Managing Catastrophic Risk Through Insurance and Securitization

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In recent years, the magnitude of catastrophic property/casualty disaster risks has become a major topic of discussion. Recent catastrophic events such as Hurricane Andrew and the Northridge earthquake each cost the insurance industry in excess of $10 billion. These events illustrate the potential stresses which the insurance industry is now facing. In addition, concentration of values in catastrophe-prone coastal areas has caused an increase in the amount of damage.

When loss exposures are (partially) correlated, the design of an optimal risk sharing contract is based on a decomposition of the risk into systemic, i.e., non-diversifiable, and idiosyncratic, i.e., diversifiable, parts. This decomposition allows us to apply two complementary risk sharing rules: risk mutualization and risk securitization.

In his classic 1962 article, Borch showed that Pareto optimal risk sharing in an economy with risk-averse agents is one in which each shares the aggregate wealth. This is risk mutualization. When aggregate wealth is riskless and without transaction costs, agents can fully insure. If aggregate wealth is risky, then individuals may insure idiosyncratic risk but retain shares in aggregate wealth according to their tolerance towards risk. This mutuality principle has largely been ignored in the literature on optimal insurance design (e.g., Arrow, Raviv). Doherty and Dionne were among the first to examine alternative contracts for insuring individual risks when insurance companies are unable to eliminate risk by pooling. However, these risks can be diversified by pooling them with other economic events that are not usually the subject of insurance. This is achieved through the securitization of risk.

Risk securitization is accomplished by issuing specific conditional claims and selling them directly to financial investors. The Chicago Board of Trade launched options on natural catastrophes (cat spreads) in 1995 and catastrophe-linked bonds (cat bonds) have been issued since 1997. These innovative financial instruments are the response to the traditional insurance and reinsurance market's inability to deal with highly correlated risks. For instance, simulations conducted by modeling firms suggest that damages caused by a major hurricane in Florida could be at least $75 billion and those due to an earthquake in California could exceed $100 billion. With prospective event-losses easily exceeding $50 billion, the capitalization of the insurance and reinsurance industry is at issue. Estimates of total capital and surplus of U.S. insurers and international reinsurers are about $300 billion and $100 billion, respectively.[1] Although such catastrophic losses are large enough to place the insurance industry under severe stress, they are lower than one standard deviation of the daily value traded (about $130 billion on average) in the U.S. capital markets (Cutler and Zeckhauser). Therefore, the pool of financial capacity provided by asset markets is able to bear the most pessimistic estimated losses caused by a natural catastrophe.

From this risk decomposition, the optimal insurance policies are contracts in which the policyholder insures the idiosyncratic component of his individual risk, but receives from his insurer a dividend based on the aggregate experience of the insurance pool. This is the logic of participating policies. These contracts are proposed in several types of insurance, and especially in life insurance. The aggregate loss is then (partially) hedged on financial markets through appropriate hedging instruments.

The model and assumptions are specified in the next section, followed by a characterization of an optimal variable participating

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[1] This capital and surplus applies to all risks (property/casualty, liability, etc.), and not just catastrophes.

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insurance policy. The findings are then applied to the management of crop-yield risk in agriculture. A conclusion highlights the main results concerning risk sharing in the presence of aggregate risk and presents further research.

Assumptions and the Model

The static model is examined in the standard expected utility framework. The agent with non-random initial wealth w_0 faces a risk of loss $\tilde{l} \in [0, w_0]$.[2] The individual loss $\tilde{l}$ can be partitioned into a systemic component $\tilde{x}$ with $E\tilde{x} = 0$, E denoting the expectation operator, and an idiosyncratic component $\tilde{\varepsilon} \geq 0$ through the deterministic relationship

(1) $l = l(x, \varepsilon)$

with $l_x \geq 0$ and $l_\varepsilon \geq 0$.[3] The distribution of both risks as well as the deterministic function are assumed to be known by the policyholder and the insurance company and to be observable costlessly by the insurer.[4] The risk pool is defined by a large population of agents with identical loss distributions. The idiosyncratic random losses of two different agents of the risk pool are assumed to be independent. However, the systemic components are equal for all members of the risk pool.

The individual has the possibility to self-construct a variable participating insurance policy via the purchase of two separate contracts: a non-participating policy and a fully participating policy. The fully participating policy is described by the couple $[I(.,.), P(.)]$ where $I(x, \varepsilon)$ is the amount of indemnity payments when the realized systemic and idiosyncratic risks are x and ε, respectively. The indemnity function must be non-negative:

(2) $I(x, \varepsilon) \geq 0$ for all x and ε.

