Economic Institutions and Stability: A Relational Approach*

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Abstract

We consider a relational economy in which economic agents participate in three types of relational economic activities: autarkic activities; binary matching activities; and plural cooperative activities. We introduce a stability notion and characterize stable interaction structures, both in the absence of externalities from cooperation as well as in the presence of size-based externalities. It is shown that institutional elements such as the emergence of socio-economic roles and organizations based on hierarchical leadership structures support and promote stable economic development.

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1 Introduction

Adam Smith showed us the importance of free markets for the welfare of the people in a nation (Smith, 1776). At that time people were restricted to a local environment and governments thought they could best protect national interest by preventing too much exchange. The market has undoubtedly liberated many people from local bonds by promoting free choice. This liberation has stimulated innovation and increased welfare. Thus, the highly specialized institution of a market—specialized as it concerns the exchange of commodities only—was very successful. This success is explained by the fact that in the 18th and 19th centuries markets made it possible for people to establish more comprehensive relations to other cultures and resources. This feature surpasses the commodity dimension usually represented in a market. Commodities from the Indies, China or Egypt, and its accompanying technical innovations connected people with other cultures and stimulated their imagination. In this paper we seek to develop a theory that is founded on these institutional dimensions.

Economists elaborating on the theory of markets emanating from Smith's seminal work, have taught us that "free" means "perfect competition", which is arrived at if no individual has any observable influence on the formation of the market price. That means a person enters a market *anonymously*, and only her anonymous willingness-to-pay is recognized and communicated. Economists have also shown that the perfectly competitive market mechanism is *amoral*: it processes what is demanded or supplied, without any moral filtering, and it accepts any initial distribution of wealth, however unjust. As argued above, this seems far removed from the underlying basic relational realities.

Furthermore, modern market institutions are designed in a way that goes well beyond the description of 18th century commodity markets. Markets are established for much more complex entities as services in the health care sector and complex securities in capital markets. There is a need for another benchmark to assess the performance of such markets. To answer that question we introduce a relational economy and derive primeval institutional properties of this relational economy. Thus, we approach the economy as a complex network of relational activities generating economic values. These foundations allow us to explore rather directly economic services as well as commodity trade as sources for the generation of such economic values. As such this approach seems an urgently needed complement to the standard neo-classical approach in exploring these issues of complexity in service economies.[1]

In particular, we aim to extend the Lancasterian approach—separating a commodity from its properties and explaining the value of a commodity from the utility of its properties (Lan-

[1] In the subsequent sections of this paper we point out several applications of this relational approach to understand many service sectors of our contemporary economies.

caster, 1966)—to the performance of economic institutions. We propose to separate these trade institutions from the basic relational framework in which these institutions are embedded and supported. Thus, a trading post explicitly emerges within a structure of inter-personal trade relations, developing into a formal market based on the price mechanism. Therefore, a trading post is viewed as a cooperative activity among related participants. We introduce a relational framework in which such cooperative activities can emerge and characterize stability properties of emerging cooperation structures. Our claim is that all specialized institutional frameworks have to meet these stability properties when performing smoothly.

Within the relational framework of value-generating activities, we focus on the stability of these activity patterns. We use standard equilibrium concepts from matching theory (Roth and Sotomayor, 1990) to describe such stable patterns. We then identify conditions on the relational structure of value-generating activities that guarantee and characterize the existence of such stable activity patterns. The identified conditions point unquestionably to institutional features. This allows us to additionally interpret economic institutions as social rules that support and guarantee generic stability in an economy.[2] Instability of such patterns, on the other hand, causes eventual undermining of the institutional superstructure of the economy.

In particular we consider a generic form of stability in our framework as a state of the fundamental structure of economic relationships such that for *every* possible configuration of individuals' productiive abilities and preferences, the economy possesses at least one equilibrium state. Within such a generically stable structure, value-generating relations are essentially transformed into anonymous exchange relations and the generation of economic values can be optimized.

In Gilles, Lazarova, and Ruys (2007) we introduced an economy in which economic agents could either be autarkic or engage in a value-generating relationship with one other agent. This resulted into a *relational matching economy*.[3] Within this framework, we introduced a stable matching pattern as a set of activated relational activities such that no agent has the incentive or opportunity to change the relational activity that she participates in. In particular, we investigated *generic stability*, the property that the economy possesses at least one stable activity pattern for any distribution of economic values. The main insight emerging from this analy-

[2]In economics, through the work of North and Thomas (1973), Williamson (1975), North (1990) and Greif (2006), institutions are usually understood as devices that lower market transaction costs. Our approach does not dismiss this interpretation, but rather amends it. Institutions also have other important functions in the economy, namely as stabilizers and promotors of economic development and growth.

[3]This setting straightforwardly extends the standard matching model in the sense of Roth and Sotomayor (1990) by introducing explicit restrictions on the permissible value-generating matchings in which an agent could participate. Thus, the main hypothesis is here that these binary matching activities are *not* anonymous. If one considers agents as persons, one could say that these matchings are formed only between persons who value their interaction, or who are "acquaintances".

sis is that generically stable matching patterns emerge *if and only if* economic agents assume complementary social roles that support the formation of their value-generating relationships. Following our discussion above, such socio-economic roles form an institutional foundation that supports and promotes stable economic development.

We extend this approach to include more broadly defined cooperative economic activities. In our relational setting we assume that such cooperative activities require a *convener*, who brings together a group of economic agents to form such a value-generating cooperative economic activity. Thus, the convener facilitates or initiates the cooperative activity.[4] In this regard these cooperative activities are relational forms of clubs as introduced seminally by Buchanan (1965). Our main assumption is that the convener can only invite economic agents to participate in a cooperative activity if they have a binary matching relationship with the convener. In other words, the convener can only form cooperative activities with acquaintances with whom she interacts.

Furthermore, the economic values generated through these cooperative activities are expressed as hedonic utilities. This is a standard technique from the theory of clubs as well. We refer to the review of Scotchmer (2002) for a discussion of this technique. It allows us to reduce the analysis of the formation of relational cooperative activities to a single dimension, expressed through the hedonic utility functions of the various economic agents.

We thus arrive at a relational economy in which economic agents can engage into three types of economic activities: autarkic activities, binary matching activities, and cooperative activities formed by a convener. Each of these three types of activities generate different hedonic utility levels for its participants. We explicitly assume that there are no widespread externalities among the various activated activities; the generated values are solely the outcome of the activities themselves. (This does not, however, exclude various forms of externalities among the members of a cooperative.)

Again, we device a standard equilibrium concept in which each agent participates in exactly one permissible economic activity. In equilibrium, no agent has an incentive to join another potentially accessible activity. Such an equilibrium is called a *stable activity pattern*. We distinguish two types of stability. Regular stability expresses a pattern of "open" cooperatives in which conveners cannot deny other agents access to the cooperative. Remarkably many trade institutions such as markets, trading posts and communal commons satisfy this openness condition. In an open cooperative, the convener is simply a coordinator. Strong stability expresses a pattern of "closed" cooperatives in which conveners have the ability to dismiss any of its members. Examples of such closed cooperatives are production teams in the sense of Marshak

[4]In this setting a market is now a cooperative economic activity in which the market auctioneer acts as its convener.

(1955). In these closed cooperatives, conveners act more like managers than coordinators.

We discuss the conditions under which such (strongly) stable activity patterns exist. The first existence result concerns economies in which there are no explicit externalities among the members of a multi-agent cooperative economic activity. Since these members can be separated, these activities are called *separable cooperatives* and exhibit rather standard properties. Examples of such separable cooperatives are trading posts and services such as religious sermons, collective insurance provision, and the standard concept of a commons. In all of these cases the participants of a cooperative activity do not affect each other directly through interpersonal externalities. If a relational economy only has separable cooperatives, we can show that there exists a strongly stable activity pattern for every hedonic utility profile *if and only if* the underlying relational structure exhibits a partial acyclicity property. In particular, relational economies with an acyclic relational structure and separable cooperatives only are generically stable.

Second, we investigate relational economies in which cooperatives are potentially non-separable in the sense that these cooperative activities might generate interpersonal externalities. Non-separable cooperative activities are prevalent in our contemporary service economies and include open-source software development and information services (e.g., Wikipedia), health care provision, and higher education. In general, the complexity of interpersonal externalities prevents us to identify conditions under which stable activity patterns can emerge. However, for size-based externalities, we can show that acyclicity of the underlying relational activity structure is sufficient to guarantee such generic stability. Standard examples of cooperatives exhibiting size-based externalities such as congestion and crowding are most highly used (semi-) public facilities such as beaches and parks on sunny summer days. Under these conditions, strong stability is, however, infeasible.

For these size-based externalities, our analysis shows that acyclicity of the structure of permissible matching activities is sufficient to establish generic stability. Again, this acyclicity condition can be interpreted as referring to a specific institutional setting of the economy. In particular, acyclicity is a feature in economies with hierarchical leadership structures. As a consequence, hierarchical leadership organizations can be viewed as integral in the establishment of stable economic development, a feature that has not yet been pointed out in the literature.

Other non-separable cooperatives might be subject to more complex externalities among its members. Such examples are health care providers and universities. These conveners bring together a team of highly trained professionals, in this case health care professionals and faculty, respectively, and a group of clients, in this case patients and students, respectively. In particular the team of professionals determines the quality and nature of the services provided through the cooperative. In these cases of more complex externalities, the existence of stable activity patterns can no longer be guaranteed.

The paper is organized as follows. Section 2 introduces our relational approach to pairwise economic cooperation. Section 3 extends this model to include multi-agent cooperative economic activities. Section 4 analyses the emergence of stable interaction patterns if there are no externalities and Section 5 discusses the implications of the introduction of certain externalities.

2 A relational approach to economic activities

In this section we introduce some basic definitions from social network theory and we establish the fundamental concepts of our relational approach to economic activities.[5] We use network relationships between economic agents to describe primitive economic interaction, denoted as economic activities. Thus, our main hypothesis is that economic activities are fundamentally relational. We pursue a theory founded on purely abstract economic relational activities, without explicit reference to other primitive concepts such as resources, commodities, or production technologies. Hence, an *economic activity* is an economic interaction between a group of economic agents that generates a hedonic utility value for each of its participants.

We emphasize that in our relational approach none of these various economic activities are provided through a standard market mechanism. Instead, the economy solely consists of these economic activities; as such our approach emphasize the cooperative nature of these economic activities. Such economic activities, of course, *include* trade: a market is then viewed as a value-generating cooperative activity. But a market is local rather than general; it is only open to its members, where potential membership is determined by the underlying structure of potential trade relationships.

