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Falsifying economic models

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Abstract

This paper presents a methodology for employing qualitative and econometric methods for refuting, or falsifying, the hypothesis presented by an economic model expressed in its structural form. The emphasis of the analysis is upon less than fully quantitative specifications of the structural form, e.g., (as here) where only the sign patterns of the arrays expressing the structural model are known. The method presented enables a structural model to be refuted regardless of its state of identification. It is found that different structural models can be equivalent in terms of the criteria they support that enable falsification. Procedures are presented that allow models to be refuted that are only expressed in terms of their algebraic form. Examples are provided for an early, over-identified macroeconomic model and an under-identified microeconomic model.

JEL classification: C12; C14; C52

1. Introduction

The degree to which economics is an "empirical" science can be deceptively complicated to pin down. The issue, in summary, concerns the degree to which economics provides hypotheses about its subject matter that can be "refuted," or following Popper (1934), *falsified.* This issue was addressed in Samuelson's (1947) *Foundations* and has

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been reiterated since, e.g., Silberberg (1990). The methodological viewpoint taken is that of comparative statics. A "model" is proposed of the generic form:

$$f^i(\boldsymbol{Y}, \boldsymbol{Z}) = 0, \qquad i = 1, 2, \ldots, n \tag{1}$$

where $\boldsymbol{Y}$ is an n-vector of endogenous variables and $\boldsymbol{Z}$ an m-vector of exogenous variables (Samuelson calls these "parameters"). The properties of the model are studied in terms of the linear system of differentials with the partial derivatives involved evaluated at a reference solution:

$$\sum_{j=1}^{n} \frac{\partial f^i}{\partial y_j} y_j + \sum_{k=1}^{m} \frac{\partial f^i}{\partial z_k} z_k = 0, \qquad i = 1, 2, \ldots, n. \tag{2}$$

In econometrics, the system (Eq. (1)) is often assumed to be linear (or the first order terms sufficiently preemptive) and Eqs. (1) and (2) can both be represented by the linear system (exclusive of an error term),

$$\boldsymbol{\beta} \boldsymbol{Y} = \boldsymbol{\gamma} \boldsymbol{Z}, \tag{3}$$

where $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are appropriately dimensioned matrices. In econometrics Eq. (3) is usually called the *structural form*.

The issue of "falsification" concerns the assembly of relevant data and the assessment of the presence, or absence, of the relationships (Eq. (3)). Although simply said, the problem of falsification (or not) becomes immediately problematical. An overwhelmingly critical issue is to state, specifically, just what it is about the system (Eq. (3)) that constitutes the hypothesis to be tested.[1] The answer is virtually never the specific magnitudes of the entries of the arrays $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$. Instead, it is usually something less. Examples include the sign patterns of the matrices $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ (calling for what is termed a "qualitative" analysis), or perhaps the relative size of some of the entries, or the claim that entries lie within specific ranges of values, or some of each.[2] The issues at stake are further complicated by the fact that, in general, the relationships (Eq. (3)) cannot be directly estimated by ordinary least squares (OLS) without suffering from a variety of statistical problems.[3] Alternative estimation strategies, such as two and three stage least squares, generalized method of moments, and limited and full information maximum likelihood, may be implemented in order to sidestep the shortcomings of OLS, but place their own restrictions on the structure embodied in the set of relations (Eq. (3)).[4]

[1] *Journal of Econometrics*, Vol. 67, No. 1 (May, 1995) was devoted to issues of testing and falsification in applied economics. An epistemology of how the concepts presented here relate to testing issues drawn in somewhat different venues is beyond the scope of this paper. For example a specification test for, say, the structure of the error covariance matrix of a model is not the same as falsifying a theory (see Spanos, 1995). Happily, a test of specification may occasionally be coincident with falsification. Unhappily, the same data in the hands of two researchers may lead to different specifications and no falsification (see Granger et al., 1995). As Cartwright (1995) argues, as do we, a variety of incompatible hypotheses can imply the same outcome. Kim et al. (1995) assert that there is no inference from a test outcome to validity of a theory. Our qualitative approach questions that assertion.

[2] Exclusion restrictions could constitute part of the hypothesis to be tested.

[3] Although the large sample properties of the simultaneous equation estimators are understood, the small sample properties and the circumstances under which one prefers one estimation strategy over another are not.

[4] First, these estimators do not have known small sample properties except in special cases. Second, they cannot be used with under-identified models, and hence cannot be used to falsify such models.

Instead, the essential inference of a specification of Eq. (3) can be represented by the *reduced form*,

$$Y = \pi Z, \qquad \text{for } \pi = \beta^{-1}\gamma.$$

The unknowns of the reduced form can always be estimated by OLS, that estimator being best linear unbiased. Taking the most felicitous case for $\gamma = I$, the $n \times n$ identity matrix, the analysis called for might amount to seeking to derive properties of β^{-1}, i.e., the signs of some or all of its entries, based upon the sign pattern of β. Once found, the hypothesis at issue could be potentially falsified by estimating π and finding that some, or all, of the called for signs of entries are not present.

Samuelson (1947) did not find the potential for such analyses to be promising, based simply (as he put it) on the improbability that sign pattern information could be extrapolated through the process of inverting a matrix to reach conclusions about the signs of entries of the inverse matrix. Given this, he proposed three sources for falsifiable hypotheses:

(a) The assumption that Eq. (1) corresponds to an optimization problem with the corresponding requirements for the matricial forms involved.
(b) Invoking particular functional forms on the relationships in Eq. (1).
(c) Assuming that the system (Eq. (1)) is "stable", i.e., the Correspondence Principle.

The truth is that these sources are very austere in terms of providing the basis for falsification. Not surprisingly, when a particular model is proposed and estimated, these considerations are not usually investigated as an explicit component of the analysis. More likely, π is estimated, and the structural system is derived therefrom (for the exactly identified case at least, see below).[5]

Part of the problem is that the burden of derivation is in some respects more severe than initially proposed. Specifically, the array γ need not be, indeed generally will not be, the $n \times n$ identity matrix. Given this, the issue of *identification* becomes part of the analysis. For the exactly identified case, the estimate of π can be used to recover the structural form uniquely. For the under-identified case, some portions of the structural relationships cannot be recovered. For the over-identified case, there is more than one route of derivation to recover the structural form, and these alternatives may not be in agreement.[6] The disarray is sufficient to somewhat discourage econometrics in general from being considered as hypothesis testing, per se. Instead, the emphasis falls at least equally on the problem of forecasting. For this, as has been often noted, the fit and statistical significance of the estimations of the reduced form are a sufficient result.[7] The burden of testing the structural hypothesis in terms of qualitative or related characteristics is simply not attempted.

[5] There would of course be issues as to errors in the data and/or the statistical significance of the findings, but this would be the general idea.

[6] All instrumental variable estimators make some use of the reduced forms, but for the over identified case there is no reason for the resulting estimates to be the same. All of the estimators are weighted averages of the observations on Y, but the weights are chosen differently for each. For example, see the multiplicity of coefficient estimates for Klein's Model I in either Greene (2000) or Berndt (1991).

[7] Indeed, Liu (1960), Leamer (1981), and Hendry (1980) have all argued this point in one form or another.

The purpose of this paper is to reconsider the issue of evaluating structural hypotheses through estimation. The strategy of the analysis is to concentrate upon the requirements for the outcome of estimating π, as based upon a qualitative specification of the arrays $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$. Part of the motivation for the approach is that the feasibility for conducting qualitative analyses is somewhat more likely than originally supposed.[8] Further, we note that the conditions for using the techniques are even more favorable in that they can be applied to the adjoint of $\boldsymbol{\beta}$. In this frame of reference, although the signs of entries of π might not be found, it is still possible to specify entries in π that must have the same, or opposite, signs, and this constitutes a falsifiable hypothesis.[9]

This approach provides a number of important results. Signs in the reduced form can be found and falsified for under-, as well as exactly and over-identified structural systems. Since all of the signs in the reduced form may not be signable, alternative structural hypotheses may lead to the same configuration of signable entries (we call such structural models *falsifiably equivalent*). Under these circumstances, the alternatives cannot be differentiated based on data, i.e., the estimated reduced form cannot lead to the rejection of one and the acceptance of the other. Even if not providing identical configurations of signable entries in the reduced form, alternative structural models can be consistent with each other, i.e., not propose opposite signs for the same signable entry in the reduced form (we call such structural models *falsifiably compatible*). In this event, both structural models could be consistent with the same estimated reduced form; or, one or the other; or, neither. The approach even enables the refutation of the Boolean form of the structural model, i.e., the specification of which entries of $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ are zero and which are not. All possible sign patterns for $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ can be generated (we call the set of such structural models *falsifiably feasible*). It is possible that all feasible structural models are falsified by an estimated reduced form.

The results presented in the paper are intended to suggest two important conclusions. First, the falsification of economic models, based upon the qualitative specification of the structural form, may be far more possible, and in a finer detail, than reflected by current econometric practice.[10] And second, using the principles of falsifiable equivalence, compatibility and feasibility, the concept of a hypothesis in economics may be more general, and in some ways more interesting, than has been previously supposed.

The paper is organized as follows. The next section presents examples of falsifiable hypotheses based upon qualitative specifications of the structural form. The examples are chosen to highlight the falsifiability of models independent of identification and to demonstrate the characteristic of falsifiable equivalence. In the section to follow examples are provided using over- and under-identified systems. The last section provides

[8] An extensive literature review and presentation of methods for conducting qualitative and related analyses is presented in Hale et al. (1999).

[9] As we note below, the correspondence principle might be used to at least provisionally resolve the sign of the determinate, i.e., for $\boldsymbol{\beta}$ stable and in a standard form sgn det $\boldsymbol{\beta}=(-1)^n$.

[10] Although beyond the scope of what is presented here, other, less than fully quantitative, information beyond sign patterns can also be used to establish falsifiable characteristics of the reduced form.

conclusions. An appendix follows with a brief discussion of techniques for conducting a qualitative analysis.