The premium is variable and it depends on the realization of the systemic loss. Idiosyncratic losses are assumed to be insurable at zero transaction cost and the risk pool is assumed to be sufficiently large so that, from the law of large numbers, deviations of the average loss from the expected loss can be neglected. The premium is thus equal to the expected indemnity conditional on the realization of the systemic risk:

(3) $P(x) = EI(x, \tilde{\varepsilon})$.

The non-participating insurance contract is described by the couple $[J(.,.), Q]$. It differs from the fully participating policy in two ways. First, the indemnity schedule depends on the index that is not perfectly correlated with the systemic component of the risk pool. The contract is thus exposed to basis risk. To model this, I assume that $\tilde{x} = \alpha + \beta\tilde{z} + \tilde{b}$, with $\alpha \geq 0$, $\beta > 0$, $E\tilde{b} = 0$ where the $\tilde{b}$ basis risk is assumed to be independent of the index $\tilde{z}$, the risk pool's systemic component $\tilde{x}$ and the idiosyncratic risk $\tilde{\varepsilon}$. For example, $\tilde{x}$ denotes the aggregate losses of a regional pool and the $\tilde{z}$ index represents the national aggregate losses. To simplify the notation, I further assume that $\alpha = 0$ and $\beta = 1$. Thus $\tilde{x} = \tilde{z} + \tilde{b}$. It is easy to relax these assumptions, but no additional insights are gained. The indemnity function of the non-participating policy must be non-negative:

(4) $J(z, \varepsilon) \geq 0$ for all z and ε.

Second, by definition of a non-participating policy, the insurance premium is fixed. In addition, the cost of insurance is unfair. The risk premium can be justified by the presence of undiversifiable risk and, thus, it is the payment required by the risk-averse shareholders. This can also be the consequence of firm-specific costs of risk bearing, e.g., convex tax schedule. For the sake of simplicity, the tariff is assumed to be sustained by a competitive insurance market with risk-neutral insurers and transaction costs that are proportional to claims:

(5) $Q = (1 + \lambda)EJ(\tilde{z}, \tilde{\varepsilon})$

where the loading factor is positive, $\lambda > 0$.

The variable insurance participating policy thus provides indemnity payments $[I(x, \varepsilon) + J(z, \varepsilon)]$ and it is sold at a price equal to $[P(x) + Q]$ when the realized values of the systemic and the idiosyncratic components and of the index are x, ε and z, respectively. By purchasing this contract, the policyholder

[2] "Tildes" are used to denote random variables and the same variables without the tildes to denote realizations of random variables.

[3] Variables in subscript denotes the partial derivative with respect to this variable.

[4] Problems associated with informational asymmetries, i.e., moral hazard and adverse selection, not analyzed here.

will have final wealth of

$$(6) \quad \tilde{w} = w_0 - l(\tilde{x}, \tilde{\varepsilon}) + I(\tilde{x}, \tilde{\varepsilon}) + J(\tilde{z}, \tilde{\varepsilon}) - P(\tilde{x}) - Q.$$

The problem of the policyholder with a von Neumann-Morgenstern utility function u, where $u' > 0$ and $u'' < 0$, is to determine the indemnity schedule of the variable participating contract that maximizes his expected utility of final wealth, $Eu(\tilde{w})$, subject to constraints (2) to (5).

Optimal Variable Participating Insurance Contract

The design of an optimal variable participating insurance policy is closely related to the partition of individual risk into systemic and idiosyncratic components. Additive and multiplicative relationships between both parts are successively examined.

Additive Risk Components

The deterministic function l expressed in equation (1) is rewritten as

$$(7) \quad l(x, \varepsilon) = x + \varepsilon.$$

This means that if $x = 10$ then losses are 10 higher for everyone than the long-run average. The optimal variable participating insurance contract is designed in two steps. The optimal fully participating insurance contract is first designed, then the optimal non-participating insurance policy is derived. Since the fully participating contract is assumed to be sold at an actuarially fair price whereas the premium of the non-participating policy is unfair, the policyholder purchases full insurance on the idiosyncratic risk

$$(8) \quad I^*(x, \varepsilon) = x + \varepsilon$$

with $P = x + E\tilde{\varepsilon}$. From equations (7) and (8), his final wealth expressed in equation (6) becomes

$$(9) \quad \tilde{w} = w_0 - E\tilde{\varepsilon} - \tilde{z} + J(\tilde{z}, \tilde{\varepsilon}) - Q - \tilde{b}.$$

