Modelling a Poisson Process: Strike Frequency in Great Britain

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I. Introduction

In regression analysis one often encounters linear models for which the dependent variable is inherently discrete. Furthermore, it may be that the data for such a model are in panel form, i.e., a cross section of time series. The discrete nature of the dependent variable makes ordinary least squares (OLS) an inappropriate estimator.[1] Any alternative estimator should reflect the underlying probability model and any possible correlations between individuals in the cross section.

As a case in point, this paper considers strike activity in Great Britain. In a previous paper [Buck, 1982] a model of strike activity was derived and subsequently tested using quarterly strike data for the period 1959-1976. The model parameters were estimated using Zellner's [1962] Seemingly Unrelated Regression, an OLS technique. A more appropriate estimator would have involved the use of an estimator derived from a multivariate discrete probability distribution. The present paper suggests modelling the dependent variables as pairs (e.g., coal mining and construction) of bivariate Poisson-distributed random variables for which there are T observations.

The bivariate Poisson estimator is illustrated using strikes per quarter in Britain for three industries; coal mining, manufacturing, and construction, for the period 1959-1977. The model parameters are estimated using a maximum likelihood technique.

The plan of the paper is as follows: The estimation framework is discussed in Section II and was developed from the work of Jorgenson [1961], Campbell [1933], and Mahamunulu [1967]. The empirical findings are generally in accord with the findings of Pencavel [1970], Bean and Peel [1974], Sapsford [1975], and Buck [1982] and are presented in Section III. The final section presents conclusions drawn from the analysis.

II. The Poisson Probability Model

Representative models of bargaining often begin with utility or wealth maximization with demands for wage increases/concessions which decay over time [Buck, 1982; Cross, 1965; Farber, 1978]. With detailed knowledge of firm and union behavior, one could solve explicitly for the time to settlement. In the event that either the firm or the union do not revise their concession schedules quickly enough the negotiations will fail, i.e., there will be a strike.

An indirect test of such bargaining models may be constructed as follows: The failure of the firm or unoin to revise their concession rates quickly enough will result in failing to reach agreement before the strike deadline.[2] Thus the researcher records either success or failure of the negotiation process. As a statistical matter, while the number of failures of wage negotiations in a given quarter is binomial,

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[1] Least squares estimators are linear unbiased when, among other things, the dependent variable is continuous, the error term is continuous with zero mean and scalar diagonal covariance matrix. Furthermore, OLS will yield fitted values that lie between the permitted integer values of the dependent variable, or outside the permitted range.

[2] Lancaster [1972] has modelled the duration of strikes. A more complete analysis would combine the present work with his. Namely, the strike-no strike outcome is binomial, as suggested here, and in the event of a strike its duration is modelled as an Inverse Gaussian.

the limiting form is Piosson.[3] The Poisson rate parameter may then be modelled as a linear combination of the relevant economic variables.

While strikes in Great Britain are not rare events, there do seem to be a large number of wage related disputes, i.e., trials of the experiment. It does not seem unreasonable to believe that the probability of a no-strike settlement does not change appreciably from one trial of the experiment (dispute) to another. Thus, as a limiting form, the Poisson would appear to be appropriate.

The distribution function of the univariate Poisson is quite familiar and is given by:[4]

$$P(X = k \text{ strikes}) = [\exp(-\frac{T}{\tau})] \frac{(T/\tau)^k}{k!} \qquad (2.1)$$

Ordinarily, the length of the interval for which one computes the probability of k strikes is taken to be $T = 1$, a convention adopted in this paper. In situations in which some discrete form of behavior is being studied, e.g., strikes, auto accidents, etc., it is suggested that the rate parameter, τ, be modelled as a linear combination of relevant explanatory variables. This approach has been used by Jorgenson [1961], Gart [1964], Frome, Kutner and Beauchamps [1973] ,and Gustavsson and Svensson [1976].[5]

However, in the present case there remains an additional aspect of the appropriate statistical model. Zellner [1962] observed that in many models the dependent variables (in the present case the number of strikes per quarter in different industries) of two or more seemingly unrelated regression equations may be related through their error terms. (The analogy to SUR is made more complete following equations 2.3 and 2.4). Strike frequencies in coal mining, construction, and manufacturing certainly are not independent. Thus, one is led to conclude that a multivariate Poisson model is the appropriate specification. The multivariate Poisson process has been derived elsewhere as either the limiting form of the multivariate binomial or a generalization of the bivariate Poisson [Krishnamoorthy, 1951; Teicher, 1954; Campbell, 1933]. Unfortunately, it does not have a closed form representation.[6]

To resolve this shortcoming, strike frequencies are modelled as three bivariate Poisson models. The estimation of the fixed rate parameters (i.e., the treatment variable is a constant) of the bivariate Poisson probability function is discussed by Holgate [1964], Campbell [1933], and Mahamunulu [1967].