2. Falsification

Consider the structural model,

$$\boldsymbol{\beta} \boldsymbol{Y} = \boldsymbol{\gamma} \boldsymbol{Z},$$

where the arrays and vectors are as specified earlier. The issue to be addressed in this section is the manner of falsifying Eq. (3). The general idea of falsification is to determine characteristics of data implied by Eq. (3) and then see if the characteristics are there. It is encouraging if data are found for which the characteristics are present; however, this finding, per se, does not qualify Eq. (3) as testable. Instead, it must be shown that it is feasible for data to be found that would not embody the called for characteristics. Specifically, it must be shown that Eq. (3) is falsifiable (Popper, 1934). Only then would data consistent with Eq. (3) be viewed as a vindication of the hypothesis, all within the limits of induction.

Falsifiability in this sense is a strict standard. It would not be unusual for econometric method to propose as a "test" the determination of whether or not an entry in the reduced form is "significantly" different from zero. If the relationship at issue can be derived from the hypothesized structural model, then it could be proposed that finding the entry to not be significantly different from zero "falsifies" the structural hypothesis. Our point is that this is not necessarily so. Specifically, unless certain entries in the adjoint of the hypothesized $\boldsymbol{\beta}$ are signable, then the magnitudes in the structural model could be such that the entry of π at issue actually would be zero. Since it can be that only some entries in π are falsifiable in this sense, our point additionally is that a variety of structural specifications could lead to the same entries of π being falsifiable or at least be consistent with the outcome of a particular estimated reduced form. In this case, one specification could not, or at least might not, be selected over the other due to the outcome of the estimated reduced form.

For economics at least, the issue as to what, just exactly, it is about the system (Eq. (3)) that constitutes the hypothesis to be tested can also be problematic. As already said, the answer is almost never the particular, full bodied values of the entries of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$. In fact, it is typically a goal of "fitting" the model to data to find out just what the values of these entries might be estimated to be. Instead, the hypothesis represented by Eq. (3) is usually something less.[11] The menu of possibilities is suggested by the categories of information associated with measurement scales (that economists often confront when considering alternative measures of "utility"). Candidate categories would include:

Boolean: A specification of which entries of $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ are zero and which nonzero.
Qualitative: Boolean, plus, the sign (+ or $-$) of the nonzero entries.

[11] Exceptions include definitional or accounting relationships, e.g., GDP$=C+I+G+X-M$, where the coefficient values in the equation are all "1" or "-1."

Ordinal: Qualitative, plus, the rank order of the entries.
Interval: Ordinal, plus, the ranges of values each entry lies within.[12]

Although it is by no means unimportant to consider how the information from each category, or from several categories at the same time, might be processed, we will limit the examples here to that of qualitative information. Indeed, sign pattern information may be insufficient to determine any signable entries in the reduced form requiring the use of other information. Nevertheless, all of our points can be sustained under this assumption. Accordingly, a hypothesis (Eq. (3)) is understood here to comprise the sign pattern {+, −, 0} of the entries of $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$.

For our purposes, the system is manipulated into its "reduced form,"

$$\boldsymbol{Y} = \pi\boldsymbol{Z}, \qquad \text{where } \pi = \boldsymbol{\beta}^{-1}\boldsymbol{\gamma}.$$

Strictly, the issue then becomes: What does a specification of the sign pattern of $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ require about the outcome of the estimation of the entries of π? The burden of the analysis falls upon the determination of which entries in $\boldsymbol{\beta}^{-1}$ can be signed, based only upon the signs of the entries of $\boldsymbol{\beta}$, i.e., entries that will have the same sign for the inverse of any matrix with $\boldsymbol{\beta}$'s sign pattern. Once this is determined, it is easy to inspect the operation $\pi = \boldsymbol{\beta}^{-1}\boldsymbol{\gamma}$ to determine if any entries in the reduced form can be signed.[13]

An example for which all of the entries of the reduced form can be signed is

$$sgn\,\boldsymbol{\beta} = \begin{bmatrix} - & 0 & 0 & - \\ + & - & 0 & 0 \\ 0 & + & - & 0 \\ 0 & 0 & + & - \end{bmatrix}; \text{ and, } sgn\,\boldsymbol{\gamma} = \begin{bmatrix} + & 0 & 0 & 0 \\ 0 & + & 0 & 0 \\ 0 & 0 & + & 0 \\ 0 & 0 & 0 & + \end{bmatrix}.$$

In this example and throughout the paper a normalization rule has been chosen such that the main diagonal entries of $\boldsymbol{\beta}$ are negative, but of no particular value. Any normalization rule is arbitrary. Typically, in econometric work, the normalization rule specifies that the main diagonal elements each be equal to one. The normalization rule used here is that called for in the literature when presenting conditions for finding qualitative inverses (see Appendix A).

[12] "Interval" measure is usually defined to include a unit of measure, but no distinct assignment of an origin. Such measures are unique up to linear transformations and are used in measuring utility under conditions of risk. A venerable reference is Stevens (1946). The usage here is somewhat different and is an intuitive terminology that is not pursued. For our usage, qualitative information is also "interval" in that entries are specified in the open positive or negative intervals of value.

[13] Kennedy (2003, pp. 397–402) presents ten reasons for the "wrong sign" being found when estimating a single equation model. Six of these relate to problems with the data, functional form of the estimator, or statistical significance. The remaining four relate to specification problems which, as here, falsify the particular model proposed by the single equation.

The sign pattern[14] for β has been chosen to be "qualitatively invertible," i.e., a configuration of signs such that the signs of at least some (and in this case all) of the entries of β^{-1} can be found based only upon the sign pattern of β, entirely independent of the magnitudes of the entries of β.[15] A qualitative analysis of β's sign pattern reveals that,

$$sgn\,\beta^{-1} = \begin{bmatrix} - & + & + & + \\ - & - & + & + \\ - & - & - & + \\ - & - & - & - \end{bmatrix} = sgn\left(\beta^{-1}\gamma\right) = sgn\,\pi.$$

In these terms, the structural model $\{\beta, \gamma\}$ is falsifiable, since π can be estimated and the signs of its entries inspected for consistency with the above. It should be noted that the inference of the structural form, in terms of its sign pattern, cannot be recovered from the sign pattern of π in this example, or in general. That is, the sign pattern found for β^{-1} cannot be manipulated, independent of the magnitudes of its entries, to find the sign pattern hypothesized for β.[16] For notation, we wish to distinguish outcomes of this kind, i.e., structural hypotheses in terms of the sign pattern of $\{\beta, \gamma\}$ that can be falsified in the sense given above. We will express this as,

$$\mathrm{FAL}\{sgn\,\beta, sgn\,\gamma\} = \begin{bmatrix} - & + & + & + \\ - & - & + & + \\ - & - & - & + \\ - & - & - & - \end{bmatrix} = \mathrm{FAL}\{sgn\,\pi\},$$

where in this case $\{\mathrm{sgn}\,\beta, \mathrm{sgn}\,\gamma\}$ are as given above.

For another example let,

$$sgn\,\beta = \begin{bmatrix} - & 0 & 0 & + \\ + & - & 0 & 0 \\ 0 & + & - & 0 \\ 0 & 0 & + & - \end{bmatrix};\ \text{and,}\ sgn\,\gamma = \begin{bmatrix} - & 0 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & - \end{bmatrix}.$$

For this case sgn β is not qualitatively invertible and no entry of its inverse can be signed, independent of magnitudes. Nevertheless, a qualitative analysis reveals that all

[14] Strictly, sgn $a = 1, 0, -1$ as $a > 0$, $a = 0$, $a < 0$. We will use the signs alone, simply to facilitate presentation.

[15] A presentation of method and review of the literature on detecting and working with qualitatively invertible matrices is in Hale et al. (1999). A brief discussion of the required derivations is provided here in the Appendix at the end of the paper.

[16] This is true for $n > 2$ and β irreducible, in which case π would contain no zero entries. An $n \times n$ matrix with no zero entries cannot be qualitatively invertible for $n > 2$ (Lancaster, 1966). Still, as we note below, it is possible to compile a roster of sign patterns for β that *could* result in the given sign pattern for (in this case) π: the class of *falsifiably compatible* structural models.

of the entries of $\boldsymbol{\beta}$'s adjoint can be signed, independent of magnitudes, and is given by,

$$sgn\ adj\, \boldsymbol{\beta} = \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}.$$

The actual signs in $\boldsymbol{\beta}^{-1}$, and ultimately sgn π, will depend upon the sign of det $\boldsymbol{\beta}$, and for this case the outcome can differ, depending upon the magnitudes of $\boldsymbol{\beta}$'s entries. Nevertheless, all of the entries in sgn π will be the same, and this is a falsifiable characteristic of the outcome of estimating π.

In a situation such as this, it is useful to invoke one of Samuelson's bases for falsification: the Correspondence Principle. Assume for the sake of the example that it is also proposed that $\boldsymbol{\beta}$ is stable.[17] Given this, det $\boldsymbol{\beta}>0$ and the signs of $\boldsymbol{\beta}$'s adjoint and inverse are the same. Taken altogether, the results for the example are given by,

$$\mathrm{FAL}\{sgn\, \boldsymbol{\beta}, sgn\, \boldsymbol{\gamma}, S\} = \begin{bmatrix} + & + & + & + \\ + & + & + & + \\ + & + & + & + \\ + & + & + & + \end{bmatrix} = \mathrm{FAL}\{sgn\, \pi\},$$

where "*S*" represents the additional assumption of stability. As before, the hypothesis is falsified by estimating the entries of π and comparing the sign pattern of results to the above. For this case the hypothesis is falsified if any negative entries are estimated.[18]

The configuration of $\boldsymbol{\gamma}$ in the above examples was chosen such that the corresponding system (Eq. (3)) is identified. Consider another system with the same sign pattern for $\boldsymbol{\beta}$, but with $\boldsymbol{\gamma}$ chosen such that the corresponding system (Eq. (3)) is under-identified,

$$sgn\, \boldsymbol{\beta} = \begin{bmatrix} - & 0 & 0 & + \\ + & - & 0 & 0 \\ 0 & + & - & 0 \\ 0 & 0 & + & - \end{bmatrix};\ \text{and,}\ sgn\, \boldsymbol{\gamma}' = \begin{bmatrix} - & - & - & - \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Standard econometric practice would take a stern view of such a specification. Most likely, the system would be amended and not dealt with in this under-identified form. This would have to be done if the goal were to recover an estimate of the structural form. Yet, from the standpoint of falsification, the structural form, as given, can be falsified; and

[17] Let β be an $n \times n$ matrix with an all negative main diagonal. β is *a stable matrix* if the real parts of the characteristic roots of β are negative (Samuelson, 1941). If β is stable, then sgn det $\beta = (-1)^n$.