The variability of his final wealth comes from the random index $\tilde{z}$ and the uninsurable and unhedgeable basis risk $\tilde{b}$. Therefore, the indemnity schedule of the optimal fully participating insurance contract will be only contingent on the index, $J^*(z, \varepsilon) \equiv K(z)$ for all z and ε. From Mahul (1999), the optimal unfair non-participating policy in the presence of an independent and additive background risk $\tilde{b}$ displays full insurance on the index above a deductible $D > 0$:

$$(10) \quad J^*(z, \varepsilon) = K(z) = \max[z - D, 0]$$

with $Q = (1 + \lambda)E \max[\tilde{z} - D, 0]$. The optimal hedging strategy with index-based contract thus requires a long call position at a strike index D and a hedge ratio equal to unity.

Therefore, the optimal variable participating insurance policy provides full insurance on the $\tilde{\varepsilon}$ idiosyncratic risk and full insurance above a deductible on the $\tilde{z}$ index. The policyholder bears part of the systemic risk through the partial coverage on the $\tilde{z}$ index, and the $\tilde{b}$ basis risk. This optimal insurance strategy can be replicated by two contracting patterns. First, a mutual insurance company proposes a contract that provides full insurance on the idiosyncratic risk, through risk mutualization, and partial insurance on the systemic risk. This company partially reinsures the systemic component of its portfolio risk through index-based contracts. In the second alternative, two separate non-participating contracts are available. The first contract is sold at a fair price and it provides a coverage against the idiosyncratic risk $\tilde{\varepsilon}$. The second one is unfair and it provides a coverage against the random index. It is straightforward to show that the agent fully insures the idiosyncratic component of his risk and the systemic component is partially hedged through index-based call options provided by the financial markets. Securitization can thus be handled independently from the insurance contract.

Multiplicative Risk Components

The deterministic relationship between the systemic and idiosyncratic components is now expressed by

$$(11) \quad l(x, \varepsilon) = (1 + x)\varepsilon.$$

For instance, if $x = 0.1$, then everyone's realized individual loss is 10% higher than the long-run average. Assuming an insurance market for the $\tilde{\varepsilon}$ idiosyncratic risk with actuarially fair pricing, the optimal fully participating policy displays full insurance on $\tilde{\varepsilon}$:

$$(12) \quad I^*(x, \varepsilon) = (1 + x)\varepsilon$$

with $P = (1 + x)E\tilde{\varepsilon}$. The policyholder's final wealth becomes

$$(13) \quad \tilde{w} = w_0 - E\tilde{\varepsilon} - \tilde{z}E\tilde{\varepsilon} + J(\tilde{z}, \tilde{\varepsilon}) - Q - \tilde{b}E\tilde{\varepsilon}$$

where $\tilde{b}E\tilde{\varepsilon}$ can be viewed as an independent background risk. From Mahul (1999), the indemnity schedule of the optimal nonparticipating policy is

$$(14) \quad J^*(z, \varepsilon) = K(z) = E\tilde{\varepsilon}\max[z - D, 0]$$

with $Q = (1+\lambda)E\tilde{\varepsilon}E\max[\tilde{z} - D, 0]$. The optimal hedging strategy with the index-based contract is to buy call options at the strike index D with a hedge ratio equal to $E\tilde{\varepsilon}$.

The optimal variable participating policy thus provides full insurance on the idiosyncratic risk and partial insurance on the systemic risk. The main difference with the additive case is the impact of the expected idiosyncratic component on the optimal hedge ratio and on the uninsurable basis risk.

This variable participating insurance contract can be replicated via insurance and financial markets as follows. A mutual company offers the optimal variable participating policy. The idiosyncratic component of each individual risk is pooled and the systemic risk is partially hedged on financial markets through securitization. Contrary to the additive case, the consumer is not able to replicate the variable participating policy by purchasing two separate non-participating contracts on idiosyncratic risk via insurance markets and on systemic risk via financial markets. This comes from the fact that the optimal hedge ratio should be equal to the random variable $\tilde{\varepsilon}$. Such a stochastic hedge ratio is not available on real-world financial markets. Consequently, the insurance company plays a central role in the management of individual risks by eliminating the idiosyncratic risk through mutualization and thus allowing to select a hedge ratio based on a deterministic value, the expectation of the idiosyncratic loss.