Throughout we work with a finite set of economic agents denoted by $N = \{1, ..., n\}$. These economic agents can engage in three different relational economic activities that generate individual values for the participants. The first and most primitive form of economic activity is that of *economic autarky*. In this case an agent $i \in N$ is self-sufficient and generates a subsistence level of economic value for Malthusian survival.

The second level of economic activity is that of an economic *matching* in the sense that two agents i and j engage into some interaction that generates (hedonic utility) values for both of these agents. This form of relational activity is purely binary. The most basic of such an economic matching activity is that of an exchange relationship. Other forms of binary interactions can also be modelled as matchings, including master-apprentice relations, service delivery

[5]Here we refer to Jackson and Wolinsky (1996), Jackson (2003), Gilles and Sarangi (2008) and Bala and Goyal (2000) for a comprehensive overview of network theory.

from a single provider to a single client,[6] and families consisting of two parents (as the engaging agents) and young children.

The third level of economic activity is that of a (relational) *cooperative activity* in which an agent is a member of a value-generating coalition of economic agents. The generated economic value can be based on the provision of a specific club good in the sense of Buchanan (1965) or by the execution of some coalitional production activity. In our relational approach we apply a specific relational design to these cooperative activities. We emphasize that the notion of a cooperative only applies to a relational structure with three or more economic agents.[7]

We develop this theory in two states. First, we discuss economies with only autarkic and matching activities, denoted as "matching economies". Subsequently we introduce cooperative activities in the developed setting to arrive at a complete model of a "relational economy".

2.1 Relational economic activities

We first develop a network model of simple economic activities among the agents in N. For an individual economic agent $i \in N$ we denote by ii the agent's ability to engage autarkically in home production to survive on a subsistence level denoted by $u_i(ii) \geqslant 0$. Thus we arrive at the *class of all permissible autarkic activities* denoted by

$$\Gamma^1 = \{ii \mid i \in N\}. \tag{1}$$

Next, consider any $i, j \in N$ with $i \neq j$. Now we denote by the mathematical expression ij a binary economic activity involving agents i and j. The binary economic activity ij is called the *matching* of agents i and j. We remark here that $ij = ji$. Note that if $i = j$, the relational activity ii represents again the economic autarky of agent i. If $i \neq j$, then ij denotes a matching activity between the two agents i and j. Now we denote

$$\Gamma^2 \subseteq \Gamma_N = \{ij \mid i, j \in N \text{ and } i \neq j\} \tag{2}$$

such that for every $i \in N$ there is some $j \in N$ with $ij \in \Gamma^2$ as the *class of the permissible matching activities*. It should be clear that not every potential matching $ij \in \Gamma_N$ is necessarily permissible. Let Γ^2 be a given class of permissible matching activities, then $\Gamma^m = \Gamma^1 \cup \Gamma^2$ is the resulting *structure of permissible simple activities*.

[6] Such service relationships include many simple services such as hair cuts and visits to one's family physician or one's dentist.

[7] Local markets are exactly examples of such cooperative economic activities. From this perspective, a local market is simply a group of traders, who gather around some auctioneer, who conducts the trading processes among the gathered members.

A permissible matching activity $ij \in \Gamma^2$ has a purely potential nature in the sense that the participants in the activity ij have to decide to willingly participate in this activity before it is realized. Since Γ^2 is a subset of the set of all possible pairings Γ_N, it is designed to capture physical, institutional, or any other constraints that may prohibit the occurrence of economic matching activities between certain agents. In this regard the structure Γ^2 reflects the *relational trust* that is present among the agents in the economy. These restrictions, however, cannot preclude opting out of the engagement in any economic relationship with others in the sense that each $i \in N$ can always decide to initiate her autarkic activity $ii \in \Gamma^1$. (Such a restriction would reflect a form of slavery or serfdom.)

Hence, it is essential that the class of permissible autarkic activities Γ^1 is taken into account fully. In terms of our application one can think of the pair (Γ^1, Γ^2) as the foundation of a social activity structure in the economy. This introduces the permissible simple activity structure Γ^m as defined. Indeed, as stated earlier we refer to an activity as "simple" if it is either an autarky or a matching activity; this is exactly captured by the structure Γ^m.

For any sub-structure $H \subseteq \Gamma^2$ we denote

$$N(H) = \{i \in N \mid \text{There is some } j \in N \text{ such that } ij \in H\} \tag{3}$$

as the set of economic agents that participate in the matching structure H. Clearly, for every $H \subseteq \Gamma^2$, if $H \neq \varnothing$, then $N(H) \neq \varnothing$.

We first discuss some graph theoretic concepts that address the description of features of any sub-structure of permissible matching activities $H \subseteq \Gamma^2$.

We define a *path* between any two distinct agents $i \in N$ and $j \in N$ in $H \subseteq \Gamma^2$ as a sequence of distinct agents $P_{ij}(H) = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = j$, $i_k \in N$ and $i_k i_{k+1} \in H$ for all $k \in \{1, \ldots, m-1\}$. We define a *cycle* in H to be a path of an agent from herself to herself which contains at least two other distinct agents, *i.e.*, a cycle in H from i to herself is defined as a path $C = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = i$, $m \geqslant 4$, $i_k \in N$, and $i_k, i_{k+1} \in H$ for all $k \in \{1, \ldots, m-1\}$. The length of the cycle C is denoted by $\ell(C) = m - 1 \geqslant 3$.

A sub-structure $H \subseteq \Gamma^2$ is called *acyclic* if H does not contain any cycles. A sub-structure $H \subseteq \Gamma^2$ is called *odd-acyclic* if H does not contain any cycles $C \subset H$ of an uneven length $\ell(C)$.

Furthermore, there may be agents in N between whom there is no path in a set of permissible matchings Γ^2; such agents are located in different components of the structure Γ^2. These components are exactly the maximally connected sub-structures within Γ^2. A sub-structure $H \subseteq \Gamma^2$ is a *component* of Γ^2 if (1) for all $i \in N(H)$ and $j \in N(H)$ there is a path $P_{ij}(H)$ connecting agents i and j and (2) for all $i \in N(H)$ and $j \in N(H)$, $ij \in \Gamma^2$ implies that $ij \in H$. The *set of* *all components* in a network Γ^2 is denoted by $\mathsf{c}(\Gamma^2) = \{H \mid H \subseteq \Gamma^2 \text{ is a component }\}$. Note that as a consequence of the requirements in the definition of a permissible matching structure, $\Gamma^2 = \cup_{H \in \mathsf{c}(\Gamma^2)} H$.

The location of a player within a network is an important characteristic. Let Γ^2 be some permissible matching structure and let $G \subseteq \Gamma^m = \Gamma^1 \cup \Gamma^2$ be some structure of permissible activities. The *set of connected agents* in structure G is denoted by

$$N(G) = \{i \in N \mid \text{ There is some } j \neq i \text{ with } ij \in G\}. \tag{4}$$

Obviously from the definitions it holds that $N(\Gamma^m) = N(\Gamma^2) = N$.

Furthermore, we define agent i's *neighborhood* in G as $N_i(G) = \{j \in N \mid ij \in G\}$. Note here that if $i \in N_i(G)$, agent i's autarkic activity ii is listed in G, i.e., $ii \in G$. Also, by the definition of a permissible matching structure, it holds that $N_i(\Gamma^2) \neq \varnothing$ for any $i \in N$. We can also express the neighborhood of an agent within an arbitrary structure $G \subset \Gamma^m$ in terms of its link based analogue, *i.e.*, $L_i(G) = \{ij \in G \mid j \in N_i(G)\} \subset G$. For example, $L_i(\Gamma^m) = \{ii\} \cup L_i(\Gamma^2)$ is the set of permissible simple activities that agent i can potentially participate in.

2.2 Institutions and stability in matching economies

Thus far we introduced autarkic and matching activities. Based on these "simple" economic activities Gilles, Lazarova, and Ruys (2007) introduced the notion of a matching economy. We present the main insights from this analysis.

Throughout we assume that every individual $i \in N$ has complete and transitive preferences over the permissible simple activities $L_i(\Gamma^m) \subset \Gamma^m = \Gamma^1 \cup \Gamma^2$ in which she can engage. Thus, by finiteness of Γ^m, these preferences can be represented by a *hedonic utility function* given by $u_i^m : L_i(\Gamma^m) \to \mathbb{R}$. Let $u^m = (u_1^m, \ldots, u_n^m)$ now denote a *hedonic utility profile*.

Definition 2.1 *A* ***matching economy*** *is defined as a triple $\mathbb{E}^m = (N, \Gamma^m, u^m)$ in which N is a finite set of individuals, $\Gamma^m = \Gamma^1 \cup \Gamma^2$ is a permissible simple activity structure on N, and $u_i^m : L_i(\Gamma^m) \to \mathbb{R}$, $i \in N$, is a hedonic utility profile on Γ^m.*

Within the context of a matching economy we investigate stability of an interaction pattern as the main concept describing an allocation of activities in a matching economy. The main hypothesis in the definition of stability is that in a matching economy $\mathbb{E}^m$ each individual $i \in N$ activates *exactly one* of her permissible matchings in $L_i(\Gamma^m)$. This hypothesis is founded on the fact that a matching economy is not endowed with the presence of advanced economic or social institutions. In such a primitive setting—in which one problem is addressed at a time—it seems natural to assume that individuals only interact with a single other individual at a time.

Definition 2.2 *A **matching pattern** in a matching economy $\mathbb{E}^m$ is a subset of the permissible activity structure $\pi^m \subset \Gamma^m$ such that every individual is either paired with exactly one other individual or remains relationally autarkic, i.e., $\pi^m \subset \Gamma^m$ is such that $|L_i(\pi^m)| = |N_i(\pi^m)| = 1$, for all $i \in N$.*
We denote by $\Pi^m(\Gamma^m) = \Pi^m$ the class of all permissible matching patterns within Γ^m.

We remark that by the applied hypotheses and definitions, the set of permissible matching patterns is non-empty. In particular, $\Gamma^1 \in \Pi^m \neq \emptyset$. Moreover, according to the assumptions on Γ^2, there exists a sufficient number of other matching patterns in which agent $i \in N$ is engaged with some other agent $j \neq i$; indeed, there is some $\pi \in \Pi^m$ with $ij \in \pi$ for any $ij \in \Gamma^2$.