[18] We expect all 16 entries in π to be positive. What do we do if one of them is negative, but not statistically significantly so? In one of the case studies in the next section we consider this problem and apply the notion of *relatively falsifiably compatible* as defined later in this section.

further, presents the same characteristics for the reduced form as the previous identified system:

$$\text{FAL}\{sgn\,\boldsymbol{\beta}, sgn\,\boldsymbol{\gamma}', S\} = \begin{bmatrix} + & + & + & + \\ + & + & + & + \\ + & + & + & + \\ + & + & + & + \end{bmatrix} = \text{FAL}\{sgn\,\pi\}.$$

It is our contention that the two structural hypotheses, $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ and $\{\boldsymbol{\beta}, \boldsymbol{\gamma}'\}$, are not really different in terms of the essential pattern of inference that they propose. Since the conditions for falsification for each are identical to the other, the two hypotheses do not propose "scientifically" different models of the subject matter. Instead, each hypothesis is a member of a class of hypotheses all proposing the same pattern of inference among the variables of the model, as expressed by refutable characteristics of the reduced form. In fact, the reader can readily confirm that any system with the same $\boldsymbol{\beta}$ and any $\boldsymbol{\gamma}^*$ with all nonpositive entries and a nonzero entry in each column is also a member of the same class, i.e.,

$$\text{FAL}\{sgn\,\boldsymbol{\beta}, sgn\,\boldsymbol{\gamma}^*, S\} = \begin{bmatrix} + & + & + & + \\ + & + & + & + \\ + & + & + & + \\ + & + & + & + \end{bmatrix} = \text{FAL}\{sgn\,\pi\}.$$

We term hypotheses that lead to the same falsifiable characteristics of the reduced form *falsifiably equivalent* hypotheses. That is,

Falsifiable equivalence. The structural hypotheses $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ and $\{\boldsymbol{\beta}', \boldsymbol{\gamma}'\}$ are *falsifiably equivalent* if and only if $\text{FAL}\{\text{sgn}\ \boldsymbol{\beta}, \text{sgn}\ \boldsymbol{\gamma}\} = \text{FAL}\{\text{sgn}\ \boldsymbol{\beta}', \text{sgn}\ \boldsymbol{\gamma}'\}$.

The different systems above were based upon the same structural hypothesis for $\boldsymbol{\beta}$ with different hypotheses for $\boldsymbol{\gamma}$ leading to a class of falsifiably equivalent structural models. Falsifiable equivalence can also be obtained for alternative hypotheses about $\boldsymbol{\beta}$. Consider initially,

$$sgn\,\boldsymbol{\beta} = \begin{bmatrix} - & 0 & 0 & + \\ + & - & 0 & 0 \\ 0 & + & - & 0 \\ + & 0 & + & - \end{bmatrix}; \text{ and, } sgn\,\boldsymbol{\gamma} = \begin{bmatrix} - & 0 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & - \end{bmatrix}.$$

The sign pattern proposed for $\boldsymbol{\beta}$ is not qualitatively invertible; and further, although some entries of its adjoint can be signed, all cannot be. A qualitative analysis reveals that,

$$\text{FAL}\{sgn\,\boldsymbol{\beta}, sgn\,\boldsymbol{\gamma}, S\} = \begin{bmatrix} + & + & + & + \\ + & ? & + & + \\ + & ? & ? & + \\ + & + & + & + \end{bmatrix} = \text{FAL}\{sgn\,\pi\}.$$

The "?"'s in the above identify entries of the adjoint of $\boldsymbol{\beta}$ that cannot be signed, independent of magnitudes. Given this, consider the alternative hypothesis with the same

$\boldsymbol{\gamma}$ but an additional nonzero element proposed for $\boldsymbol{\beta}$, i.e., change β_{31} from zero to positive.

$$sgn\,\boldsymbol{\beta}' = \begin{bmatrix} - & 0 & 0 & + \\ + & - & 0 & 0 \\ + & + & - & 0 \\ + & 0 & + & - \end{bmatrix}; \text{ and, } sgn\,\boldsymbol{\gamma} = \begin{bmatrix} - & 0 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & - \end{bmatrix}.$$

Nevertheless,

$$\mathrm{FAL}\{sgn\,\boldsymbol{\beta}', sgn\,\boldsymbol{\gamma}, S\} = \begin{bmatrix} + & + & + & + \\ + & ? & + & + \\ + & ? & ? & + \\ + & + & + & + \end{bmatrix} = \mathrm{FAL}\{sgn\,\pi'\},$$

and the two systems are shown to be falsifiably equivalent. Accordingly, it is one of our particular points that the two structural systems, identical except for the hypothesis concerning the sign of the entry β_{31} are in fact the *same* from the perspective of being confronted with the outcome of estimating the reduced form. If either system is consistent with the data, then both are; and, if one is falsified, then both are. The hypotheses about the sign of β_{31} (0 or +) cannot be differentiated by the outcome of the estimation of the reduced form.

Consider yet another variation on basically the same structural model, this time setting β_{31} negative.

$$sgn\,\boldsymbol{\beta}'' \begin{bmatrix} - & 0 & 0 & + \\ + & - & 0 & 0 \\ - & + & - & 0 \\ + & 0 & + & - \end{bmatrix}; \text{ and, } sgn\,\boldsymbol{\gamma} = \begin{bmatrix} - & 0 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & - \end{bmatrix}.$$

A qualitative analysis reveals that,

$$\mathrm{FAL}\{sgn\,\boldsymbol{\beta}'', sgn\,\boldsymbol{\gamma}, S\} = \begin{bmatrix} + & + & + & + \\ + & ? & + & + \\ ? & ? & ? & ? \\ ? & + & + & + \end{bmatrix} = \mathrm{FAL}\{sgn\,\pi''\}.$$

This outcome is not the same as for the two systems above. As a result, if the estimated reduced form presented negative signs for any of the entries: (3,1), (3,4), or (4,1), the structural models that embody $\boldsymbol{\beta}$ and $\boldsymbol{\beta}'$ would both be falsified, but that with $\boldsymbol{\beta}''$ would not be. Nevertheless, if the estimated reduced form had (say) all positive entries, then all three models would be consistent with the data, i.e., not be falsified by the sign pattern of the estimated reduced form. In general, there are a number of outcomes for the estimated reduced form that would not falsify any of these models. We have termed structural models with implied reduced forms that can be consistent with the same, estimated reduced form *falsifiably compatible*.

Falsifiable compatibility. Let FAL(sgn π) and FAL(sgn π^*) be the signable entries in π for two alternative structural models. The two models are *falsifiably compatible* if and only if for each entry, (i,j), signable for *both* of them, sgn π_{ij} = sgn π^*_{ij}.

Falsifiably equivalent models are also falsifiably compatible; however, the implied reduced forms of a class of falsifiably compatible models can be very different. The key feature is that signable entries in the reduced forms that are common among more than one model have the *same* sign. Otherwise, a signable entry for one model may not be signable for the others and can have any sign. Given this, there exist outcomes for the estimated reduced form that would falsify no models in the class, or some but not others, or all models in the class. In contrast, an estimated reduced form would falsify all, or none, but not just some, of a class of falsifiably equivalent models.

Consider further the Boolean specification of $\{\beta, \gamma\}$ with (possibly) additional constraints added as to the signs of entries. For example some could be allowed to be any sign, including zero. The main diagonal of β might be (usually is) required to be all negative. The signs of some nonzero entries might be set for theoretical or other reasons, e.g., entries in accounting or definitional equations. Nevertheless, many nonzeros might be allowed to be either sign, a priori. Given this, all possible structural models, i.e., sign patterns for $\{\beta, \gamma\}$, conforming to the Boolean specification (with constraints) can be defined with a simple algorithmic principle, and actually generated up to whatever might be "too many" as related to the computing hardware available. We have termed the set of structural models defined in this way to be the set of *falsifiably feasible* structural models.

Given this, and given the estimated reduced form, the class of corresponding falsifiably compatible models can be exhaustively specified by finding those structural models in the feasible set that are not falsified by the sign pattern of the estimated reduced form. The outcome of such an analysis can have decisive results. For example, it could be found for a given estimated reduced form that the class of falsifiably compatible structural models is empty, i.e., all feasible structural models are falsified by the estimated reduced form. This could be considered to be a rather fundamental basis for the rejection of a structural hypothesis. Alternatively, it could be found that for the class of falsifiably compatible structural models that certain entries of $\{\beta, \gamma\}$ have the same sign. At the least, this would be a firm rejection of the hypothesis that the entry could have the opposite sign, all within the framework of the proposed Boolean specification (as constrained).

Finally, the discussion so far has accepted the outcome of the estimated reduced form's sign pattern uncritically. Of course actual outcomes can present very different levels of significance associated with the estimate of each entry. For sufficiently "insignificant" outcomes for a particular entry, the analysis could be conducted with the entry considered to be unsigned. All of the principles outlined above would be pursued accordingly, with the entry(ies) at issue simply ignored. We term such circumstances "relative" (to a given estimated reduced form). For example, two structural models are *relatively falsifiably equivalent* if the entries of their implied reduced forms are the same, ignoring the entries in the estimated reduced form that are considered unsigned. "Relative" falsifiable compatibility and feasibility would be defined in the same way.

In the next section, a number of proposed structural models are considered and the various concepts developed in this section are applied to them.

