Temple University

Department of Economics

Econ 3503 – Introduction to Econometrics

Chapter 2 Exercises – Simple Regression – Answer Key

2.1 Given some observations on a demand curve:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Y Pounds | X Dollars | xi-xbar | (xi-xbar)2 | (xi-xbar)yi | e\_hat, LS resids, y-yhat |  |  |  |
| 17 | 2 | -6 | 36 | -102 | -0.357 |  |  |  |
| 15 | 4 | -4 | 16 | -60 | -0.571 |  |  |  |
| 14 | 6 | -2 | 4 | -28 | 0.214 |  |  |  |
| 13 | 8 | 0 | 0 | 0 | 1 |  |  |  |
| 11 | 10 | 2 | 4 | 22 | 0.785 |  |  |  |
| 8 | 12 | 4 | 16 | 32 | -0.428 |  |  |  |
| 6 | 14 | 6 | 36 | 36 | -0.642 |  |  |  |
| 84/7=12 | 56/7=8 |  | 112 | -100 |  |  |  |  |

1. Find the slope and intercept:

$$b\_{2}=\frac{\sum\_{i=1}^{n}\left(x\_{i}-\overbar{x}\right)y\_{i}}{\sum\_{i=1}^{n}\left(x\_{i}-\overbar{x}\right)^{2}}=\frac{-100}{112}=-0.89286$$

$$b\_{1}=\overbar{y}-b\_{2}\overbar{x}=12-\left(-.89\right)8=19.14$$

1. Plot the data and the least squares line.
2. The point (xbar, ybar) is shown as the red square sitting on the regression line.
3. The coefficient b2 is the slope of the fitted line. For every one unit increase in X there will be a decrease in Y of -0.89 units.
4. The set of LS residuals are shown in the so labeled column of the table.
5. The error variance is

$$\hat{σ}^{2}=\frac{\sum\_{}^{}\hat{e}\_{i}}{n-2}=\frac{2.71}{7-2}=0.542$$

1. The standard error of the coefficient estimator is given by

$$Var\left(b\_{2}\right)=\frac{\hat{σ}^{2}}{\sum\_{}^{}\left(x\_{i}-\overbar{x}\right)^{2}}=\frac{0.542}{112}=.0048$$

$$se\left(b\_{2}\right)=\sqrt{.0048}=0.0695$$

2.2 An ice cream parlor reports $\hat{y}=-72+1.6x$

(a) Below a temperature of **72/1.6 = 45** no ice cream will be sold.

(b) Sketch a graph of the relationship:

(c) If the temperature is 75 degrees then we expect to sell -72+1.6\*75 = **48** lbs of ice cream.

2.3 Mean income and high school grads.

(a) The error variance equals 3.941. What is the sum of the squared residuals?

$$3.941=\frac{SSR}{51-2}$$

SSR = 3.941\*49 = **193.109**

(b) The estimated variance of b2 is 0.0086.

The standard error of b2 is SQRT(Var(b2)) = 0.0927

From $\hat{Var\left(b\_{2}\right)= \frac{\hat{σ}^{2}}{\sum\_{}^{}\left(x\_{i}-\overbar{x}\right)^{2}}}$ we get 0.0086 = 3.941/SSTx (sum of squares for x). Therefore

$$\sum\_{}^{}\left(x\_{i}-\overbar{x}\right)^{2}=\frac{3.941}{.0086}=458.25$$

(c) The economic meaning of b2 = 0.57 means that as the proportion of HS grads increases by 1% then mean income among males 18 and over will increase by $570.00

(d) The intercept is b1 = 39.134 - .57(78.285) = **-5.488**

(e) For yi = 36.255 and xi = 73.139 the LS residual is 36.255 – (-5.488+.57(73.139)) = **0.05377**

(f) The state’s mean income for PMHS = 81 is -5.488+.57(81) = **40.682**

2.4 True or False

(a) Although the sum of all least squares residuals equals zero, each individual residual does not.

This is true. By construction the sum of the LS residuals is zero. Each individual could be zero only if the regression line gave a perfect fit. That is, all of the data points would have to lie on the line.

(b) Since least squares estimators are the best, the sum of squared residuals can be made equal to zero.

False. Although the LS estimators are best in the sense of minimum variance in the class of linear unbiased estimators, it does not mean that the sum of squared errors is zero.

(c) If, from an estimated regression line, the dependent variable *y* is a constant, the line is vertical.

False, the line would be horizontal.

(d) If the means of *x* and *y* are zero, the estimated *y*-intercept is zero, too.

True, since the OLS line goes through the sample means.