Stability can now be introduced on a matching pattern and is based on the standard assumptions of Individual Rationality and a no-blocking condition from matching theory (Roth and Sotomayor, 1990). Here we refer to this no-blocking condition as "pairwise stability" with reference to stability in network formation theory, seminally introduced by Jackson and Wolinsky (1996). This is summarized in the following definition of matching stability:

Definition 2.3 *A matching pattern $\pi^m \in \Pi^m$ is **stable** in the matching economy $\mathbb{E}^m = (N, \Gamma^m, u^m)$ if all matchings in π^m satisfy the **individual rationality** (IR) and **pairwise stability** (PS) conditions:*

IR *$u_i^m(\pi^m) \geqslant u_i^m(ii)$ for all $i \in N$, where $u_i(\pi^m) = u_i(ij)$ with $ij \in \pi^m$ denotes the utility of agent i from her unique activity ij in which she participates in the matching pattern π^m and*

PS *there is no blocking matching with regard to π^m, i.e., for all $i, j \in N$, $i \neq j$, with $ij \in \Gamma^m \setminus \pi^m$:*

$$u_i^m(ij) > u_i^m(\pi^m) \text{ implies that } u_j^m(ij) \leqslant u_j^m(\pi^m). \tag{5}$$

For an economy to have persistent access to gains from organization, the social structure of the economy has to *generically* admit stable matchings. Hence, whatever capabilities and desires of the individuals—represented by their (hedonic) utility functions and (possibly) other individualistic features—a stable matching pattern has to exist in the matching economy.

Definition 2.4 *A structure of permissible simple activities Γ^m on N is a **generically stable** if for every hedonic utility profile u^m on Γ^m there exists at least one stable activity pattern in the matching economy $\mathbb{E}^m = (N, \Gamma^m, u^m)$.*

The term "generic" refers here in principle to a state of socio-economic organization in the matching structure Γ^m such that for every pattern of individual capacities—represented by the hedonic utilities u^m—there emerges at least one stable matching pattern. Such a generically stable matching structure thus reflects that the organizational and institutional structure of the economy supports stability regardless of the exact individual preferences of the agents in the economy, i.e.; in this regard it reflects a "healthy" relational structure of the economy, which promotes and enhances the economic activities selected by the agents in the economy.

The next result identifies the conditions for generic stability.

Theorem 2.5 (Gilles, Lazarova, and Ruys, 2007, Corollary 5.4)
A structure of simple activities $\Gamma^m = \Gamma^1 \cup \Gamma^2$ on N is generically stable if and only if the matching structure Γ^2 is bipartite in the sense that there exists a partitioning $\{N_1, N_2\}$ of N such that

$$\Gamma^2 \subseteq N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}. \tag{6}$$

For a complete proof of this result we refer to Gilles, Lazarova, and Ruys (2007).

The generic existence result Theorem 2.5 has a clear interpretation. Any generically stable matching structure has to be based on two socio-economic roles such that economic matching activities can only occur between pairs of agents of different roles. These roles can be indicated as "demand" and "supply" with reference to an interpretation of Γ^2 as a (general) matching market.

3 The relational economy

Next we expand the scope of our model to include cooperation among multiple economic agents. These cooperative economic activities essentially represent collectives through which economic agents interact taking into account the underlying structure of economic relationships. In this regard a cooperative activity can be viewed as a club in the sense of Buchanan (1965) that is formed between matched individuals only.

Formally, consider a structure of permissible matching activities Γ^2 on agent set N. Now, a cooperative is understood as a certain combination of permissible matchings in Γ^2. This is founded on the idea that agents base their interactions on the simplest building blocks, their relationships. We assume that a cooperative economic activity is formed around some "convener". The idea is that a convener brings together a number of economic agents with whom she already has an established economic relationship—in the form of a permissible matching. This is formalized as follows:

Definition 3.1 *Let $\Gamma^2 \subseteq \Gamma_N$ be a permissible matching structure on N. A* ***cooperative activity****—or simply a "cooperative"—is defined as a sub-structure $G \subseteq \Gamma^2$ of permissible matchings such*

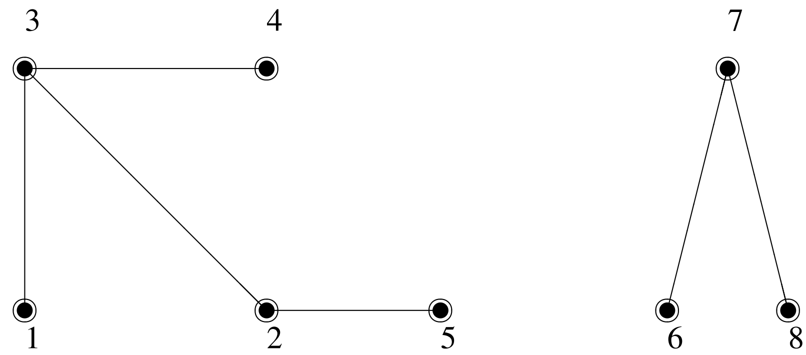

Figure 1: An example of a permissible matching structure

that $|N(G)| \geqslant 3$ and there is a unique agent $i \in N(G)$ such that $N_i(G) = N(G) \setminus \{i\}$ and that for all other agents $j \in N(G) \setminus \{i\}$ it holds that $N_j(G) = \{i\}$. The agent i is called the ***convener*** *of the cooperative G and denoted by $N^\star(G) \in N(G)$.*

A cooperative is constituted of a set of permissible matchings and as such it is seen as a combination of permissible relational activities. Our definition of a cooperative imposes that a cooperative has at least three member agents.[8] Furthermore, a cooperative has an explicit star structure in the network of permissible matching activities Γ^2. This implies that the cooperative has a relational center, representing a "middleman" or "convener" who essentially binds all constituting matchings of the cooperative. In particular, all matchings that constitute a cooperative are controlled by its convener.

To illustrate this important definition consider the agent set $N = \{1, \dots, 8\}$ and the permissible matching structure $\Gamma^2 \subset \Gamma_N$ depicted in Figure 1.

The permissible matching structure Γ^2 contains two components given as 12345 and 678. Here the permissible matching activities in this structure are given by $\Gamma^2 = \{13, 34, 23, 25, 67, 78\}$. To illustrate the notion of a cooperative, we refer to $G_1 = \{23, 25\}$ and $G_2 = \{13, 23, 34\}$ as two examples of such permissible cooperatives. Here the conveners of these cooperatives are given by $N^\star(G_1) = 2$ and $N^\star(G_2) = 3$.

Using the definition of cooperative activities, we can introduce some auxiliary concepts.

Definition 3.2 *Let $\Gamma^2 \subseteq \Gamma_N$ be some permissible matching structure.*

(a) *The collection of all (potential) cooperative activities is now defined by*

$$\Sigma(\Gamma^2) = \{G \mid G \subset \Gamma^2 \text{ is a cooperative}\} \tag{7}$$

[8]This distinguishes cooperatives from regular matching activities, $ij \in \Gamma^2$, consisting of two matched agents only.

A collection $\Gamma^3 \subseteq \Sigma(\Gamma)$ *is denoted as a* ***permissible cooperative structure*** *on the structure of permissible matching activities* Γ^2.

A triple $(\Gamma^1, \Gamma^2, \Gamma^3)$ *is referred to as a* ***permissible activity structure*** *on N consisting of all autarkies* $G_1 \in \Gamma^1$*, permissible matchings* $G_2 \in \Gamma^2$*, and permissible cooperatives* $G_3 \in \Gamma^3$*. The union of a permissible activity structure,* $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Gamma^3$*, serves as its alternative description.*

(b) *Let* $\Gamma^3 \subseteq \Sigma(\Gamma^2)$ *be some permissible cooperative activity structure. The* ***set of conveners*** *in* Γ^3 *is defined by*

$$N^\star(\Gamma^3) = \{i \in N \mid i = N^\star(G) \text{ for some } G \in \Gamma^3\}. \tag{8}$$

In Figure 1 a permissible cooperative structure can be selected and listed[9] as

$$\Gamma^3 = \{3124, 342, 768\} \subsetneq \Sigma(\Gamma^2) = \{235, 3124, 312, 324, 314, 768\}.$$

The set of conveners in Γ^3 can now be computed as

$$N^\star(\Gamma^3) = \{i \in N \mid i \text{ is the convener of some cooperative } G \in \Gamma^3\} = \{3, 7\} \subset N.$$

Let $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Gamma^3$ be a permissible economic activity structure on the agent set N. An agent $i \in N$ has complete and transitive preferences over the activities in which he or she potentially can participate. We assume that these preferences can be represented by a hedonic utility function. The hedonic utility function is in principle an *indirect* utility function that captures the utility of the value generating activities.

Let $\mathcal{A}_i(\Gamma)$ be the set of all permissible activities in which agent i participates. Formally, for $i \in N$ we introduce

$$\mathcal{A}_i(\Gamma) = \{ii\} \cup \{ij \mid ij \in \Gamma^2\} \cup \{G \mid G \in \Gamma^3 \text{ is a cooperative and } i \in N(G)\}. \tag{9}$$

For every economic agent $i \in N$, preferences are now introduced through the hedonic utility function $u_i \colon \mathcal{A}_i(\Gamma) \to \mathbb{R}$. Let $u = (u_1, \ldots, u_n)$ be a profile of hedonic utility functions for all agents in N. Let $\mathcal{U}$ be the set of all profiles of hedonic utility functions on Γ.

We now have introduced the fundamental notions needed to define a relational economy with cooperative economic activities. Such an economy is defined to be the set of permissible

[9]Here we use shorthand notation in the form of triplets ijh to denote a permissible cooperative consisting of the three agents i, j and h, where i acts as its convener. Similarly we use the quadruplet $ijhk$ to describe a four-agent cooperative with convener i.

relational activities (autarky, matchings, and cooperatives) and a hedonic utility function that assigns a value to every agent in each of these permissible activities. This is formalized as follows.

Definition 3.3 *A **relational economy** is introduced as a triple $\mathbb{E} = (N, \Gamma, u)$ in which N is a finite set of economic agents, $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Gamma^3$ is a permissible activity structure on N, and $u \in \mathcal{U}$ is a profile of hedonic utility functions with for every $i \in N$ that $u_i \colon \mathcal{A}_i(\Gamma) \to \mathbb{R}$.*

Finally we address an equilibrium concept for such relational economies. To analyze stability we again adapt the notion of pairwise stability in the same fashion as formalized for matching economies in Definition 2.3.