3. Case studies

3.1. Klein Model I

For an example of an over-identified model, consider Klein's Model I as assessed for pedagogical purposes in Berndt (1991).[19]

$$\boldsymbol{\beta Y} = \boldsymbol{\gamma Z}$$

$$\begin{bmatrix} -1 & 0 & a_1 & 0 & a_2 & 0 & 0 \\ 0 & -1 & 0 & 0 & b_1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & c_1 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} C \\ I \\ W_1 \\ Y \\ P \\ W \\ E \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 & -a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -b_2 & -b_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_2 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} W_2 \\ P_{-1} \\ K_{-1} \\ E_{-1} \\ Year \\ TX \\ G \end{bmatrix}.$$

In Klein's model the endogenous variables are: private consumption (C), investment (I), the private wage bill (W_1), income (Y), profits or nonwage income (P), the sum of private and government wages (W), and private product (E); and, the exogenous variables are: the government wage bill (W_2), lagged profits (P_{-1}), end of last period capital stock (K_{-1}), lagged private product (E_{-1}), years since 1931 (Year), taxes (TX), and government consumption (G).

[19] Analysis of the model's qualitative properties was presented in Lady (2000). Earlier, Maybee and Weiner (1988) also used the model to illustrate the use of additional information, as well as the signs of the model's coefficients, in studying the invertibility of $\boldsymbol{\beta}$ (as notated here). The analysis here only uses sign pattern information. As already said, and for that matter as pointed out in both of the references cited above, other information can also be used. Once the limits of using such additional information have been reached, the interpretation of results with respect to falsification remains the same as presented here.

The sign patterns proposed by Klein for β and $\boldsymbol{\gamma}$ are as follows,[20]

$$sgn\,\beta\begin{bmatrix} - & 0 & + & 0 & + & 0 & 0 \\ 0 & - & 0 & 0 & + & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & + \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix} \text{ and } sgn\,\boldsymbol{\gamma} = \begin{bmatrix} - & - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & + & - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & 0 & 0 & 0 & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & - & 0 \end{bmatrix}.$$

The proposed β-matrix is not qualitatively invertible; however, many of the entries in the adjoint can be signed. The corresponding signable entries in its inverse will be determined by additionally assuming stability, i.e., that the determinant is negative. Given this, the β-inverse (assuming stability) and signable entries of the corresponding reduced form are as follows:

$$sgn\,\beta^{-1} = \begin{bmatrix} ? & ? & ? & ? & - & + & ? \\ ? & ? & ? & ? & ? & ? & ? \\ - & - & ? & - & - & + & ? \\ - & - & ? & - & - & + & ? \\ ? & ? & ? & ? & ? & ? & ? \\ - & - & ? & - & - & ? & ? \\ - & - & ? & - & - & + & ? \end{bmatrix}$$

with

$$\mathrm{FAL}(\beta, \boldsymbol{\gamma}, S) = \begin{bmatrix} ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & - & ? & ? & ? & + \\ ? & + & - & ? & ? & ? & + \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & - & ? & ? & ? & + \\ ? & + & - & ? & ? & ? & + \end{bmatrix} = \mathrm{FAL}(\pi).$$

The question then is whether the proposed sign pattern for {β, $\boldsymbol{\gamma}$} agrees with those found from the data; and, how the signs in the estimated reduced form agree with those implied by any of these.

Based on the original Klein data for 1920–1941, Berndt considers seven alternative estimation strategies for finding the coefficient values of the model's structural form. Three different versions, in terms of sign pattern, of the structural β-matrix were found, depending upon the estimation strategy used. In standard econometric practice the researcher would attribute the differences to such things as model misspecification

[20] The discussion below deals with alternative models, all expressed in the generic notation $\beta Y = \boldsymbol{\gamma} Z$. Alternative versions of the models are designated by distinguishing the symbols for β and $\boldsymbol{\gamma}$, e.g., β, β1, …. As the modeling venue of the discussion is changed, these same designations will be reused, rather than a more elaborate notation.

attributable to too many exclusion restrictions, errors in variables, variables incorrectly categorized as endogenous, or maybe even something as mundane as rounding errors. At the two extremes the researcher may reject the model or leave it to the reader to decide whether the model is tenable. But one need not be so pessimistic about the validity of the model, as the multiplicity of results may be falsifiably equivalent.

The three observed sign patterns found for β were as follows: for OLS, 2SLS, OLS-AR1, and 2SLS-AR1 (call it Klein Model Hypothesis #1 and also the same as initially proposed by Klein as given above),

$$sgn\,\beta 1 = \begin{bmatrix} - & 0 & + & 0 & + & 0 & 0 \\ 0 & - & 0 & 0 & + & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & + \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix};$$

for 3SLS and I3SLS (call it Klein Model Hypothesis 2 for which only the (2,5)th entry is changed to negative compared to $\beta 1$),

$$sgn\,\beta 2 = \begin{bmatrix} - & 0 & + & 0 & + & 0 & 0 \\ 0 & - & 0 & 0 & - & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & + \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix};$$

and, for FIML(call it Klein Model Hypothesis 3, for which, in addition to the (2,5)th entry changed to negative, the (1,5)th entry also is changed to negative compared to $\beta 1$),

$$sgn\,\beta 3 \begin{bmatrix} - & 0 & + & 0 & - & 0 & 0 \\ 0 & - & 0 & 0 & - & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & + \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix}.$$

For all of the estimation methods $\boldsymbol{\gamma}$ had the same sign pattern (also as initially proposed by Klein),

$$\boldsymbol{\gamma} = \begin{bmatrix} - & - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & + & - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & 0 & 0 & 0 & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & - & 0 \end{bmatrix}.$$

None of the estimated β-matrices is qualitatively invertible; however, many of the signs in the adjoint of each of them could be signed, independent of magnitudes. As above, for illustrative purposes, the corresponding signable entries in their inverses will be determined by additionally assuming stability, i.e., that the determinant is negative. Given this, the β-inverses and signable entries of the corresponding reduced forms are as follows:

$$sgn\,\beta 1^{-1} = \begin{bmatrix} ? & ? & ? & ? & - & + & ? \\ ? & ? & ? & ? & ? & ? & ? \\ - & - & ? & - & - & + & ? \\ - & - & ? & - & - & + & ? \\ ? & ? & ? & ? & ? & ? & ? \\ - & - & ? & - & - & ? & ? \\ - & - & ? & - & - & + & ? \end{bmatrix};$$

with,

$$\mathrm{FAL}(\beta 1, \boldsymbol{\gamma}, S) = \begin{bmatrix} ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & - & ? & ? & ? & + \\ ? & + & - & ? & ? & ? & + \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & - & ? & ? & ? & + \\ ? & + & - & ? & ? & ? & + \end{bmatrix} = \mathrm{FAL}(\pi 1).$$

$$sgn\,\beta 2^{-1} = \begin{bmatrix} ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ - & - & ? & - & ? & ? & ? \\ - & - & ? & - & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ - & - & ? & - & ? & ? & ? \\ - & - & ? & - & ? & ? & ? \end{bmatrix};$$

with,

$$\mathrm{FAL}(\beta 2, \boldsymbol{\gamma}, S) = \begin{bmatrix} ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & - & ? & ? & ? & + \\ ? & + & - & ? & ? & ? & + \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & - & ? & ? & ? & + \\ ? & + & - & ? & ? & ? & + \end{bmatrix} = \mathrm{FAL}(\pi 2)$$

$$sgn\,\beta 3^{-1} = \begin{bmatrix} ? & ? & - & ? & + & - & - \\ ? & ? & ? & ? & ? & ? & ? \\ - & - & - & - & + & - & - \\ - & - & - & - & + & - & - \\ ? & ? & ? & ? & ? & ? & ? \\ - & - & - & - & + & ? & - \\ - & - & - & - & + & - & - \end{bmatrix};$$

with

$$\mathrm{FAL}(\beta 3, \boldsymbol{\gamma}, S) = \begin{bmatrix} ? & ? & ? & + & + & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & - & + & + & ? & + \\ ? & + & - & + & + & ? & + \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & - & + & + & ? & + \\ ? & + & - & + & + & ? & + \end{bmatrix} = \mathrm{FAL}(\pi 3).$$

In the above, $\pi 1$, $\pi 2$, and $\pi 3$ are implied sign patterns for the reduced form corresponding to the structural models that include, respectively, β1, β2, and β3.

Inspection of the results shows that hypotheses #1 and #2 are falsifiably equivalent; i.e., the sign patterns in $\pi 1$ and $\pi 2$ are identical. For hypothesis #3, all of the falsifiable entries of the reduced form for #s 1 and 2 remain falsifiable for #3; plus, ten additional entries are falsifiable. Given this, if the data, i.e., the estimated reduced form, are consistent with hypothesis #3, they are also consistent with hypotheses #1 and #2; and as a result, the three structural models are falsifiably compatible. Thus, if Klein hypothesis #3 is not falsified by the estimated reduced form, the three hypotheses presented in terms of sign pattern information only, cannot be differentiated in terms of the data. Alternatively, it is at least possible for data to be assembled that falsifies hypothesis #3, but remains consistent with hypothesis #1 or #2. Hypothesis #3 is falsified, if #1 and #2 are. In any case, the point here is that for any of the three sign patterns taken as the initial, structural hypothesis, the finding of the alternative sign patterns of the structural model through different routes of estimation need not be viewed as "inconsistent" with each other. Instead, the data assembled for the reduced form may leave all three estimated structural models as feasible alternatives. At the level of sign patterns at least, the essential, falsifiable, implication for the inference among the variables involved can be consistent with the data, i.e., the estimated reduced form, for all three hypotheses.