The closed form representation of the bivariate Poisson is

$$P(X, Y) = \exp[-(a + b - d)] \qquad (2.2)$$

$$\sum_{u=0}^{\min x,y} \frac{(a-d)^{x-u}(b-d)^{y-u} d^u}{(x-u)!(y-u)!\,u!}.$$

The derivation of the bivariate Poisson has a rather intuitive starting point and is presented here for reasons that will become obvious.

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[3] Rigorously, a binomial or Bernoulli process consists of n independent and identical trials of an experiment with only two mutually exclusive outcomes and constant probability of "success" on all trials. While the success of wage negotiations is not binomially distributed in the strictest sense, such an assumption does less violence to the observed facts than the assumption of a continuous distribution implied by OLS.

[4] Each firm is treated as a repetition of the experiment.

[5] The specification in (2.1) and the resulting estimator rely on the crucial assmption that the number of strikes per quarter is the sum of k independently distributed Poisson random variables. This assumption is considerably more rigorous than the usual regression analysis assumption of full column rank for the matrix of independent variables.

[6] Lacking a closed form representation means that the joint probability distribution function of three or more Poisson variates involves an infinite number of terms [see Krishnamoorthy, 1951]. To estimate the parameters of such a function would require arbitrary truncation beyond a given number of terms. Dropping a sufficient number of appropriate terms results in the bivariate Poisson. As the bivariate and univariate results were not appreciably different from one another, the former seemed a low cost approximation of the multivariate model. While it might be argued that the univariate model is a lower cost, adequate approximation, explicit incorporation of the correlation between industries is more appropriate.

(X, Y) has a bivariate Poisson distribution, if

$$X = X^* + U,\ Y = Y^* + U \qquad (2.3)$$

and X^*, Y^*, U are independent Poisson variates with means $a' = a - d$, $b' = b - d$, and d respectively. Thus, in modelling strike frequencies in different industries with the use of both industry specific and common explanatory variables, it is appealing to use the bivariate specification, where a and b, the rate parameters of the two industries, are linear combinations of industry specific variables and d is a linear function of common variables.

In the usual SUR model, the T observations on the k industry specific variables would be used to construct an $NT \times NK$ block diagonal design matrix, the common causal variables are subsumed in the unobservable error term. In the bivariate Poisson model, the common variables are observable and appear in the common Poisson variate U in (2.3).

The analysis of the previous work suggests that the time-to-settlement, and hence the Poisson rate parameter in a given industry (i.e., a and b), depends on the wage bill relative to profits in the previous period, the elasticity of demand for labor, and the reaction functions of both the firm and the union as well as factors common to all industries. Thus, for each of the coal mining, construction, and metals manufacturing industries the industry unemployment rate (U_{jt}), the overall unemployment rate ($U_{\cdot t}$), gross trading profits as a percent of wages and salaries lagged on quarter ($\pi_{\cdot t}$)the rate of change of industry real wages lagged one quarter (R_{jt-1}), and the rate of change of overall real wages ($R_{\cdot t}$) are included as treatment variables.[7]

One approach to the estimation problem is to consider the task of finding the mean X corresponding to a fixed Y. The locus of the means of the X's corresponding to any Y is given by Campbell [1933].

$$E(X \mid Y) - a = \frac{d}{b}(Y - b). \qquad (2.4)$$

The maximum likelihood estimates of a and b are the sample means of X and Y, respectively. The second step in the estimation is to determine d.

The strike frequency (time-to-failure) model was specified by allowing the parameter a to be a linear function of unemployment and real wage changes in the X^{th} industry ($X^* \alpha$) and making b a linear combination of unemployment and real wage changes in the Y^{th} industry ($Y^* \beta$). The coefficients in the linear specification were estimated using an iterative *ML* technique based on Jorgenson.