Definition 3.4 *Let $\mathbb{E} = (N, \Gamma, u)$ be a relational economy and let $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Gamma^3$ be the corresponding permissible activity structure.*

(a) *A listing $\Lambda = (G_1, \ldots, G_m)$ is an **activity pattern** in Γ if $G_p \in \Gamma$ for all $p \in \{1, \ldots, m\}$, $N(G_p) \cap N(G_q) = \varnothing$ for all $p \neq q$, and $\cup_{p=1}^{m} N(G_p) = N$. (Noting that $N(ii) = \{i\}$ for all $ii \in \Gamma^1$.) We denote the family of all activity patterns in Γ by $\mathcal{P}(\Gamma)$.*

(b) *The activity pattern $\Lambda^\star = (G_1^\star, \ldots, G_m^\star) \in \mathcal{P}(\Gamma)$ is **stable** in the economy $\mathbb{E}$ if for every $p \in \{1, \ldots, m\}$ the activity $G_p^\star \in \Lambda^\star$ satisfies the* individual rationality *[IR] and two* pairwise stability *conditions [PS] and [PS*] as specified below:*

IR *for all $i \in N(G_p^\star)$ it holds that $u_i(G_p^\star) \geqslant u_i(ii)$;*

PS *for all distinct agents $i \in N(G_p^\star)$ and $j \in N(G_q^\star)$ with $q \in \{1, \ldots, m\}$ and $ij \notin \Gamma^2 \cap G_p^\star$:*

$$u_i(ij) > u_i(G_p^\star) \quad \textit{implies} \quad u_j(ij) \leqslant u_j(G_q^\star); \tag{10}$$

PS* *and for all distinct agents $i \in N(G_p^\star)$ and $j \in N(G_q^\star)$ with $q \in \{1, \ldots, m\}$ with $ij \in \Gamma^2$, $ij \notin G_p^\star \cap G_q^\star$ and either $j \in N^\star(G_q^\star)$ or $G_q^\star \in \Gamma^2$:*

$$u_i(G_q^\star \cup \{ij\}) > u_i(G_p^\star) \quad \textit{implies} \quad u_j(G_q^\star \cup \{ij\}) \leqslant u_j(G_p^\star). \tag{11}$$

(c) *The activity pattern $\Lambda^\star = (G_1^\star, \ldots, G_m^\star) \in \mathcal{P}(\Gamma)$ is **strongly stable** in the economy $\mathbb{E}$ if $\Lambda^\star$ is stable in $\mathbb{E}$ and for every $p \in \{1, \ldots, m\}$ the activity $G_p^\star \in \Lambda^\star$ satisfies additionally the following* Reduction Proofness *condition [RP]:*

RP *If $G_p^\star$ is a cooperative economic activity, i.e., $G_p^\star \in \Gamma^3 \cap \Lambda^\star$, it holds that for every sub-structure $G \subset G_p^\star$*

$$u_i(G) \leqslant u_i(G_p^\star) \tag{12}$$

where $i = N^\star(G^\star_p)$ is the convener of that cooperative economic activity.

An activity pattern is an assignment of exactly one activity to every economic agent. As for the case of matching economies, it is again assumed that agents participate only in a single activity. An activity pattern is defined to be stable if it satisfies certain standard stability conditions from game theory, in particular matching theory (Roth and Sotomayor, 1990), network formation theory (Jackson and Wolinsky, 1996), and core theory for Tiebout and club economies (Gilles and Scotchmer, 1997).

The condition IR is a standard individual rationality condition that allows an individual to opt out of an economic activity if she is better off being autarkic. The first pairwise stability condition PS rules out blocking opportunities for pairs of agents who are not connected to each other in the same cooperative. It requires that there are no pairs of such agents who prefer to be linked to each other rather than to the agents with whom their are linked in the present activity pattern. Condition PS has already been applied in Definition 2.3 of a stable matching pattern for matching economies.

The second pairwise stability condition PS* rules out blocking opportunities for pairs of agents at least one of whom can add a link without severing his existing links in the present activity pattern. Hence, such an agent is either a convener in the present pattern or is linked in a matching with another distinct agent and not member of a cooperative. This condition requires that there are no two distinct agents who want to be linked to each other in a cooperative in which one of them is a convener.[10]

Both PS and PS* are concerned with the re-structuring of the prevailing activity pattern. These conditions still do not allow the convener of a cooperative to block access to this cooperative by third parties if it is to their gain. Hence, stability of an activity pattern defines a notion of cooperatives that are principally "open" in the sense that any outsider can join the cooperative. There are numerous cooperative activities that satisfy the principle of openness such as trading posts (stores) and markets, open source communities, and many economic service provision cooperatives (clubs). In most of these cases, if entrants follow the house rules of the cooperative in question, they will not be excluded from participation.

The stronger notion of strong stability excludes the possibility of open cooperative activities. Condition RP explicitly "closes" a cooperative in the sense that the convener is allowed to exclude participation of third parties based on her own preferences. In economic practice we encounter many of such closed cooperatives as well. We mention many team production situations (e.g., health care provision) and exclusive clubs (guilds and unions).

[10]Note that a convener and an agent linked in a matching with another distinct agent have multiple blocking opportunities available: such agents can add a link with or without severing their current links. Such agents are subject to both (no blocking) conditions PS and PS*.

Under (regular) stability, a convener is merely a coordinator of a cooperative economic activity who is open to participation, while under strong stability a convener is considered to be a manager of the cooperative activity under consideration. We emphasize that strong stability implies stability, i.e., management implies coordination, but that the reverse is not true.

4 Separability: The absence of externalities

After having established a theory of relational economic activities, we investigate the existence of equilibria in these economies in the form of stable activity patterns. We have to distinguish two types of relational economies: economies with relational externalities affecting the performance of cooperatives and economies without such relational externalities. We first investigate economies without relational externalities. Such economies are denoted as “separable”.

Definition 4.1 *Let $\mathbb{E} = (N, \Gamma, u)$ be a relational economy.*

(i) *The hedonic utility function $u_i : \mathcal{A}_i(\Gamma) \to \mathbb{R}$ **exhibits no (cooperative) externalities** if for all $G_i \in \mathcal{A}_i(\Gamma)$ and $H_i \in \mathcal{A}_i(\Gamma)$ with $N_i(G_i) = N_i(H_i)$, it holds that $u_i(G_i) = u_i(H_i)$.*
The collection of all utility profiles exhibiting no externalities is denoted by $\mathcal{U}_n \subset \mathcal{U}$.

(ii) *The relational economy $\mathbb{E} = (N, \Gamma, u)$ is **separable** if $u_i \in \mathcal{U}_n$ for every agent $i \in N$.*

The non-externality property on a hedonic utility function imposes that an agent derives value only from matchings with agents with whom she is linked *directly*. Thus, changes in cooperatives regarding third parties do not affect the hedonic utility value of a member of that cooperative. Although this seems to be a very severe condition, it is a common assumption in traditional public economics, where the public good itself acts as a convener in our terms.[11]

Besides separability, we introduce a second property that hedonic utility functions have to satisfy. This is the standard superadditivity condition.

Definition 4.2 *Let Γ be a permissible activity structure on N. For agent $i \in N$, the hedonic utility function $u_i : \mathcal{A}_i(\Gamma) \to \mathbb{R}$ is **superadditive** if for any $G_i \in \mathcal{A}_i(\Gamma)$ and $H_i \in \mathcal{A}_i(\Gamma)$ with $G_i \cup H_i \in \mathcal{A}_i(\Gamma)$ and $G_i \cap H_i = \varnothing$ it holds that $u_i(G_i \cup H_i) \geqslant u_i(G_i) + u_i(H_i)$.*
Furthermore, we say that a utility profile $u \in \mathcal{U}$ on Γ is superadditive if the hedonic utility function u_i is superadditive for every agent $i \in N$. The collection of all superadditive utility profiles is denoted by $\mathcal{U}_s \subset \mathcal{U}$.

[11] In this regard if all cooperatives exhibit such non-externalities towards its members, the activities represented through these cooperatives are separable and, thus, can in principle be evaluated objectively. This is the principle of pricing membership of clubs in a club economy (Gilles and Scotchmer, 1997), or the Samuelson conditions in the efficient provision of a pure public good (Samuelson, 1954).

The superadditivity property reflects synergies which are assumed to be allocated to the central agent who acts as a coordinator in the value generation process.

4.1 Generic stability under separability

Within the context of separable relational economies we address the generic existence of stable activity patterns. This refers to the existence of such a stable activity pattern for any hedonic utility profile exhibiting no externalities. Formally, we define the permissible activity structure $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Gamma^3$ to be generically stable if it admits a stable activity pattern for every permissible hedonic utility profile u on Γ.

Definition 4.3 *Let $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Gamma^3$ be a permissible activity structure such that $\Gamma^3 = \Sigma(\Gamma^2)$ and let $\mathcal{U}^\star \subseteq \mathcal{U}$ be some given class of permissible utility profiles on the permissible activity structure Γ. The permissible activity structure Γ is* ***generically (strongly) stable*** *on the class $\mathcal{U}^\star$ if for every utility profile $u \in \mathcal{U}^\star$ there exists a (strongly) stable activity pattern $\Lambda^\star$ in the relational economy $\mathbb{E} = (N, \Gamma, u)$.*

We denote by $\overline{\mathcal{U}} = \mathcal{U}_s \cap \mathcal{U}_n$ the class of all hedonic utility profiles that satisfy the superadditivity as well as the non-externality properties.

Theorem 4.4 *The permissible activity structure $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Gamma^3$ with $\Gamma^3 = \Sigma(\Gamma^2)$ is generically strongly stable on the class $\overline{\mathcal{U}}$ of superadditive hedonic utility profiles exhibiting no externalities if and only if the permissible matching structure Γ^2 is acyclic or only contains cycles $C \subset \Gamma^2$ with length $\ell(C) = 3s$ where $s \in \{1, 2, \dots\}$.*

The proof of Theorem 4.4 is given in Appendix A.

Theorem 4.4 states that under some regularity conditions, a permissible activity structure is generically strongly stable for hedonic utility profiles without externalities if and only if the relational structure exhibits a certain acyclicity property. Unfortunately, the partial acyclicity condition on the permissible activity structure stated in the assertion has no convenient or obvious interpretation, in contrast to the condition stated in Theorem 2.5. However, from Theorem 4.4 we may derive some more directly interpretable conclusions. In particular, if the relational structure is acyclic, then the permissible activity structure is generically stable for utility profiles exhibiting no externalities.