In the case of the estimations cited for this example, Berndt reported two reduced forms, one using OLS and the other using ML incorporating the over-identifying restrictions.

$$\textit{OLS estimated sgn } \hat{\pi} = \begin{bmatrix} + & + & - & +^* & +^* & - & + \\ - & + & - & - & + & - & + \\ - & +^* & -^* & +^* & +^* & - & +^* \\ - & +^* & -^* & +^* & +^* & - & +^* \\ - & + & - & + & + & + & - \\ + & +^* & -^* & +^* & +^* & - & +^* \\ - & +^* & -^* & +^* & +^* & + & +^* \end{bmatrix}.$$

An "*" indicates that the entry is falsifiable for the structural FIML estimation technique and is consistent with those derived for FAL($\pi 3$), and therefore also with FAL($\pi 1$) and

FAL(π2). When the reduced form is estimated incorporating the over-identifying restrictions the sign pattern is:

$$ML\ restricted\ estimated\ sgn\ \hat{\pi} = \begin{bmatrix} + & + & - & +^{*} & +^{*} & - & + \\ - & + & - & - & -^{**} & - & + \\ - & +^{*} & -^{*} & +^{*} & +^{*} & - & +^{*} \\ +^{**} & +^{*} & -^{*} & +^{*} & +^{*} & - & +^{*} \\ - & + & - & + & -^{**} & + & - \\ + & +^{*} & -^{*} & +^{*} & +^{*} & - & +^{*} \\ - & +^{*} & -^{*} & +^{*} & +^{*} & + & +^{*} \end{bmatrix}.$$

Although the two reduced forms differ for a small number of entries (the three indicated with "**"), none of these are among those that are falsifiable as determined above. Accordingly, at the level of the estimated reduced form, the three structural models found, if treated as individual hypotheses, are not distinguishable from one another based on the observed data. To this extent, they have each been found to be compatible in the refutable or falsifiable characteristics of their reduced forms with the results estimated from the data. This is in contrast to the more customary interpretation of the alternative structural outcomes, in which the researcher might be inclined to conclude that one or another of the estimation outcomes embodies some statistical problem or the model (Eq. (3)) was incorrect in the sense that the over identifying restrictions may be incorrect.[21] Apparently, rejecting the over identifying restrictions is not sufficient to falsify Klein's original model.

Klein's Model I, and its results, as discussed above was first fit to 1921–1941 data and appeared in print in 1950. Since that time it has been the basis for extensive modeling efforts and has been a valuable pedagogical tool. The years of subsequent research and pedagogy is nicely summed up by Berndt's (1991) remarks regarding a specification test of the model.

> . . . a result suggesting that the overidentifying restrictions are not consistent with the data. This result is not surprising, since Klein's Model I is obviously a highly aggregated and simplified model, useful for pedagogical purposes but not necessarily an accurate model of the U.S. economy. (pp. 553–554, *The Practice of Econometrics: Classic and Contemporary*, Ernst R. Berndt, Addison-Wesley, 1991, New York)

For the original Klein Model I sample we demonstrated that the empirical results for the structural form estimation strategies produced two sign patterns for β that were falsifiably equivalent. The third sign pattern, while not falsifiably equivalent to the first two,

[21] Berndt (1991) reports that the likelihood ratio test statistic for the 12 over-identifying restrictions is 39.398, "...a result suggesting that the over-identifying restrictions are not consistent with the data." Greene (2000) reports that the over-identifying restrictions are rejected for the wage equation and that the results are mixed for the consumption equation.

produced a sign pattern in its implied reduced form that did embody the first two and was therefore falsifiably compatible. None of the implied reduced form sign patterns were falsified by the fitted reduced forms.

In the years since the development of Klein's Model I the world has undergone remarkable changes. It is one thing to find that Klein's model is not falsified for a short time span and quite another to find that it is not falsified when the sample spans most of the 20th century. Using data for the period 1921–2000,[22] we re-estimated Klein's structural model using a variety of methods with the results presented in Table 1.

The signs of some of the coefficients depend on the estimation strategy. Current profits in the consumption equation; current profits, lagged profits and lagged capital in the investment equation; and current private product in the wage equation all have signs that depend on the estimation strategy.

Our structural estimates produce four distinct sign patterns for the matrix β of endogenous coefficients: for OLS and OLS corrected for autocorrelation the structural form sign pattern is

$$sgn\,\beta 1 = \begin{bmatrix} - & 0 & + & 0 & + & 0 & 0 \\ 0 & - & 0 & 0 & + & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & + \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix}.$$

For 2SLS and 2SLS corrected for autocorrelation the structural form sign pattern is

$$sgn\,\beta 2 = \begin{bmatrix} - & 0 & + & 0 & + & 0 & 0 \\ 0 & - & 0 & 0 & - & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & - \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix}.$$

For 3SLS

$$sgn\,\beta 3 = \begin{bmatrix} - & 0 & + & 0 & - & 0 & 0 \\ 0 & - & 0 & 0 & - & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & + \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix}.$$

[22] The data are available in Carnero et al. (2002). They assembled the data from the National Income and Product Accounts of the BLS. The data are in real values, with 1996 as the base year. They are consistent with the original 1921–1941 data.

Table 1
Structural estimates of Klein's Model 1: 1921–2000 (*t*-statistics in parentheses)

Structural Unknown	OLS	2SLS	3SLS	I3SLS	OLSAR( )	2SLS AR(1)
$\mathbf{a}_0$	−61.255 (−1.763)	−83.623 (−1.59)	−89.397 (−1.99)	−184.523 (−2.46)	2.374 (.29)	−38.014 (−0.78)
$\mathbf{a}_1$	0.797 (8.511)	0.700 (4.67)	4.643 (2.99)	8.784 (3.23)	1.040 (11.60)	0.884 (6.98)
$\mathbf{a}_2$	0.820 (2.437)	3.595 (2.00)	−3.514 (−2.37)	−6.671 (−2.41)	0.176 (1.16)	3.011 (2.48)
$\mathbf{a}_3$	0.092 (.251)	−20.504 (−1.49)	0.673 (5.83)	0.271 (2.94)	0.051 (0.40)	−2.408 (−2.21)
$\mathbf{b}_0$	−67.518 (−2.511)	−22.465 (−0.41)	8.203 (0.20)	565.616 (3.14)	−45.032 (−3.50)	−26.294 (−0.62)
$\mathbf{b}_1$	0.617 (3.314)	−10.94 (−1.42)	−30.692 (−3.76)	−17.652 (−3.04)	0.868 (8.11)	−.958 (−1.19)
$\mathbf{b}_2$	−0.295 (−1.509)	2.201 (1.65)	3.911 (4.04)	16.549 (2.78)	−0.229 (−1.90)	1.204 (1.56)
$\mathbf{b}_3$	−0.021 (−1.785)	0.003 (0.11)	0.020 (1.25)	0.317 (7.36)	−0.069 (−3.30)	0.0006 (0.035)
$\mathbf{c}_0$	−102.794 (−5.941)	−133.503 (−3.48)	−116.839 (−5.13)	−110.321 (−4.60)	−30.252 (−3.17)	−131.157 (−3.82)
$\mathbf{c}_1$	0.434 (10.082)	−0.252 (−0.93)	0.048 (−0.31)	−0.086 (−0.52)	0.499 (22.50)	−0.077 (−0.39)
$\mathbf{c}_2$	0.207 (4.602)	0.928 (3.32)	0.711 (4.45)	0.748 (4.39)	0.145 (6.21)	0.747 (3.66)
$\mathbf{c}_3$	−0.382 (0.725)	−1.287 (−1.11)	−0.859 (−1.30)	−0.631 (−0.89)	−0.647 (−0.54)	−1.194 (−1.15)
					C-AR(3) I-AR(2) W1-AR(2)	

And for iterative 3SLS

$$sgn\,\beta 4 = \begin{bmatrix} - & 0 & + & 0 & - & 0 & 0 \\ 0 & - & 0 & 0 & - & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & - \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix}.$$

Using the sample for the period 1921–2000 there are three distinct sign patterns for the $\boldsymbol{\gamma}$ matrix. For OLS and OLS corrected for autocorrelation the matrix is

$$sgn\,\boldsymbol{\gamma}1 = \begin{bmatrix} - & - & 0 & 0 & 0 & 0 & 0 \\ 0 & + & + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & + & 0 & 0 \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & 0 & 0 & 0 & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & - & 0 \end{bmatrix}.$$

For iterative 3 stage least squares and 2 stage least squares

$$sgn\,\boldsymbol{\gamma}2 = \begin{bmatrix} - & + & 0 & 0 & 0 & 0 & 0 \\ 0 & - & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & + & 0 & 0 \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & - & 0 \end{bmatrix}.$$

For 3 stage least squares and 2 stage least squares corrected for autocorrelation

$$sgn\,\boldsymbol{\gamma}3 = \begin{bmatrix} - & - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & + & 0 & 0 \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & 0 & 0 & 0 & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & - & 0 \end{bmatrix}.$$

Combining the appropriate results we obtain four distinct realized sign patterns for the reduced forms. For the OLS and OLS corrected for autocorrelation the signable entries are

$$\mathrm{FAL}(\pi_{\mathrm{OLS}}) = \begin{bmatrix} ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ ? & ? & - & ? & ? & ? & + \\ ? & ? & - & ? & ? & ? & + \\ ? & ? & ? & ? & ? & ? & ? \\ ? & ? & - & ? & ? & ? & + \\ ? & ? & - & ? & ? & ? & + \end{bmatrix}.$$

Although the OLS results of the structural unknowns are not the same as the model as originally proposed by Klein, the data are falsifiably compatible with Klein's hypothesized model at the level of the reduced form; compare the sign pattern of $\mathrm{FAL}(\pi_{\mathrm{OLS}})$ with $\mathrm{FAL}(\boldsymbol{\beta}, \boldsymbol{\gamma})$ in Eq. (4).

The reduced form sign pattern implied by the two stage least squares and 2 SLS corrected for autocorrelation is

$$\mathrm{FAL}(\pi_{\mathrm{2SLS}}) = \begin{bmatrix} ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & - \\ ? & ? & - & ? & ? & ? & - \\ ? & ? & + & ? & ? & ? & + \\ ? & ? & + & ? & ? & ? & + \\ ? & ? & - & ? & ? & ? & - \\ ? & ? & + & ? & ? & ? & + \end{bmatrix}.$$

The reduced form sign patterns implied by 3 SLS and iterative 3 SLS are

$$\mathrm{FAL}(\pi_{\mathrm{3SLS}}) = \begin{bmatrix} ? & ? & ? & + & - & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & + & + & - & ? & + \\ ? & + & + & + & - & ? & + \\ ? & ? & ? & ? & ? & ? & ? \\ ? & + & + & + & - & ? & + \\ ? & + & + & + & - & ? & + \end{bmatrix}$$

$$\mathrm{FAL}(\pi_{I\mathrm{3SLS}}) = \begin{bmatrix} ? & - & - & + & - & ? & - \\ ? & + & + & ? & ? & ? & - \\ ? & ? & - & + & - & ? & - \\ + & ? & + & + & - & - & + \\ ? & ? & + & ? & ? & ? & + \\ ? & ? & - & + & - & ? & - \\ ? & ? & + & + & - & ? & + \end{bmatrix}$$

The sign patterns derived from the 3SLS estimates are also not falsifiably compatible with the outcomes using the earlier data. For the longer time span none of the estimated structural models are falsifiably equivalent. Further, the derivations from OLS and 3SLS are not falsifiably compatible with any of the other derivations; however, the derivations for 2SLS and I3SLS are falsifiably compatible.

