1),$$

which we used throughout.

For estimation of m', m'' we use blockwise parabolic fitting. As an estimate of $V = \int V(x)$, we obtain

$$\hat{V} = \sum_{j=1}^{N} 2r_b \hat{V}(c_j), \tag{A.8}$$

using notation from (A.7) and the quartic kernel given before. The bias $B(c_j)$ for the quartic kernel in each block B is estimated via

$$\hat{B}(c_j) = \tfrac{1}{2}\tfrac{1}{7}\, 2\hat{\beta}_{3j} + 2[2\hat{\beta}_{3j}(c_j) + \hat{\beta}_{2j}](\hat{f}'/f)(c_j), \tag{A.9}$$

where $\hat{\beta}_{2j}$ and β_{3j} are least squares estimates in the model

$$\beta_{1j} + \beta_{2j}x + \beta_{3j}x^2 \tag{A.10}$$

over block β_j. The final estimate of $B_2 = \int B^2(x)\mathrm{d}x$ is obtained by summing up the squares of these quantities,

$$\hat{B}_2 = \sum_{j=1}^{N} 2r_b \hat{B}^2(c_j). \tag{A.11}$$

This leads finally to

$$\hat{h}_1 = n^{-1/5}[\hat{V}/4\hat{B}_2]^{1/5}.$$

A.3. Blocks for a local smoothing parameter function

The estimates of variance and squared bias over each block provide an easy bandwidth choice for each block, given by

$$\hat{h}_0(c) = n^{-1/5}[\hat{V}(c)/4\hat{B}^2(c)]^{1/5}, \tag{A.12}$$

using the notation from (A.7) and (A.10). The logs (base 2) of these values are the heights of the dotted step function in the upper left inset of Fig. 4a. The bandwidth $\hat{h}_1$ is represented by the dotted and dashed constant function in this inset bandwidth plot. This provides a useful reference for understanding the relative sizes of the local bandwidths. The average of the $\hat{h}_0(c_j)$ does *not* give the global bandwidth $\hat{h}_1$ because the variance and bias terms need to be summed separately for the latter!

The local bandwidth estimates are best in the centre of the blocks. For the points away from the centres we use a smooth, represented by the solid curve, of the step function. This smooth is computed on a fixed grid of x's by the formula (A.1) with the X_i replaced by the *bin* centres and the Y_i replaced by the height of the step function. The kernel used is the quartic, and the bandwidth is the block radius r_b. This choice of bandwidth guarantees that the smooth coincides with the step function at the points where it is most accurate, i.e., at the *bin* centres.

A.4. Diagnostic plots

The inset plots in Fig. 4a are intended to show visually how well our methods are performing. For example, Fig. 4a shows in the left upper corner the fact that a bigger bandwidth has to be chosen for larger quantities.

The raw data plot in the lower right was also a useful diagnostic in the choice of $N = 2$ blocks for the sardine data example presented in Fig. 4a. Our initial choice of $N = 5$ blocks gave visually poor performance because too many 'corners' appeared, as can be seen in Fig. 4b. This gave a visually poor parabolic fit, which resulted in an oversmoothed global choice $\hat{h}_i$ and a less effective local bandwidth function $\hat{h}_0(x)$.

The bandwidth plots enhance the understanding of the performance of the local smoothing method by showing the amount of smoothing done at each point. The effective bandwidth is shown on the log scale, because this parameter is multiplicative in character.

A further diagnostic device, which we find useful, is to calculate the observed significance level of the parabolic fit on each block. The numbers shown in the top part of the bandwidth plot are p-values for testing, within each block, the null hypothesis of linearity,

$$\mathrm{H}_0: \quad \beta_{3,j} = 0,$$

in the local parabolic model. When these are small, there is strong evidence of curvature in the data, so our local bandwidth estimate should be reliable. Note that in most of our examples the local method works well in many cases, even when the p-value is large.

Appendix B

Stability of price distributions

Recall that the distribution of prices is given by

$$f(p) = \frac{\sum \textit{quantities sold at prices in the jth interval}}{\textit{Total quantities sold}}.$$

Let us denote the pairs $\{X_i, Y_i\}_{i=1}$ as observations of X_i = price of the transactions and Y_i = quantity of ith transactions. We shall assume that the data $\{X_i, Y_i\}_{i=1}$ are independent and identically distributed observations.

This can be justified in several ways. Firstly, not all players are present all the time (randomness). Second, buyers do not stick to 'their' seller (independence) and buy different quantities on different occasions (identical distribution). Third, sellers do not stick to 'their' prices, as empirical analysis of late transactions shows. We are aware of the fact that this is an important assumption, but nevertheless pose it in order to be able to analyse the stability of the distribution.