Corollary 4.5 *If the permissible matching structure Γ^2 is acyclic, then the permissible activity structure $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Gamma^3$ with $\Gamma^3 = \Sigma(\Gamma^2)$ is generically strongly stable on the class $\overline{\mathcal{U}}$ of superadditive hedonic utility profiles exhibiting no externalities.*

One particular interesting class of acyclic permissible matching structures is that of the *hierarchical* structures. Within a hierarchical structure, multiple levels can be distinguished in which agents in a certain level can only communicate with agents in lower and higher levels. It is well-accepted that hierarchical structures are common institutional features of any contemporary society. In particular, social roles are usually assigned to correspond to the various levels within the hierarchical power structure in the economic and political sphere of a society.

The main conclusion from the assertion stated in the corollary is that if a society is hierarchically structured, it is generically strongly stable. In this regard a hierarchical organization structure is a "mode of governance" and as such the corresponding social role and authority patterns steer the society towards a (strongly) stable state. As such, a hierarchical organization of a society supports and promotes economic development and stability.

4.2 Examples of separable cooperatives

We illustrate the abstract theoretical discussion with some practical examples of cooperatives that exhibit the non-externality condition imposed above. This implies that such cooperatives are not affected by size and other third-party externalities for the value extracted by members other than the convener. A surprising large number of institutions in our contemporary economies satisfy this rather strict condition. We discuss some of these in no particular order and compare the standard view with our relational approach.

Church service. A church organization, when restricted to its traditional services, is a prime example of a separable and open cooperative. Here, agents are natural persons only. The pastor of the church acts as its convener, while the parishioners obtain economic values only from a direct interaction with the pastor. In this reduced approach the value of a church visit of a parishioner only emanates from the sermon delivered by the pastor. In this regard a church is thus a cooperative in which there are no direct externalities among the parishioners. It rarely happens that parishioners are excommunication; if parishioners submit to the rules of conduct, they will not be excluded from the services. As such a church is an open cooperative, i.e., its convener (pastor) does not explicitly exclude members from participation even if the pastor obtains a lower hedonic utility due to their participation.

We contrast this view with the traditional perspective of the church as a firm (Ekelund, Hébert, and Tollison, 1989). In this view churches provide services in a particular market and many activities of parishioners are interpreted from a purely market perspective. For example, cathedral building can be interpreted as an entry deterrent (Bercea, Ekelund, and

Tollison, 2005). However, cathedral building can also be interpreted as a core activity of the relational cooperative that forms the church-going community.

The commons. The standard view of a commons is that it is a local public good provided to the members of a community. Examples are public parks, infrastructure, and local government agencies. For a traditional theoretical approach to the problem of the provision of a commons we refer to, e.g., Falk, Fehr, and Fischbacher (2001).

Here we argue that the relational view defining a commons as a separable and open cooperative is required for stability in the traditional theory of the commons problem. First, in our approach the commons is represented as an economic cooperative. Indeed, the convener is simply the local authority that provides a common service and the members of the cooperative are the users of that service. (In this regard, the convener may also be an appointed or elected official.) Only members of the local community have potential access to the commons, and as such this can be represented in the permissible matching structure Γ^2, where $ic \in \Gamma^2$ if i is a member of the community c. (Here c represents the convener for the provision of the commons.) The decision to participate is made solely by the community member; the convener will only deny access to the commons if there is misconduct. Thus, each community member decides to participate or not after receiving institutional information about the convener and the quality of the service provided.

The cooperative formed by c and certain community members $i \in N_c(\Gamma^2)$ is separable under the standard assumptions. The number of actual users of the commons is usually not assumed to affect the utility enjoyed by a member i who uses the commons.

Second, the main difference with the standard view of a commons as a pure public good is the reciprocity of the relational structure. A commons generates (directed) utility from the convener as the maintainer of that commons, and allows for free-riding if it is not managed satisfactorily. As such, free-riding is a consequence of managerial qualities in the cooperative.

Insurance provision. A special case of a separable commons provision is that of financial loss insurance. In this case the insurance provider is the convener of the cooperative in question and its members are the policy holders. The policy holders bring in capital through their contributed premiums to cover potential financial losses of certain policy holders from calamities.

First, we argue that the collective of an insurance provider and her clients can be described as a separable cooperative. The direct relation between the insurance provider and a client is focused on the characteristics of both of them, as expressed by the insurer's policy and the member's risks profile. Their relation may be a unique one, based on trust or private information, but it is a separable cooperative only as far as the member's activities do not influence the provider's policy towards all other members.

Second, according to the standard view, the insurer's policy is a given offer—comparable to offers by other insurance companies—to a market of anonymous insurance searchers. An insurance searcher selects subjectively the best offer. Both parties know that they can act strategically, but this behavior is not typical for a specific insured party, it applies to all. The neoclassical framework ignores the role of non-market social interactions in determining individual and collective behavior and in shaping economic and social outcomes.[12] This standard view compares to the relational view in that both are separable, but the standard view implies additional requirements, in particular the anonymity of the parties' properties. These extra requirements facilitate institutional arrangements, of course, such as the market mechanism.

Third, these additional conditions may not be fitting with other cultures. Such societies have sought solutions that fit the relational framework rather than the traditional viewpoint. We discuss the local solutions, as is suggested by the example of the *Mahber* system in East Africa.

A Mahber is an informal association where members of non-kinsmen participate in communion. Each member of the Mahber makes a periodic contribution of a specific amount of money and benefits are paid out to members in money, or in kind, in the event of the loss of a job, an accident, illness or death. Mahber provides also financial credits to its members at times when they face serious crises. The logic of joining Mahber has more to do with reciprocity and credit systems in that everyone expects to receive benefits sometime during her/his membership.

The question addressed in Habtom and Ruys (2007) is whether—in the absence of health insurance and other formal safety nets—the traditional Eritrean community can transform and extend the traditional Mahber system to cover also unexpected costs of health care and related social costs. To answer this question four central aspects of social capital are used in the analysis: relations of trust; reciprocity and exchanges; common rules, norms and sanctions; connectedness, networks and groups. Our concept of a separable cooperative fits this type of analysis.

Trading posts. Trading posts are important elements of the trade infrastructure in our contemporary economies. These trading posts usually assume the form of stores, shopping malls, and local markets. We argue here that such a trading post can be viewed as a separable

[12] In the real world, there are many examples of how individual and collective behavior is shaped by non-market social influences in the form of culture, norms, and social structure. The central idea of social capital reflects this since this concept incorporates that social networks are valuable assets (Bourdieu and Wacquant, 1992).

as well as open cooperative. The convener is the owner of the trading post—in the form of a store or local market. She announces which commodities will be traded and initiates contacts with potential providers of these commodities. Demanders for these commodities join the trading post and indirectly trade with the providers of these commodities. The trading post is "open" in the sense that any potential customer has access; the convener will rarely deny access to the trading post. In principle there are no externalities assumed in this trading process and, thus, there are no externalities emanating from the interactions between the various members of the trading post.

Furthermore, this description of a trading post explicitly puts it into the context of a larger relational context. The convener is assumed to have explicit permissible relationships with the providers and demanders will only access those trading posts that are available in their locality. As such the trade infrastructure has an explicit spatial dimension, which is described in the permissible matching structure Γ^2.[13]

The relational approach to trade infrastructure also conforms with relationship-building of a retailer with its clients through information gathering regarding the purchasing behavior of each client. This is now standard practice for online retailers such as Amazon.com and for supermarket retailers through a system of "bonus cards". In these cases the retailer-client relationship is strengthened through targeted advertising and promotions.

The examples of separable cooperatives are only a few of the numerous economic institutions and structures that have been devised throughout history to facilitate certain economic activities. From our discussion it should be clear that we view these cooperatives as semi-public entities and as such the club nature of these cooperatives is emphasized.

5 Non-separability: Introducing externalities

Next we address the question under which conditions stable activities patterns emerge in the presence of externalities. From our analysis we conclude that different types of externalities result in different stability requirements. Consequently a varied picture emerges in which a case-based approach is more fruitful than a general one.

We investigate two prevalent types of externalities related to cooperative economic activities. The first one is a simple size-based formulation of externalities. The more members a

[13]For elaborations on these points of trading post owners as conveners or "middlemen" in a trade process or infrastructure we also refer to Bose and Pingle (1995) and Rubinstein and Wolinsky (1987). We refer to Diamantaras, Gilles, and Ruys (2003) and Gilles and Diamantaras (2003) for further discussion and modeling of the provision of such trade infrastructures as a collective good problem.

cooperative has, the more it affects the resulting value for its members. Such size-based externalities are very common as every bounded facility is subject to crowding. In the literature on Tiebout and club economies such crowding externalities have been investigated extensively. We refer here to the seminal paper by Conley and Wooders (1997) and the subsequent work by Conley and Konishi (2002). For the case of size-based externalities we are able to state a rather general result concerning existence of stable activity patterns.

A second type of externalities are the ones based on the hedonic utility values generated. Here the externality is related to the difference of the convener's extracted value from the cooperative and all its other members. This refers to the notion of envy and equal treatment as the underlying source of such externalities. For a general theoretical treatment of such externalities we refer to the seminal paper Fehr and Schmidt (1999). Unfortunately, even for rather specific formulations of such value-based externalities, we cannot provide a general statement concerning the existence of stable activity patterns.

5.1 Size-based externalities

For utility profiles with size-based externalities, the number of agents in a cooperative is determining the size of the externality. The identity of the convener of the cooperative determines whether the externality is positive or negative, but the identity of the remainder of the cooperative membership is irrelevant for the amount of externality generated.

Definition 5.1 *Let $\mathbb{E} = (N, \Gamma, u)$ be a relational economy with $\Gamma^3 = \Sigma(\Gamma^2)$. Then the utility function u* ***exhibits a (linear) size-based externality*** *if for every cooperative $G \in \Gamma^3$:*

$$u_i(G) = \sum_{j \in N_i(G)} u_i(ij) + \alpha_c \cdot [\,\#N(G) - 2\,] \tag{13}$$

for all $i \in N(G)$, where $c = N^*(G)$ and $\alpha_c \in \mathbb{R}$.

If a convener c has an externality parameter $\alpha_c > 0$, she brings about a positive externality in the cooperative. This refers to "economies to club size" based on the total size of the cooperative gathered around this convener. If, on the other hand, this convener has an externality parameter $\alpha_c < 0$, she causes a negative externality in the cooperative. This can be referred to as "crowding" (Conley and Wooders, 1997).