The unknowns of the reduced form were estimated directly from the data, providing the following sign pattern

$$\textit{estimated sgn } \hat{\pi} = \begin{bmatrix} + & - & + & + & + & + & - \\ + & - & - & + & + & + & - \\ + & - & + & + & + & + & + \\ + & - & + & + & + & + & + \\ - & + & - & + & + & - & + \\ + & - & + & + & + & + & + \\ - & - & + & + & + & + & + \end{bmatrix}.$$

Note that the estimated reduced form is the transpose, less the intercept, of Table 2.

A comparison of this outcome with the sign patterns of the reduced forms implied by each of the structural models found reveals that *all* of the structural models are falsified. Further, the implied reduced forms of the models found from the data for the shorter timeframe are also falsified.

From a qualitative standpoint, the difference in conclusion contingent on the sample period regarding the falsification of Klein's model 1 is quite striking. For the short, early sample none of the estimated structural models were falsified, nor was Klein's originally proposed sign pattern. Using the larger data set, all of the estimates produced by the structural estimators were falsified by the data. Furthermore, the sign pattern originally proposed by Klein was falsified. A natural question to ask is whether any variant of the Boolean specification[23] of Klein's model is not falsified by the data. The question can be answered by the method of enumeration.

The feasibly falsifiable set of structural models are those specifications of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ that preserve the Boolean structure of the model subject to user elected constraints, but otherwise place no restrictions on the signs of the unknown coefficients. For Klein's Model I the entries in $\{\boldsymbol{\beta}, \boldsymbol{\gamma}\}$ for rows (4)–(7) correspond to definitional/accounting relationships. Accordingly, the sign pattern for the nonzero entries in these rows is invariant for any possible, hypothesized structural model. For the first three rows there are nine unknown coefficients, one of which is the marginal propensity to consume. For the purpose of enumerating the set of feasibly falsifiable structures the marginal propensity to consume was constrained to be positive and the remaining eight nonzeros were allowed to be either positive or negative. Consequently, there are eight possible hypotheses for $\boldsymbol{\beta}$ and

[23] By Boolean specification we mean that the unknown coefficients can be either positive or negative, while the zero entries in $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ must remain zero, as amended by user elected constraints.

Table 2
Klein's Model 1 (1921–2000) reduced form estimates corrected for 1st order serial correlation

	C	I	W_1	Y	P	W	E
Intercept	403.795 (2.12)	−71.674 (−1.17)	140.596 (1.63)	264.555 (1.78)	136.152 (1.88)	124.428 (1.54)	264.555 (1.78)
W_2	1.253 (1.62)	0.153 (0.237)	0.036 (0.053)	0.765 (0.63)	−0.289 (−0.48)	0.959 (1.41)	−0.235 (−0.19)
P_{t-1}	−0.851 (−5.04)	−0.256 (−2.01)	−0.826 (−5.94)	−1.220 (−4.98)	0.288 (2.72)	−0.867 (−6.35)	−1.220 (−4.98)
K_{t-1}	0.279 (3.91)	−0.050 (−1.37)	0.117 (2.34)	0.131 (1.51)	−0.356 (−2.92)	0.105 (2.19)	0.131 (1.51)
E_{t-1}	0.505 (3.21)	0.229 (2.25)	0.583 (4.76)	0.912 (4.26)	0.022 (0.53)	0.626 (5.29)	0.912 (4.26)
Year	5.139 (6.27)	1.533 (2.21)	4.887 (6.73)	5.968 (4.64)	3.028 (4.71)	4.003 (5.55)	5.968 (4.64)
T	0.543 (1.90)	0.275 (0.98)	0.105 (0.40)	0.053 (0.114)	−0.087 (−0.37)	0.121 (0.46)	1.053 (2.24)
G	−0.368 (−1.74)	−0.316 (−1.74)	0.132 (0.69)	0.357 (1.06)	0.816 (12.72)	0.151 (0.80)	0.826 (13.19)
Rho	0.963	0.682	0.837	0.826	0.816	0.813	0.826

32 possible hypotheses for $\boldsymbol{\gamma}$, yielding a total of 256 alternative structural models and corresponding reduced forms. Furthermore, if the presumption of stability is relaxed then there are a total of 512 implied reduced forms, i.e., 256 assuming det $\boldsymbol{\beta}>0$ and 256 assuming det $\boldsymbol{\beta}<0$.

The signable entries in the implied reduced forms for each possible pair of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ matrices were constructed and compared to the estimated reduced form. Of the 512 implied reduced forms, 473 were falsified and 39 were not. All of the systems that were not stable, i.e., the assumption that det $\boldsymbol{\beta}>0$, were falsified. Accordingly, the nonfalsified systems are only consistent with the assumption of stability. Of the four $\boldsymbol{\beta}$ sign patterns found via alternative estimation strategies, those found using 2SLS, 2SLS-AR, and Iterative 3SLS were not included in any nonfalsified structural system. Those $\boldsymbol{\beta}$ sign patterns that are included in nonfalsified structural systems were found using OLS, OLS-AR, and 3SLS. Of the three $\boldsymbol{\gamma}$ sign patterns found via alternative estimation strategies, those found using OLS, OLS-AR, 2SLS-AR, and 3SLS were not included in any nonfalsified structural system. 2SLS and I3SLS both produce estimates of gamma that can be part of a nonfalsified structure. As noted above, none of the structural models found via any of the 6 estimation strategies were nonfalsified. This is confirmed by the enumeration since at least part of each estimation strategy is falsified by the data.

The 39 nonfalsified models are falsifiably compatible. That is, their reduced form sign patterns are not falsified by the observed reduced form. For each falsifiably compatible system the number of signable entries in the implied reduced form was tabulated. This enumeration is reported in Table 3. Inspection of the implied reduced forms for each group with the same number of signable entries found all in the group to be falsifiably equivalent.

Given the class of falsifiably compatible systems, it is immediate to ask if any of the signs in the structural models are the same. For the class of 39 systems that were found for the Klein estimated reduced form there were common signs in the structure for two of the eight unconstrained structural coefficients. For $\boldsymbol{\beta}$, $\beta_{37}=\mathrm{d}W_1/\mathrm{d}E>0$ for all of the feasible structural models. This confirms Klein's original speculation. Yet, for $\boldsymbol{\gamma}$, $\gamma_{23}=-\mathrm{d}I/\mathrm{d}K_{-1}<0$ for all of the structural models and this is contrary to what was originally proposed.

The outcome of the analysis is (perhaps) understandable from an inspection of the three behavioral equations in the model. With the exception of the propensity to consume, it is possible to hypothesize economically rational behavior, or market circumstances, that

Table 3
Models and signable entries

Number of models with an equal number of reduced form signable entries	Number of signable entries in the implied reduced form
24	8
12	12
2	18
1	22

would lead to either sign for the other eight behavioral coefficients. Any structural model involving these would be potentially transitory and the prospect for substantial variation in the implied behaviors over the better part of a century is clear. Not surprisingly, not many individual structural hypotheses are even consistent with the larger data set. And even for these, many of the relationships are indeterminant. The analysis of the falsifiability of the estimated structural models, the falsifiably feasible set of structural models, and the residual class of falsifiably compatible models provided decisive insight into the disarray associated with any specific structural hypothesis based upon the basic specification of Klein's model I.

3.2. A classic model of under-identification

An oft cited study of the relationship between advertising, concentration, and price–cost margins is the work by Strickland and Weiss (1976).[24] In the econometrics literature (Amemiya, 1985; Greene, 2003; Gujarati, 2003) the paper is cited because the model fails the rank test for identification, although estimation proceeds as a result of its non-linearity in variables. The original model was specified as

$$\boldsymbol{\beta Y} = \boldsymbol{\gamma Z}$$

$$\begin{bmatrix} -1 & a_3 & a_4 & a_1 \\ b_1 & -1 & 0 & 0 \\ c_5 & c_3 & 0 & -1 \end{bmatrix} \begin{bmatrix} Ad/S \\ C \\ C^2 \\ M \end{bmatrix}$$

$$= \begin{bmatrix} -a_5 & -a_6 & 0 & 0 & 0 & -a_2 \\ 0 & 0 & b_2 & 0 & 0 & 0 \\ -c_2 & 0 & -c_6 & -c_1 & -c_4 & 0 \end{bmatrix} \begin{bmatrix} Gr \\ Dur \\ MES/S \\ K/S \\ GD \\ CD/S \end{bmatrix}.$$

In the Strickland and Weiss model the endogenous variables are: advertising divided by sales (Ad/*S*), concentration and its square (*C*), and the price cost margin (*M*); and the exogenous variables are growth of the industry (Gr), a dummy for durable goods (Dur), minimum efficient scale divided by sales (MES/*S*), capital stock divided by sales (K/S), geographic dispersion of the industry (*GD*), and the percent of industry sales that is represented by consumer demand (*CD*/*S*). For purposes of illustrating the power of our approach to falsifiability the C^2 variable is dropped from the Advertising equation so that the model is not only under-identified but its

[24] The model seems to have withstood the test of time. Textbook citations in the industrial organization literature include Greer (1984), Scherer and Ross (1990), and Carlton and Perloff (2000).

structural unknowns cannot be estimated. With the re-specification the model becomes

$$\begin{bmatrix} -1 & a_3 & a_1 \\ b_1 & -1 & 0 \\ c_5 & c_3 & -1 \end{bmatrix} \begin{bmatrix} Ad/S \\ C \\ M \end{bmatrix} = \begin{bmatrix} -a_5 & -a_6 & 0 & 0 & 0 & -a_2 \\ 0 & 0 & -b_2 & 0 & 0 & 0 \\ -c_2 & 0 & -c_6 & -c_1 & -c_4 & 0 \end{bmatrix} \times \begin{bmatrix} Gr \\ Dur \\ MES/S \\ K/S \\ \mathrm{GD} \\ CD/S \end{bmatrix}.$$

The sign pattern proposed by Strickland and Weiss is

$$sgn\,\boldsymbol{\beta} = \begin{bmatrix} - & + & + \\ + & - & 0 \\ + & + & - \end{bmatrix} \text{ and } sgn\,\boldsymbol{\gamma} = \begin{bmatrix} - & ? & 0 & 0 & 0 & - \\ 0 & 0 & - & 0 & 0 & 0 \\ - & 0 & - & - & + & 0 \end{bmatrix}.$$

The "?" for the coefficient on the durable goods dummy enters the advertising equation since Strickland and Weiss had no a priori belief about the sign. Consequently, falsifiability will be considered for all three possibilities: the coefficient is −, 0 or +.