The price distribution at $p = x$ can be re-written as

$$n^{-1}h^{-1}\tfrac{1}{2}\sum_{i=1}^{n} Y_i I(|x - X_i| \leqslant h)\Big/ n^{-1}\sum_{i=1}^{n} Y_i,$$

when we have essentially rescaled by $2h$, the length of the interval over which we are computing the price distribution. This information is essentially a kernel estimator.

$$\hat{r}_h(x)/\bar{y} = n^{-1}\sum_{i=1}^{n} K_h(x - X_i)Y_i/\bar{y},$$

for kernel $K(u) = \frac{1}{2}I(|u| \leqslant 1)$, $K_h(\cdot) = h^{-1}K(\cdot/h)$.

A kernel is a symmetric probability density. Examples are

$$K(u) = \tfrac{3}{4}(1 - u^2)I(|u| \leqslant 1) \qquad \text{Epanechenkov,}$$

$$K(u) = \tfrac{15}{16}(1 - u^2)I(|u| \leqslant 1) \qquad \text{Quartic,}$$

$$K(u) = (1/\sqrt{2u})\exp(-u^2/2) \qquad \text{Gaussian.}$$

The parameter h controlling the 'window' over which we are averaging is called bandwidth (see Härdle, 1990, Ch. 3). What is $\hat{r}_h(x)$ estimating? Denote by $m(x) = \mathrm{E}(y \mid Y = x)$ and (in contrast to earlier notation) $f(x)$ the density of the prices of all transactions.

Thus, following well-known arguments of Härdle (1990),

$$\begin{aligned}\mathrm{E}\hat{r}_h(x) &= \textstyle\int K_h(x - u)m(u)f(u)\,\mathrm{d}u \\ &= m(x)f(x) + \frac{h^2}{2}\textstyle\int u^2K(u)\mathrm{d}u\,(mf)''(x) + \mathrm{o}(h^2) \quad \text{as} \quad h \to 0.\end{aligned}$$

The bias is thus of order $\mathrm{O}(h^2)$. The variance of $\hat{r}_h(x)$ is given by

$$\begin{aligned}\mathrm{var}[\hat{r}_h(x)] &= n^{-1}\mathrm{var}[K_h(x - X)Y] \\ &= n^{-1}h^{-1}\mathrm{E}(Y^2 \mid X = x)f(x)\textstyle\int K^2(u)\,\mathrm{d}u + \mathrm{O}(n^{-1}h^{-1}) \\ &\qquad \text{as} \quad nh \to \infty.\end{aligned}$$

Hence the mean squared error can be written with constraints C_1 and C_2 as

$$MSE(x) = u^{-1}h^{-1}C_1 + h^4C_4.$$

The optimum for h is reached when $h \sim n^{-1/5}$, yielding a spread of $MSE(x) \sim n^{-4/5}$. Therefore positive intervals have length (and 'shrinking rate') $n^{-2/5}$. More precisely

$$\sqrt{nh}\left(\frac{\hat{r}_h(x)}{\bar{y}} - \frac{r(x)}{\mu}\right) \xrightarrow{L} \mathrm{N}(0, M_2(x)f(x)\int K^2),$$

for $nh5 \to 0$, $\mu = \mathrm{E}(Y)$, $r = mf$.

Of course, pointwise confirmation intervals do not help us in testing the stability of the price distribution, we need uniform confirmation bounds. They can be developed as follows. With

$$\hat{r}_h(x) - r(x) = \iint K_h(x - n)v\,\mathrm{d}(F_u - F)(n, v),$$

when F_n diverts the empirical distribution function of the data $\{(X_i, Y_i)\}_{i=1}^{4}$ and F their theoretical distribution. Suppose the data has been rescaled so that $X \in [0, 1]$. The process $\sqrt{n}(F_n - F)$ can be approximated by Brownian bridges, so that asymptotically as the number of transactions becomes very large,

$$\sqrt{uh}(\hat{r}_h(x) - r(x)) \approx [M_2(x)f(x)]^{1/2}\, h^{-1/2} \int K\left(\frac{x-u}{h}\right) \mathrm{d}W(u),$$

for a Wiener process W and $M_2(x)$: $\mathrm{E}(Y^2 \mid X = x)$.

The desired uniform confidence bound can be constructed from the following statement:

$$P\{(2\delta \log n)^{1/2}[(uh/\int K^2)^{1/2} \sup(\hat{M}_2(x)\hat{f}_h(x))^{-1/2}| - d_u] < z\}$$
$$\to \exp(-2\exp(-z)) \quad \text{as} \quad u \to \infty\,.$$

Here,

$$d_u = (2\delta \log n)^{1/2} + (1/2\delta \log n)^{1/2}\{\log(C_3/2\pi)\},$$
$$h = n^{-\delta}, \qquad \delta > \tfrac{1}{5},$$
$$C_3 = \int [K^1(u)]^2\, \mathrm{d}u / 2\int [K(u)]^2\, \mathrm{d}\bar{u}.$$

For an algorithm for these confidence bounds see Härdle (1990).

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