First we report that there exist relational economies exhibiting size-based externalities in which there is no stable activity pattern. An example is presented below.

Example 5.2 Let $N = \{1, 2, 3, 4\}$ and $\Gamma^2 = \{12, 23, 34\}$. Let $\alpha_2 = 200$ and $\alpha_3 = -50$. Let the utility function be such that $u_1(12) = u_2(22) = u_3(33) = -100$, $u_1(11) = u_2(12) = 0$,

$u_2(23) = u_4(34) = 100$, $u_4(44) = 90$, $u_3(23) = 60$, and $u_3(34) = 300$. Using the linear size-based externality function, we can compute the utility levels in the two possible cooperatives 213 and 314 in a straightforward manner: $u_1(213) = 100$, $u_2(213) = 300$, $u_3(213) = 260$, $u_2(314) = 50$, $u_3(324) = 310$, and $u_4(324) = 50$.
We now claim that in this example there is no stable activity pattern. First, consider the activity pattern $(12, 34)$. It is not stable because [PS*] is not satisfied: $50 = u_2(324) > u_2(12) = 0$ and $310 = u_3(324) > u_3(34) = 300$. Also, since $-100 = u_2(22) < u_2(12) = 0$, the activity pattern $(11, 22, 34)$ is not stable either. Next, consider $(1, 324)$, which is not stable since [IR] for agent 4 is not satisfied: $50 = u_4(324) < u_4(44) = 90$. Moving on, $(11, 23, 44)$ is not stable due to a violation of [PS*]: $0 = u_1(11) < u_1(213) = 100$ and $100 = u_2(23) < u_2(213) = 300$. Finally, $(213, 44)$ is not stable due to a violation of [PS]: $260 = u_3(213) < u_3(34) = 300$ and $90 = u_4(44) < u_4(34) = 100$. Using the same reasoning, we find that $(12, 33, 44)$ and $(11, 22, 33, 44)$ are not stable as well. ♦

Second, stable activity patterns may not exist even when we impose *uniform* linear size-based externalities on all conveners. The following two examples illustrate this point. The first example imposes uniform, but negative, size-based externalities.

Example 5.3 Let $N = \{1, 2, 3\}$ and let $\Gamma^2 = \{12, 23\}$. Now consider $\alpha_2 = -2$. Let the utility function be such that $u_i(ii) = 0$ for all $i = 1, 2, 3$ and $u_3(23) = 1$, $u_1(12) = u_2(12) = 3$ and $u_2(23) = 4$. Using the linear size-based externality function, we can now compute the utility levels in the cooperative 213 in a straightforward manner: $u_1(213) = 1$, $u_3(213) = -1$, and $u_2(213) = 4$.We now claim that there is no stable activity pattern in this economy.
To show this, first, consider $(12, 33)$. This activity pattern is not stable due to a violation of [PS]: $3 = u_2(12) < u_2(23) = 4$ and $0 = u_3(33) < u_3(23) = 1$. Similarly $(11, 22, 33)$ is not stable. Next, $(11, 23)$ is not stable due to a violation of [PS*]: $0 = u_1(11) < u_1(213) = 1$ and $4 = u_2(23) < u_2(213) = 5$. Finally, (213) is not stable due to a violation of [IR] for agent 3: $-1 = u_3(213) < u_3(33) = 0$. ♦

Finally, we consider a 5-agent circular permissible matching structure. Here, uniformity of the the size-based externality for conveners is positive. However, the emergence of a Condorcet-like cycle in the economy prevents the desired stability.

Example 5.4 Let $N = \{1, 2, 3, 4, 5\}$ and let $\Gamma^2 = \{12, 15, 23, 34, 45\}$. Furthermore, let $\alpha_c = \alpha = 2$ for all potential conveners $c \in N^\star\left(\Sigma(\Gamma^2)\right) = N$. The utility levels for each matching is given as follows: $u_i(ii) = 0$ for all $i \in N$, $u_1(12) = u_2(23) = u_3(34) = u_4(45) = 2$, $u_1(15) = u_2(12) = u_3(23) = u_4(34) = u_5(45) = 10$ and $u_5(15) = -1$. The utility levels in all possible cooperatives are computed in a straightforward manner from the linear size-based externality function: $u_5(125) = 1$, $u_1(213) = u_2(324) = u_3(435) = u_4(514) = 4$, $u_5(514) = 11$, $u_1(514) = u_2(125) = u_3(213) = u_4(324) = u_5(435) = 12$, and $u_1(125) = u_2(213) = u_3(324) = u_4(435) = 14$. We now claim that also in this example there is no stable activity structure. ♦

We conclude from these three examples that size-based externalities prevent the emergence of a stable activity pattern if there are non-uniform externalities, there are negative size-based externalities, or there are cycles in Γ^2. However, if these three conditions are excluded, stability can still be established.

Theorem 5.5 *Let $\mathbb{E} = (N, \Gamma, u)$ be a relational economy such that $\Gamma^3 = \Sigma(\Gamma^2)$ and u exhibits size-based externalities such that $\alpha_c = \alpha > 0$ for all potential conveners $c \in N^\star\left(\Gamma^3\right)$. If Γ^2 is acyclic, then $\mathbb{E}$ admits a stable activity pattern.*

For a proof of this existence result we refer to Appendix B of this paper.

This assertion cannot be strengthened to cover strong stability rather than regular stability. The next example devices a simple case satisfying the conditions of Theorem 5.5 in which no strongly stable activity pattern can be constructed.

Example 5.6 Let $N = \{1, 2, 3\}$. Consider the permissible matching structure $\Gamma^2 = \{12, 23\}$ and the resulting permissible cooperative structure $\Gamma^3 = \Sigma(\Gamma^2) = \{213\}$. We consider the hedonic utility profile with size-based externalities generated by $\alpha = 2$ and $u_1(11) = u_3(33) = 0$, $u_2(22) = -4$, $u_1(12) = u_2(23) = -3$, and $u_2(12) = u_3(23) = 1$. Now we derive that $u_1(213) = -1 + 2 = 1$, $u_2(213) = 1 - 3 + 2 = 0$, and $u_3(213) = 1 + 2 = 3$.

We now check that in this economy there is no strongly stable activity pattern: $\Lambda_1 = \{11, 23\}$ is not stable since agent 1 wants to join agent 2 in the cooperative 213 and its convener, agent 2, agrees; $\Lambda_2 = \{12, 33\}$ is not stable since IR is not satisfied for agent 1; $\Lambda_3 = \{213\}$ is not strongly stable since its convener, agent 2, will prefer 12 over 213 and thus severs the participation of agent 3; and $\lambda_4 = \{11, 22, 33\}$ is not stable since agents 2 and 3 prefer the matching 23 over being autarkic.

Although there is no strongly stable activity pattern in this relational economy, $\Lambda_3 = \{213\}$ forms a regularly stable activity pattern. ♦

Example 5.6 confirms that the presence of simple size-based externalities prevents the emergence of strongly stable activity patterns. Thus, in the presence of externalities only economies with "open" cooperative economic activities can achieve stability.

5.2 Other types of externalities

We have shown in our previous discussion that under certain size-based externalities it is still possible to establish stable activity patterns. However, size-based externalities form a rather specific category, with significant differences from other types of externalities. In this section we investigate various examples of surplus based externalities such as envy that show that in general it is not possible to establish existence of stable activity patterns. The purpose of these examples is to point out that, generally, stability cannot be established under other types of externalities.

Definition 5.7 *The utility function u exhibits a* ***surplus based externality*** *if for every cooperative $G \in \Gamma^3$ and every member $i \in N(G)$, her utility function u_i is of the form given by*

$$u_i(G) = \sum_{j \in N_i(G)} u_i(ij) + \frac{1}{\#N(G)} \cdot \left[u_c(G) - \sum_{j \in N(G)} u_j(cj) \right] \tag{14}$$

where $c = N^\star(G)$ is G's convener and $u_c(G) \in \mathbb{R}$ is the hedonic utility of this convener c from participating in G.

The next example shows that there are relational economies exhibiting surplus based externalities with equal division, in which there is no stable activity pattern.

Example 5.8 Let $N = \{1, 2, 3\}$ and let $\Gamma^2 = \{12, 23\}$. Let the utility function be such that $u_1(11) = 9$, $u_2(22) = u_3(33) = 0$, $u_1(12) = u_2(12) = 10$, $u_2(23) = u_3(23) = 7$, and $u_2(213) = 11$. The utility levels for agents 2 and 3 in the cooperative 213 are calculated in a straightforward fashion to be $u_1(213) = 8$ and $u_3(213) = 5$. It can easily be determined that in this economy there is no stable activity pattern. ♦

From the example of a relational economy exhibiting surplus based externalities, it is clear that the establishment of stable activity patterns in relational economies with non-size-based externalities requires careful case-based analysis.

5.3 Examples of non-separable cooperatives

The examples of relational economies with non-separable cooperatives discussed thus far have had a purely theoretical nature. However, in our contemporary economies there are numerous institutions and organizations that exhibit the properties of cooperatives with externalities among its members. Here we discuss some important cases.

Open source communities. Online communities have been established for the development and provision of open source projects such as *Wikipedia* (Anthony, Smith, and Williamson, 2007) and *Linux*. The commonality of these online resources is that they are provided through voluntary contributions from members of online communities established for the specific purpose of the resource at hand.

The traditional way to view the emergence of open source software is that these systems are able to solve problems caused by strategic behavior of agents. These agents have private information about their abilities, which must somehow be communicated to the financier of the network enabling him to decide whether to start the network and whom to engage. They decide positively when the screening costs are sufficiently low (Prüfer, 2006). For further discussion and issues concerning open source production we refer also to Johnson (2002) and Lerner and Tirole (2002).

We argue that these open source online communities are essentially non-separable cooperatives. The convener is usually a person who takes the initiative for the development of the resource in question such as Jimmy Wales in the creation of Wikipedia and Linus Thorvald in the case of the development of Linux. The members who make voluntary contributions to the open source project, in this regard gather around the convener. Over time the resource grows organically in a process of development through voluntary contributions by the members of the community. The relational view focuses on the production externalities obtained from participation. In contrast to the traditional view, it also allows for voluntary membership and participation.

Even though open source communities are non-separable, they are prime examples of open cooperatives. Indeed, anyone who makes a contribution is considered a full participant of the community. This is most striking in the Wikipedia project where even malicious "contributions" can be made. A form of self-policing has emerged to counter such destructive behavior.