The original Strickland and Weiss model is examined here using data for 1992.[25] Their original paper used data for 1963.[26] Instead of the four firm concentration ratio we have used the Herfindahl index. Instead of the geographic dispersion measure developed by Collins and Preston (1969) as used by Strickland and Weiss, we use an information measure. The information measure compares the distribution of shipments of the given industry across the 50 states to the distribution that would be seen if the industry was the same as the distribution of all industries in the aggregate. Let S_j, $j=1,2,\ldots 50$ be a vector of the distribution of shipments by state and S_{ij} be the corresponding value for the distribution of shipments by the i-th industry. An information measure of the geographic dispersion of shipments is then $\sum_{j=1}^{50} s_{ij}\ln\left(s_{ij}/s_j\right)$.

The unknowns of the advertising and concentration equations can be estimated by 2SLS. Questions of identification and simultaneity aside, the price–cost margin equation can be fit by OLS. These results are presented in Table 4.

All of the slope coefficients in the structural advertising equation should be positive according to Strickland and Weiss. This does not appear to be the case,

[25] All of the variables used in the model are derived from the 1992 Census of Manufactures and from the corresponding National Input–Ouptut Tables, both available from the U.S. Department of Commerce. Except for the two variables noted in the text all variables were constructed as described in Strickland and Weiss (1976) and Collins and Preston (1969).

[26] We did not try to replicate the Strickland and Weiss results for 1963. Their data set, although derived from public sources, is not in the public domain. Further, their durability dummy was assigned on the basis of judgment, so we would have been guessing in the construction of that variable.

Table 4
Coefficient estimates

	Dependent variables		
	2 SLS		OLS
	Advertising	Herfindahl	Price–cost margin
Constant	−4.3316(−1.48)	521.9331(8.90)	−0.2531 (−1.61)
Advertising		46.9925(1.45)	0.0202(6.64)
Herfindahl	−0.000567(−1.32)		0.000029 (2.85)
Price cost margin	−0.6684 (−0.16)		
Growth	5.3236(1.58)		0.4779(3.20)
Durables	−0.0673 (−0.31)		
Minimum efficient scale		113.9034(3.47)	0.0050 (1.44)
Capital			0.0787 (4.01)
Geographic dispersion			−0.0192 (−1.49)
Consumer demand	3.0602 (6.34)		
R^2	0.17	0.06	0.29
Degrees of freedom	389	392	395

Heteroscedasticity corrected standard errors.

although the negative coefficients are not statistically significant. The results for the concentration equation conform to the original hypothesis. The price–cost margin equation also conforms to the proposed model, again overlooking the statistical shortcomings of OLS in the present circumstance. All in all, it appears that the Strickland and Weiss model may not be falsified by the data. There remains the caveat of the properties of the estimators in simultaneous equations models. That caveat suggests that the reduced forms should be examined for the possibility of falsification.

Assuming stability, the expected sign pattern of the reduced form based on Strickland and Weiss' conjectures can be determined from

$$sgn\,\boldsymbol{\beta}^{-1}\begin{bmatrix} - & - & - \\ - & ? & - \\ - & - & ? \end{bmatrix} \text{ with } \mathrm{FAL}(\boldsymbol{\beta},\boldsymbol{\gamma},S)=\begin{bmatrix} + & * & + & + & - & + \\ + & * & ? & + & - & + \\ ? & * & ? & ? & ? & + \end{bmatrix}=\mathrm{FAL}(\pi).$$

The asterisks in the second column of FAL(π), the matrix of signs for the reduced form coefficients, is a consequence of the inability to sign a_6 in the original model. When $a_6>0$ then all three asterisks are −, when $a_6=0$ then all three entries are 0, and when $a_6<0$ then all three are +. More importantly, the third row, corresponding to the price–cost equation has two predicted signs in it. Recall that in the original structural model the price–cost equation was under-identified.

The reduced form unknowns can always be estimated by OLS, which has all of the usual desirable small sample properties regardless of simultaneity and lack of identification in the structural model. The estimates of the reduced form unknowns are reported in Table 5 with White's heteroscedasticity corrected standard errors.

Table 5
Reduced form estimates corrected for heteroscedasticity

	Advertising	Herfindahl	Price cost margin
Constant	−3.2151 (−1.11)	−1663.4862 (−1.92)	−0.3407 (−2.08)
Growth	3.7857 (1.40)	1911.1434 (2.35)	0.5883 (3.81)
Durables	0.0109 (0.06)	−28.9979 (−0.44)	−0.0159 (−1.60)
Minimum efficient scale	−0.1161 (−1.77)	90.8831 (2.94)	0.0046 (1.35)
Capital	−0.0078 (−0.02)	240.2515 (1.61)	0.0871 (4.42)
Geographic dispersion	−0.0514 (−0.26)	265.6275 (3.60)	−0.0184 (−1.51)
Consumer demand	3.0004 (5.99)	131.9597 (1.21)	0.0978 (6.35)
R^2	0.24	0.14	0.19
Degrees of freedom	388	393	393

FAL(π) has ten predicted coefficient signs apart from those on the Durable Goods dummy. Of the ten signable entries, the empirical results agree with seven of them. The three coefficients on the durables dummy should all have the same sign. Although this is not the case, the coefficients are not statistically different from zero. The Minimum Efficient Scale coefficient and Capital coefficient in the Advertising equation, and Geographic Dispersion in the Herfindahl equation all have incorrect signs. Although the structural estimates of the model lend support to the structure–conduct–performance paradigm, the model is falsified by our qualitative approach.

Pursuing the approach used for the analysis of the Klein model, the complete set of feasibly falsifiable structural models was enumerated. The β and γ matrices for Strickland and Weiss were created for all possible sign patterns with the following constraints: All zero entries were kept at zero. The main diagonal of the beta matrices were kept all negative. Given this, there were 32 possible β matrices. For γ, the (1,2)nd entry was varied positive, negative, and zero in keeping with the uncertainty about the effect of the Durables dummy. For the remaining nonzeros, the entries were varied positive and negative. Given this, there were 384 possible gamma matrices. Thus, there are (32 × 384=) 12,288 possible structural hypotheses. Given that the beta matrix can have a positive or negative determinant, this gives a grand total of 24,576 implied reduced forms. Each of these was compared to the estimated reduced form sign pattern. All possible structural systems were falsified by the empirical reduced form sign pattern. There apparently is no sign pattern in the Boolean specification of Strickland and Weiss that is consistent with the data.

The striking result that every possible Boolean version of the Strickland and Weiss model is falsified rests on a very strict comparison of the implied reduced forms with the estimated reduced form. Since there are four coefficients in the estimated reduced form with t-statistics less than one, it is reasonable to say that these entries in the empirical reduced form are not signable. This then weakens the criteria for including a feasible structure in the set of falsifiably compatible models, i.e, the criteria are applied as before, but the four entries of the reduced form that are unsigned are simply ignored for all of the feasible structural models. The set defined by this weakened criterion we term relatively falsifiably compatible. Under the amended criterion there were 180 falsifiably compatible structural form sign patterns. Inspection of the β and γ matrices belonging to the class of relatively compatible systems found that β_{12} and β_{21} were both positive for all of the feasible systems, in agreement with the original Strickland and Weiss speculation.

4. Summary

Traditionally, economic theory has been used to specify an algebraic model. In its sparest form the specification may include the menu of endogenous and exogenous variables, some exclusion restrictions, and possible signs of unknown coefficients. The model is then confronted with the data and statistical tests of the hypothesis are then conducted. Unfortunately, in terms of current practice, testing depends on a multiplicity of estimation strategies and a multiplicity of possible test statistics. Given this, model falsification in economics has not really been established as a distinct enterprise conducted under a commonly agreed to protocol of analysis. The problem seems to be that there are too many ways to fit the data and too many ways to test a given hypothesis. This paper provides a methodology intended to resolve these problems.

Qualitative analysis can be used to define new concepts for the falsification of models. Using the idea of falsifiable equivalence, an application of qualitative analysis as introduced in this paper was used to show that alternative versions of the same structural model may imply the same characteristics in the reduced form. A broader concept is that of falsifiable compatibility. Structural models are falsifiably compatible if there can be outcomes for the estimated reduced form that falsify none of them. For a given estimated reduced form, there can be the decisive result that some entries in β and/or γ have the same sign, notwithstanding that the structural models also exhibit significant differences. A still broader concept is the class of falsifiably feasible models. This class consists of all possible models that are consistent with the barest Boolean specification of the structural model. If all of these are falsified by the estimated reduced form, this serves as a decisive rejection of the hypothesized model.

The authors see a host of technical issues to be resolved in expanding the analysis beyond the qualitative characteristics of proposed, or derived, structural models and in reaching deeper interpretations of the outcome of the analysis. Nevertheless, we believe that the principle of falsification, and the associated concepts developed here, brings a new and decisive tool to the evaluation of hypotheses about economic activity and the assessment of statistical realizations of the hypotheses.