Managing Crop Yield Risk in Agriculture

Historical experience shows that multiple peril crop insurance programs, in which individual farm yields are used to measure yield loss, fail to operate on an actuarially sound basis. There is extensive literature investigating the causes of market failure. They are usually explained by the presence of information asymmetries such as moral hazard and adverse selection (Chambers, Quiggin). In contrast to this literature, Miranda and Glauber argue that systemic risk may be the most serious obstacle in the development of a private crop insurance market. They note that, contrary to automobile or fire risks which tend to be independent and to price or rate risks which tend to be highly correlated, crop-yield risk lies between these two extremes. This risk is a combination of systemic component stemming primarily from the impact of unfavorable weather events and of an idiosyncratic component depending on individual characteristics. The systemic part is highly correlated among individual farm-level yields (droughts or extreme temperature affect simultaneously a large number of farms), while the idiosyncratic part is almost independent among individual yields.

Several studies have been recently devoted to the optimal management of agricultural systemic risk. The limitations on the ability of the private insurance and reinsurance industry to fund catastrophic losses have led to a proposal for government involvement. Since the 1980s, the Federal Crop Insurance Corporation has acted as the primary reinsurer for crop insurers. Although government solutions to the capacity problem may have some potential, they should be used only if the private solutions are not forthcoming. A private market to provide additional capacity for financing catastrophic risk in agriculture is developing. In 1995, the Chicago Board of Trade launched an innovative contract into the agricultural markets, namely, the quantity-based crop yield futures and options contracts. The payoff of such contracts is based on the aggregate yield of a surrounding geographical area (see, e.g., Li and Vukina for a detailed description of these contracts). When a linear relationship exists between the individual yield and the area yield, Mahul (1999) shows that the optimal area yield insurance policy provides full insurance on the area yield risk above a deductible and that the marginal indemnity function is equal to the individual beta coefficient which measures the sensitivity of farm yield to area yield. Under a multiplicative relationship, the slope of the optimal indemnity schedule turns out to be less than the beta coefficient if the producer is prudent (Mahul 2000). Due to the similarity between insurance contracts and options contracts, the optimal hedge ratio is thus equal to (lower than) the beta coefficient under a linear (multiplicative) relationship between individual yield and area yield. Notice that these results can also be applied to other types of index-based contracts, such as weather-based securities. Nevertheless, these studies assume that the idiosyncratic risk is uninsurable and unhedgeable and therefore is treated like a background risk.

Variable participating contracts turn out to be useful instruments in efficiently managing both components of the individual yield risk. The idiosyncratic risk would be managed through the mutuality principle and the financial markets would deal with the systemic risk via securitization. It is interesting to notice that the idiosyncratic component could be managed not only by mutual insurance companies but also by cooperatives. Under a linear relationship between both components of risk, it has been shown that the optimal variable participating contract can be replicated by two separate contracts. The first one provides full insurance on the idiosyncratic risk and the second one provides partial insurance on the systemic risk. However, a multiplicative relationship seems to be a more natural assumption in agriculture. In this context, cooperatives could play a central role. They would manage the independent idiosyncratic risks through mutualization and use area yield insurance contracts to hedge against the systemic risk of the pool.

Nevertheless, Crop Yield Insurance contracts are facing limited trading interest since their market entry. This relative failure is usually explained by the large amount of yield basis risk they contain and therefore they are considered unattractive by most U.S. farmers. The development of participating policies should contribute to increasing the correlation between the index and the aggregate risk of the cooperative, i.e., to reducing the yield basis risk, and thus to increasing the efficiency of these instruments in the management of crop-yield risk.

Conclusion

This article has presented a normative model to investigate the role of insurance and securitization in the management of catastrophic risk. The variable participating insurance contract, defined as a linear combination of fully participating policy and non-participating policy, has turned out to be an optimal hedging tool. Individuals insure the idiosyncratic component of their loss exposure through a participating contract and then the systemic component is hedged through securitized products offered by financial markets. In the case of additive risk component, the optimal variable participating policy can be replicated with a fully participating insurance policy offered at a fair price and a separate hedging contract. For the case of multiplicative component, the key role of a mutual insurer has been stressed. This role could be played by cooperatives in agriculture. They could offer a variable participating policy to insured farmers, and pass off the systemic risk, which is not assumed by the policyholders, in the capital markets. Innovative hedging instruments, like area yield insurance futures and options contracts, provide such an opportunity.

An important assumption in the current model is that fully participating policy is sold at an actuarially fair price. Nevertheless, the existence of informational asymmetries, like moral hazard or adverse selection, or the presence of transaction costs preclude the insurer or the cooperative from offering fully participating policy at a fair price. Further research should investigate the design of an optimal variable participating policy when both fully participating and non-participating contracts face administrative costs.

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