Hometown associations. Hometown associations refer to groups of people who descend from the same village. An association helps to keep people in touch by creating a social space with others coming from the same village and having the same identity, yet living in urban areas. Members of the association contribute a fixed amount of money periodically (monthly). Such hometown associations are founded on communal bonds and the social capital that is historically created in the community.[14] These associations are used as a safety net beside their social and economic benefits, promote social relations, and

[14] Social capital generally refers to trust, concern for ones associates, a willingness to live by the norms of one's community and to punish those who do not (Bowles and Gintis, 2002).

strengthen ties among people with a common identity. These hometown associations also develop reciprocity and exchange among members, provide financial and material assistance for common festivities (e.g. wedding and other ceremonies), and provide financial assistance to the ailing and to the families of the deceased member.

A common function of hometown associations is to provide health insurance schemes to its members. Community-based micro-insurance has aroused much interest and hope in meeting health care challenges facing the poor; micro-insurance is considered to be an important financing tool for protecting the poor from adverse financial consequences in the event of sickness. Several types of community-based health insurance schemes have emerged in sub-Saharan Africa (Wiesmann and Jutting, 2000), Asia (Krause, 2000), and in other regions. Many people in these regions simply do not understand the concept of insurance. It takes time to explain insurance and risk sharing. The idea of handing over money that will be used to pay for other peoples' health care is hard to explain and to absorb. Under these circumstances standard insurance solutions might not work, while hometown associations provide a more secure remedy.

Health care providers. In our contemporary economies, the health care sector is of very significant importance. Health care services are provided through complex networks of professional organizations such as networks of family practitioners, regional hospitals, and academic research hospitals. We argue that these health care organizations can be viewed as cooperative economic activities in the context of a relational economy.

Indeed, the convener of a health care organization in this case is the management organization that brings together a team of health care professionals and a set of patients to generate economic values for all participants. We remark that such a team of health care professionals in fact acts as a "production team" in the sense of Marshak (1955). It provides services to patients, who also have to be considered explicitly to be member of the health care organization. Thus, we arrive at a view of a health care organization as a cooperative in the sense of our theory.

We recognize that there are significant externalities within a cooperative representing a health care organization. Indeed, each team of health care professionals generates the specific characteristics of the health care services provided to the patients. Such a team imposes a certain quality standard backed up by a certain reputation of the health care organization as a whole. Patients respond to these characteristics and the economic values generated within the organization are significantly impacted by these characteristics.

On the other hand, health care provision is done through "closed" cooperatives. The convener is able to prevent access of certain members if necessary. in practice this is done through dismissal and non-renewal of contracts.

6 Some concluding remarks

The main practical contribution of this paper consists of designing a basic framework for relational value-systems in economics. We derive proto-institutional properties for that framework, which are used as benchmark for concrete, specific institutional arrangements. It is evident that, if these values diverge, problems will arise. Institutional innovation causes convergence of these values. The intricate relationship between the two views has to be scrutinized continuously in order to stabilize the economy and society.

There is a significant link with the work of Burt (1992) on structural holes and the developed relational framework of economic activities. According to Burt, optimizing the number of nonredundant contacts is a way to increase the efficiency of a social network: while the presence of cycles allows for at least two distinct paths between two distinct individuals, in the absence of cycles, there is at most one path between any two distinct individuals. Thus, in an acyclic structure one does not support links that provide the same accessibility. Given that the generation and maintenance of links is costly, a structure without cycles is more efficient than such in which cycles are present. Last, we should mention some limitations of our general framework. In particular, in our work we focus on very special class of activity patterns, which consists of star structures. For complex production processes, such as hierarchies of several levels, predominant in today's economic world, these tools are inadequate. A clear goal for future work is the development of a framework where more complex patterns can be analyzed.

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A Appendix: Proof of Theorem 4.4

The following Lemma states an intermediate result that is required for the proof of existence of a strongly stable activity pattern in a relational economy without any externalities.
Throughout we let $\mathbb{E} = (N, \Gamma, u)$ be some relational economy. As before let $\Gamma^m = \Gamma^1 \cup \Gamma^2$ be a structure of feasible simple activities on N and let $u \in \mathcal{U}$ be an arbitrary profile of utility functions, we denote by

$$B_i(\Gamma^m, u) = \{\, j \in N \mid ij \in \Gamma^m \text{ and } u_i(ij) \geq u_i(ik) \text{ for all } k \in N \text{ with } ik \in \Gamma^m \,\} \tag{15}$$

the *set of most preferred partners* of agent i for all $i \in N$.[15]

Lemma 1 *Let the feasible matching structure* Γ^2 *be acyclic. Then there is an agent* $i \in N$ *such that* $i \in B_i(\Gamma^m, u)$ *and/or there is a pair of agents* $i, j \in N$ *with* $i \neq j$ *such that* $j \in B_i(\Gamma^m, u)$ *and* $i \in B_j(\Gamma^m, u)$.

Proof. If there is some agent $i \in N$ with $i \in B_i(\Gamma^m, u)$ the assertion is obviously valid. Next assume that for every agent $i \in N$ it holds that $i \notin B_i(\Gamma^m, u)$ and the second part of the assertion is not true. Then for all agents $i, j \in N$ with $i \neq j$ such that $j \in B_i(\Gamma^m, u)$ it holds that $i \notin B_j(\Gamma^m, u)$. Consider agent $i \in N$ and without loss of generality we may assume that the set of most preferred agents is a singleton, i.e., $B_i(\Gamma^m, u) = \{j\}$. So, it must hold that $j \neq i$. Next, consider the set of most preferred partners of agent j. Without loss of generality we again may assume that B_j is a singleton, say $B_j(\Gamma^m, u) = \{k\}$. It must again hold that $k \notin \{i, j\}$. Subsequently, consider the set of most preferred partners of agent k. Without loss of generality we again assume uniqueness, say $B_k(\Gamma^m, u) = \{l\}$. It must be that $l \notin \{j, k\}$, moreover $l \neq i$ otherwise Γ^2 contains a cycle. Hence, $l \notin \{i, j, k\}$. By continuing this process in a similar fashion, given that the player set N is finite, we construct a cycle. Therefore, we have established a contradiction. ∎

Proof of Theorem 4.4

If: Consider a separable relational economy $\mathbb{E} = (N, \Gamma, u)$ such that $u \in \overline{\mathcal{U}}$ exhibits no externalities and is superadditive. We consider two separate cases: (I) when Γ^2 does not contain any cycle and (II) when Γ^2 contains a cycle with a number of connected agents that is a multiple of 3. Let $M \subseteq N$ be some subset of economic agents. Then we denote by

$$\Gamma(M) = \Gamma^m \cap \{ij \mid i, j \in M\}$$

the structure of economic matching activities and autarkic positions restricted to the subset M. Using this auxiliary notation we proceed with the proof of the two cases.

Case I: Assume that Γ^2 is acyclic. We now device an algorithm to construct a stable activity pattern in the economy $\mathbb{E}$ introduced above. This construction consists of several steps and collects agents in various cooperatives such that the resulting pattern is stable.

[15] Here $i \in B_i(\Gamma^m, u)$ refers to agent i preferring to remain in autarky over being member of any matching with another agent.

We define $\Gamma_1 = \Gamma^m$, $N_1 = N$, and $\Lambda_1 = \varnothing$. We now proceed by constructing the desired strongly stable activity pattern in a step-wise fashion:

Let Γ_k, N_k, and Λ_k be given for k, emphasizing that $\Gamma_k \subseteq \Gamma(N_k)$ and that $\Lambda_k \subseteq \Gamma$ is some partial activity pattern. We now proceed by constructing these elements for step $k+1$. With application of Lemma 1 to Γ_k, there might be an agent $i \in N_k$ such that $i \in B_i(\Gamma_k, u)$. If that is the case, we define

$$\begin{aligned} N_{k+1} &= N_k \setminus \{i\} \ ; \\ \Gamma_{k+1} &= \Gamma\left(N_{k+1}\right) \ ; \\ \Lambda_{k+1} &= \Lambda_k \cup \{ii\}. \end{aligned}$$

Subsequently we proceed to step $k + 1$ in our construction process.

If that is not the case, then for every $i \in N$ it holds that $i \notin B_i(\Gamma_k, u)$, but according to Lemma 1 there exist at least two agents $i, j \in N_k$ with $i \neq j$ and $i \in B_j(\Gamma_k, u)$ as well as $j \in B_i(\Gamma_k, u)$. Take two agents $i, j \in N_k$ as indicated and define $M = \varnothing$ as well as $G = \{ij\} \in \Gamma^2$. We now check whether the activity $G = \{ij\}$ can be enhanced into a cooperative. This is done as follows.

We first introduce some auxiliary notation. Let $\Gamma_k^{-pq} = \Gamma(N_k \setminus \{pq\})$ for any feasible matching $pq \in \Gamma_k$.

If for every agent $h \in N_k \setminus \{i, j\}$ it holds that $i \notin B_h(\Gamma_k^{-jh}, u)$ as well as $j \notin B_h(\Gamma_k^{-ih}, u)$, then we proceed by defining[16]

$$\begin{aligned} N_{k+1} &= N_k \setminus \{i, j\} \ ; \\ \Gamma_{k+1} &= \Gamma\left(N_{k+1}\right) \ ; \\ \Lambda_{k+1} &= \Lambda_k \cup \{ij\}. \end{aligned}$$

Subsequently we proceed to step $k + 1$ in our construction process.

If not, then without loss of generality we may suppose there is some agent $h \in N_k \setminus \{i, j\}$ such that $i \in B_h(\Gamma_k^{-jh}, u)$. Then ih is an optimal matching for agent h in Γ_k knowing that agent j is engaged with agent i as well. In that case we make agent i a convener and we add agent h to that cooperative. Thus, we redefine $G = \{ij, ih\}$ and we let $M = \{j, h\}$.

We now follow the subsequent iterative procedure:

(♣) We first introduce $\Gamma_k' = \Gamma(N_k \setminus M) \subset \Gamma_k$. We proceed as before and check whether there is some agent $h' \in N_k \setminus (M \cup \{i\})$ such that $i \in B_{h'}(\Gamma_k', u)$. If that is not the case, then we proceed to (♦). Otherwise, we proceed to (♠).