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Appendix A. Conducting qualitative analyses

A.1. Background

The issue to be considered here is: If only the signs of the entries of an $n \times n$ matrix β are known, what, nevertheless, can be determined about the entries of the matrix β^{-1}. As noted, other information may also be assumed, e.g., the relative sizes of the entries of β, and there is nothing in principle that restricts any of the points made here about falsification to an analysis that only considers sign patterns. Further, the attributes of the entries of β^{-1} to be derived are also limited (here) to the signs of entries, but none of the points about falsification are limited to tests of sign patterns.

This said, a great deal of the literature focuses on sign patterns, since this is typically assumed to be the limit of information provided about relationships by economic theory. As noted, Samuelson (1947) did not hold much promise for deriving information about the signs in β^{-1} based upon knowledge of the signs in β. The problem, called "qualitative analysis," was reconsidered by Lancaster (1962) who showed (sufficient as it turned out) conditions under which a qualitative analysis would be successful. Significantly, Lancaster (1966) also proposed an algorithmic principle that would enable a successful qualitative analysis to be conducted, if such were possible.[27]

A constructive, necessary and sufficient, "algebraic" approach for detecting when a successful qualitative analysis could be conducted was provided by Maybee and Quirk (1969). A review of the subsequent literature and additional derivations for working with information in addition to sign patterns are presented in Hale et al. (1999). A means of conducting a qualitative analysis based upon this algebraic approach is given below.[28]

A.2. Qualitative invertibility

The first step is to show that det $\beta \neq 0$, based only on sgn β. For reasonably small, or otherwise sparse, matrices the analysis can be conducted with pencil and paper. To do this, β is transformed by reordering columns and multiplying columns by "-1" as necessary such that: $\beta_{ii} < 0$ for all i. If desired, the resulting system, as transformed, can be interpreted such that each variable is considered to correspond to the row in which it appears on the main diagonal. Call β transformed in this way in "standard form".

[27] The algorithm, called the *elimination principle*, is based upon testing all possible solution sign patterns to a linear system involving the given β and "eliminating" all that are not consistent, e.g., the sum of all nonpositive numbers cannot have a positive outcome; and hence, the associated solution would be "eliminated." For the residual of solutions not eliminated, if any variable has the same sign for all of them, then the sign of a corresponding entry of β^{-1} has been found, based only upon sgn β.

[28] A computer program, SGNSOLVE, written by one of the authors is based upon this algebraic approach and was used to support the examples given in this paper.

Next, mark and enumerate places on the page, one for each variable, i.e., column of β. The places marked are called "vertices." Draw arrows among the vertices following the convention:

$(j) \rightarrow (i)$ if and only if $\beta_{ij} \neq 0$.[29]

Place a "+" or "–" subscripting the head of each arrow as appropriate to the sign of the nonzero entry of β to which it corresponds, e.g.,

$(j) \rightarrow_{+} (i)$ if and only if $\beta_{ij} > 0$.

The arrows are called "signed directed arcs" and the entire array of vertices and signed arrows is called the "signed directed graph (SDG(β))" corresponding to β (as transformed). A traversal of the SDG(β) by following the arrows from one vertex to another without passing through an intermediate vertex more than once is called a "path." A traversal from a vertex to itself without passing through an intermediate vertex more than once is called a "cycle." The signs of cycles and paths are determined by the signs of the products of the entries of β that correspond to the embodied directed arcs. If there is a path between every pair of vertices β is called "irreducible" and the corresponding linear system must be solved simultaneously for all of the associated variables. The results below are for irreducible matrices.

Let Q_β be the set of all matrices **A** with the same sign pattern as β, i.e., sgn **A** = sgn β for all A ε $Q\beta_{fs}$.

Qualitative invertibility (Bassett et al., 1968). For β irreducible and in standard form, det **A**≠0 for all **A** Qβ if and only if the sign of all cycles in SDG(β) is negative. If β satisfies the condition, then sgn (det β)=$(-1)^n$. Call β*qualitatively invertible*, if it satisfies the condition.

A.3. Signing entries in β^{-1}

If β is qualitatively invertible, the entries of β^{-1} incident on the nonzeros in the transpose of β can be immediately signed.

Lady (1983). For β an irreducible, qualitatively invertible matrix with $\beta_{ii} \neq 0$, if $\beta_{ij} \neq 0$, then,

$$sgn\, \beta_{ji}^{-1} = sgn\, \beta_{ij}.$$

If $\beta_{ij}=0$, then sgn β_{ji}^{-1} may be determined independent of magnitudes, or not. The result depends upon the signs of the paths from vertex i to vertex j.

Maybee and Quirk (1969). For β an irreducible, qualitatively invertible matrix in standard form with $\beta_{ij}=0$, sgn β_{ji}^{-1} is the same for all members of $Q\beta$ if and only if the

[29] The transposed convention is often used, i.e., $(i) \rightarrow (j)$ if and only if $\beta_{ij} \neq 0$, and some of the literature cited here follows this convention. This would be entirely intuitive for β_{ij} (such as) the technological coefficient in an input–output model identifying the flow of a good or service from sector #i to #j. The convention here is used to highlight that the entry β_{ij} represents a flow of inference from variable #j to #i for β in standard form.

signs of all of the paths from vertex (i) to vertex (j) have the same sign. If the condition is satisfied, then sgn $\beta_{ji}^{-1} = -$sgn (path (i)→(j)).

If β is not qualitatively invertible, falsifiable attributes of β^{-1} based only upon sgn β are still available. Specifically, the arrays corresponding to each entry in the adjoint of β can be submitted to a qualitative analysis. If at least two are qualitatively invertible, then the corresponding entries of β^{-1} must have the same, or opposite signs (assuming β is nonsingular). The signs themselves in β^{-1} might be resolved (as here) by (such as) assuming stability.

Example. Consider the matrix β given in the first example in section II above,

$$sgn\,\beta = \begin{bmatrix} - & 0 & 0 & - \\ + & - & 0 & 0 \\ 0 & + & - & 0 \\ 0 & 0 & + & - \end{bmatrix}.$$

Inspection reveals that there is only one cycle and this involving all of the vertices. The cycle can be written as:

$$(1)\rightarrow_{+}(2)\rightarrow_{+}(3)\rightarrow_{+}(4)\rightarrow_{-}(1),$$

(repeating the appearance of vertex (1) to facilitate exposition). Inspection also reveals that the sign of the cycle is negative; hence, β is qualitatively invertible. Given this, from Lady (1983) above, the entries in the inverse corresponding to the transposed nonzeros in β can be immediately signed,

$$sgn\,\beta^{-1} \begin{bmatrix} - & + & ? & ? \\ ? & - & + & ? \\ ? & ? & - & + \\ - & ? & ? & - \end{bmatrix}.$$

The entries designated "?" may or may not be signable, depending upon the signs of paths. Since SDG(β) has only the single cycle, there is only one path between each pair of vertices. Accordingly, all of the entries designated "?" can be signed; and, the sign for each is equal to the negative of the sign of the corresponding path. For example,

$$sgn\,\left(\text{path}((1)\rightarrow(2))>0;\text{ so that}, sgn\,\beta_{21}^{-1}<0;\text{ while}\right),$$

$$sgn(\text{path}((4)\rightarrow(1)))<0;\ \text{ so that,}\ \ sgn\,\beta_{14}^{-1}>0.$$

The reader can readily find the signs of the other paths at issue and confirm that,

$$sgn\,\beta^{-1} = \begin{bmatrix} - & + & + & + \\ - & - & + & + \\ - & - & - & + \\ - & - & - & - \end{bmatrix}.$$

Consider the next example in section II for which,

$$sgn\,\beta = \begin{bmatrix} - & 0 & 0 & + \\ + & - & 0 & 0 \\ 0 & + & - & 0 \\ 0 & 0 & + & - \end{bmatrix}.$$

This case also has only the single cycle involving all of the variables. But now, since the sign of the (1,4)th entry has been set positive, the sign of the cycle is positive,

$$(1)\rightarrow_{+}(2)\rightarrow_{+}(3)\rightarrow_{+}(4)\rightarrow_{+}(1),$$

and β is not qualitatively invertible. Nevertheless, it is still possible to derive results concerning the signs of the entries of β^{-1}.

The method used is to analyze, as above, the arrays corresponding to β's cofactors. One approach is to isolate the arrays corresponding to each cofactor and, as above, perform a qualitative analysis to determine if the array is qualitatively invertible. If this is done for the example here, it is found that all of the cofactors are signable and negative. Although simply said, this approach is somewhat of a burden, even for the 4×4 array above.[30] A shortcut that can sometimes ease the burden of derivation can be based upon early work by Maybee (1966). Here it was shown that each term in the (j, i)th cofactor can be expressed by,

$$v(\text{path }(j\rightarrow i))(-1)^{q}\beta(path(j\rightarrow i)),$$

where v(path ($j\rightarrow i$)) is the product of the entries of β that correspond to the directed arcs of the path, q is the length of the path (i.e., the number of embodied directed arcs), and β (path($j\rightarrow i$)) is the principle minor of β corresponding to the array formed by deleting the rows and columns that correspond to the vertices in the path. It was shown that for β Hicksian (i.e., odd order principal minors negative and even order principal minors positive), all of these terms would have the same sign if all of the paths have the same sign, i.e., the cofactor is signable. For the example here, given the single positive cycle, the reader can easily confirm that all principal minors of order less than 4 are Hicksian; and further, that all paths are positive. If also det $\beta>0$ (as is the case if β is assumed to be a stable matrix), then β itself is Hicksian and all the entries of β^{-1} are negative, since all paths are positive, as given in Maybee and Quirk (1969).[31] Although β's determinate is not in fact signable, nevertheless all of the entries of β's inverse will have the same sign, an easily falsifiable attribute.

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[30] This is the algorithmic principle utilized by SGNSOLVE when deriving results for this paper.

[31] Hicks (1939). Bassett, Maybee and Quirk showed that a matrix **A** was Hicksian based upon sign pattern alone if and only if all cycles in SDG(A) are negative. Given this, the relationship between the signs of cofactors and the signs of paths in Maybee and Quirk (1969) utilizes Maybee's (1966) result.

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