In this case, the dependent variable is a $2T$ column of observations on strike frequencies. The matrix of observations on the independent variables is block diagonal of dimension $2T \times 6$. These results were then plugged into (2.4) and iterative least squares used to determine the coefficients of

$$d = \delta_1 \pi_{\cdot t} + \delta_2 R_{\cdot t} + \delta_3 U_{\cdot t} = U\delta \qquad (2.5)$$

III. Empirical Results

The sample consists of 67 quarterly observations from the fourth quarter of 1959 to the second quarter of 1976 for the coal mining, construction and metals manufacture and engineering industries. The data on wages, gross trading profits, and unemployment rates were gathered from the following British publications: *Department of Employment Gazette*, *British Labour Statisitcs: Historical Abstract*, *British Labour Statistics: Yearbook*, and the *Monthly Digest of Statistics*.

Real wage changes were calculated from

$$R_t = \left(\frac{W_{t+2} - W_{t-2}}{2W_t}\right) - \left(\frac{P_{t+2} - P_{t-2}}{2P_t}\right) \quad (3.1)$$

where, W and P are money wage and price indices. Such a centralized second difference measure of real wage changes is commonly used as a rational expectations variable. It also seemed to give the best fit in terms of the residual sum of squares.

The coefficient estimates of the bivariate

[7] Previous work has shown that offers and demands depend on profitability, wages, elasticity of demand for labor, and the reaction parameters. The latter two variables are not observable so unemployment rates have been used as proxies.

Poisson model are presented in Table 1, t-statistics are not presented for lack of a small sample distribution for the estimator.[8] However, comparison of these results with a univariate Poisson model shows that the coefficients are of similar sign and order of magnitude in most cases.[9] Table 1 also reports the residual sum of squares for each industry for each of the models.

The results of Table 1 compare favorably with those of previous authors on the basis of expected signs. The overall unemployment rate has an inverse impact on the mean number of strikes, while overall real wage changes has the opposite effect. The effects of manufacturing real wage changes and unemployment rate are contrary to both expectations and the results for the other two industries.

In summary, movements in $X\alpha$ and $Y\beta$, the means of the marginal distributions, have the expected impact on the joint probability. The impact of $U\delta$ depends on the means relative to a given frequency. For the examples provided, if both means are greater than the given frequency, then an increase in $U\delta$ causes the joint probability to increase. If one of the means is smaller than its respective frequency, then an increase in $U\delta$ causes the probability to decline.

[8] Note that the least squares estimate of a mean is also maximum likelihood. However, because of the first stage, iterative maximum likelihood procedure one can at best derive the asymptotic properties of the estimators.

[9] The univariate results are available from the author. Discrepancies are due to the fact that the independent variables are not independent of one another nor are they orthogonal.

TABLE 1

Estimates of Rate Parameters: Bivariate Poisson

Industry Pair	Intercept	R_{it}	U_{it}	$\pi_{\cdot t}$	$U_{\cdot t}$	$R_{\cdot t}$	RSS
Mean							
Coal	281.2997	-.9709	-45.814195				356431.3643
Given				-.0052	-9.7663	31.3942	
Manufacturing	173.8108	23.7509	15.739533				609121.6655
Mean							
Manufacturing	173.8108	23.7509	15.739533				599693.9840
Given				.1660	-13.9650	30.3048	
Construction	64.2121	-1.1260	-.217471				18886.7509
Mean							
Construction	64.2121	-1.1260	-.217471				14642.6231
Given				.0165	-.4342	1.4521	
Coal	281.2997	-.9709	-45.814195				351717.1088

IV. Conclusions

There are several noteworthy aspects of the work presented here. It is noted that previous parametric models of the bargaining process may be solved for the time-to-settlement of a wage dispute. It is then argued that such a specification is a model of time-to-failure of the bargaining process, or, considering frequencies of strikes in a fixed interval, a Poisson process. This is a very intuitive conclusion that reinforces the inappropriateness of OLS models of strike frequency.[10] An estimation method based on regression planes is used to estimate the model parameters of bivariate Poisson models of strike frequency in coal mining, construction, and manufacturing. The empirical results confirm the author's prior expectations about the determinants of strike frequency. [See for example Buck, 1982; Pencavel, 1970; and Sapsford, 1975.]

More specifically, as the labor market gets tighter, as measured by industry and overall unemployment rates, the expected number of strikes increases. Increasing profits relative to the wage bill causes strikes to decline in construction and manufacturing and increase in coal mining. Increases in the rate of change of overall real wages causes the expected number of strikes to decline in coal mining and increase in construction and manufacturing. The rate of change of industry real wages has a negative impact in construction, positive in mining, and mixed in coal mining.

[10] The OLS assumption is that the dependent variable, if not normally distributed, is continuous.

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