(♠) Suppose that an agent h' can be selected as identified in (♣), then we proceed by redefining $G = G \cup \{ih'\}$ and $M = M \cup \{h'\}$. In this case the identified agent h' is added to the cooperative under construction G and removed from consideration. We then return to (♣) to repeat the process described there for the redefined G and M.

[16] In this case there is no agent who has an optimal matching with agent i or j. In that case the matching ij is assigned to the activity pattern under construction.

(♦) Suppose there is no agent h' that has an optimal matching with the identified convener i of cooperative G as described in (♣). Then we proceed to the next step by defining

$$N_{k+1} = N_k \setminus (M \cup \{i\}) \ ;$$
$$\Gamma_{k+1} = \Gamma(N_{k+1}) \ ;$$
$$\Lambda_{k+1} = \Lambda_k \cup \{G\}.$$

Subsequently we proceed to step $k+1$ in our construction process.

We proceed through the procedure until for some $k = \bar{k}$ we arrive at the situation that $N_{\bar{k}} = \varnothing$. (Note that such a $\bar{k} \leq n$ always exists.) Now consider $\Lambda^\star = \Lambda_{\bar{k}}$. First, since the procedure devised above assigns every agent to either an autarkic activity, a matching activity, or a cooperative activity, $\Lambda^\star$ is an activity pattern. Furthermore, each constructed activity in $\Lambda^\star$ is based on either the optimality of an autarkic activity or the optimality of a matching activity. In the latter case, the non-externality and superadditivity properties of the hedonic utilities imply that the utilities generated in the constructed cooperatives in $\Lambda^\star$ are maximal under the imposed restrictions as well. Finally, this also guarantees that the convener of cooperative $G \in \Gamma^3 \cap \Lambda^\star$ does not have any incentives to break any relationships with members $i \in N(G)$. This implies, therefore, that the constructed activity pattern $\Lambda^\star$ is indeed strongly stable as required.
This concludes the proof of Case I.

Case II: The proof of Case II is based on the constructed proof for Case I above. Let the set of feasible matchings Γ^2 contain a cycle $C = (i_1, \ldots, i_m)$ with $i_{k-1}i_k \in \Gamma^2$ and $m \geq 4$ with $m-1 = 3s$ with $s \in \{1, 2, \ldots\}$. Depending on the utility profile, we will distinguish two sub-cases.

Case II.1 First, consider a utility functions $u_i \in \overline{\mathcal{U}}$ which satisfies superadditivity and the non-externality property, such that either (a) there exists an agent i_k with $k = 1, \ldots, m-1$ such that $i_k \in B_{i_k}(\Gamma^m, u)$; or (b) there are two consecutive agents along the cycle $i_{k-1}, i_k \in C$ for some $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(\Gamma^m, u)$ and $i_k \in B_{i_{k-1}}(\Gamma^m, u)$; or (c) there is a pair of agents one of whom is on the cycle and the other not, *i.e.*, $i_k \in C$ for some $k = 2, \ldots, m-1$ and $j \notin C$ such that $j \in B_{i_k}(\Gamma^m, u)$ and $i_k \in B_j(\Gamma^m, u)$. Then, we can use the algorithm described in Case I to construct a stable activity pattern since the utility profile ensures that in any of the three cases described above, we can identify agents that fit the requirements stated in Lemma 1.

Case II.2 Last, consider a profile of utility functions $u_i \in \overline{\mathcal{U}}$ such that there is no agent i_k with $k = 1, \ldots, m-1$ such that $i_k \in B_{i_k}(\Gamma^m, u)$, or there are no consecutive agents along the cycle $i_{k-1}, i_k \in C$ for some $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(\Gamma^m, u)$ and $i_k \in B_{i_{k-1}}(\Gamma^m, u)$, nor is there a pair of agents one of whom is on the cycle and the other not, *i.e.*, $i_k \in C$ for some $k = 1, \ldots, m-1$ and $j \notin C$ such that $i_j \in B_{i_k}(\Gamma^m, u)$ and $i_k \in B_j(\Gamma^m, u)$. Then, without loss of generality, we may assume that $u_{i_k}(i_k i_k) \leq u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k, i_{k+1})$ or $u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$.[17] Suppose, the profile of utility function is $u_{i_k}(i_k i_k) \leq u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$. Then, a partial activity pattern $\Lambda^\star$ can be introduced that consists of exactly s cooperatives of

[17] Alternatively, the profile of utility functions $u_i \in \mathcal{U}_s$ must be such that $u_{i_k}(i_k i_k) \leq u_{i_k}(i_k i_{k+1}) < u_{i_k}(i_{k-1} i_k)$ or $u_{i_k}(i_k i_{k+1}) < u_{i_k}(i_k i_k) < u_{i_k}(i_{k-1} i_k)$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$.

the type

$$\{\{i_2 i_1 i_3\}, \{i_5 i_4 i_6\}, \ldots, \{i_{m-2} i_{m-3} i_{m-1}\}\} \subseteq \Lambda^\star.$$

Next, all other agents are linked following the algorithm presented in Case I. Thus, we have constructed a (complete) activity pattern $\Lambda^\star$, which furthermore is stable: all agents who are not linked to their most preferred partner have their most preferred partner linked to her own most preferred partner. This implies that they have no incentive to sever their links; moreover, these agents are not in a matching activity and, therefore, they cannot add a link without severing an existing link.
Last, suppose, the profile of utility function is $u_{i_k}(i_{k-1}i_k) < u_{i_k}(i_k i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$. Then, a partial activity pattern $\Lambda^\star$ can be introduced that consists of exactly $m-1$ autarkic agents

$$\{\{i_1 i_1\}, \{i_2 i_2\}, \ldots, \{i_{m-1} i_{m-1}\}\} \subseteq \Lambda^\star.$$

All other agents are linked following the algorithm presented in Case I. Thus, we have constructed a (complete) activity pattern $\Lambda^\star$, which furthermore is strongly stable: all along the cycle are autarkic as the only partner whom they prefer to being autarkic prefers to be autarkic himself than to be matched with them.
This completes the proof of Case II.

Only if: Let $\Gamma = \Gamma^1 \cup \Gamma^2 \cup \Sigma(\Gamma^2)$ be a feasible activity structure and let $\overline{\mathcal{U}}$ be the collection of all superadditive and non-externality hedonic utility profiles. We show by contradiction the necessity of the condition that Γ^2 contains no cycles or if it contains a cycle it is a cycle with a number of connected agents equal $m \geq 4$ with $m-1 \neq 3s$ with $s = \{1, 2, \ldots\}$.
Let there be a stable activity pattern in the standard relational economy (N, Γ, u) for all $u \in \overline{\mathcal{U}}$. Let the set of feasible matchings Γ^2 contain a cycle $C = (i_1, i_2, \ldots, i_m\}$ with $i_k, i_{k+1} \in \Gamma^2$ for all $k = 1, \ldots, m-1$ and $m \geq 4$ and $m-1 \neq 3s$ with $s = \{1, 2, \ldots\}$.
Now, consider a utility profile $u \in \overline{\mathcal{U}}$ such that $u_{i_k}(i_k, j) < u_{i_k}(i_k, i_k) < u_{i_k}(i_{k-1}, i_k) < u_{i_k}(i_k, i_{k+1}) < u_{i_k}(i_k i_{k-1} i_{k=1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$ and all $j \in N_{i_k}(\Gamma^2) \setminus \{i_{k-1}, i_{k+1}\}$. Let $\Lambda^\star$ be a stable activity pattern in this standard relational economy. Note that in the stable activity pattern $\Lambda^\star$ the largest number of agents that can form a cooperative that satisfies the [IR] condition is three. We again consider two sub-cases.

Case A. First, suppose that $i_k i_k \in \Lambda^\star$ for some $k = 1, \ldots, m-1$. Since $\Lambda^\star$ is a stable activity pattern, the individual rationality condition is satisfied for all agents in N. Hence, agent i_{k-1} is in a state of autarky or connected to agent i_{k-2} either in the matching $g' = \{i_{k-1} i_{k-2}\}$, or in the cooperative $g'' = \{i_{k-2} i_{k-1} i_{k-3}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, and $i_{-2} = i_{m-3}$. In all three cases the [PS] condition is violated: $u_{i_k}(i_{k-1} i_k) > u_{i_k}(\Lambda^\star)$ and $u_{i_{k-1}}(i_{k-1} i_k) > u_{i_{k-1}}(g'') = u_{i_{k-1}}(g') > u_{i_{k-1}}(i_{k-1} i_{k-1})$. Since $\Lambda^\star$ is stable, then it cannot be that $\{i_k i_k\} \in \Lambda^\star$ for some $i_k \in C$.

Case B. Next, suppose that there is no agent along the cycle such that $i_k i_k \in \Lambda^\star$. Since $\Lambda^\star$ is a stable activity pattern, the [IR] condition is satisfied for all agents in N. Since $m-1 \neq 3s$ with $s = \{1, 2, \ldots\}$, $m-1 \geq 4$ and there is no agent i_k along the cycle such that $i_k i_k \in \Lambda^\star$, there must be at least two distinct agents along the cycle, i_{k-1} and i_k for some $k = 1, \ldots, m-1$ and

$k_0 = m-1$, such that the matching $\{i_{k-1}, i_k\} \in \Lambda^\star$. Then, agent i_{k-2} is connected to agent i_{k-3} either in the matching $g' = \{i_{k-2}i_{k-3}\}$, or in the cooperative $g'' = \{i_{k-3}i_{k-2}i_{k-4}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, $i_{-2} = i_{m-3}$, and $i_{-3} = i_{m-4}$. In all cases the no blocking condition [PS*] is violated: $u_{i_{k-2}}(i_{k-1}i_{k-2}i_k) > u_{i_{k-2}}(g') = u_{i_{k-2}}(g'')$ as the the matching $i_k i_{k-2} \notin \Gamma^2$ and $u_{i_{k-1}}(\{i_{k-1}i_{k-2}i_k\}) \geq u_{i_{k-1}}(i_{k-1}i_k)$ with $k_{-1} = m-2$ due to superadditivity.
Hence, when Γ^2 contains a cycle with a number of connected agents not a multiple of three, there are such utility profiles that satisfy superadditivity and non-externality properties, for which there is no stable activity pattern in the relational economy.

This completes the proof of Theorem 4.4.

B Appendix: Proof of Theorem 5.5

Before we present the proof we will introduce additional shorthand notation and some auxiliary results.

First, we introduce some new terms. Let $\mathbb{E} = (N, \Gamma, u)$ be a standard relational economy. Let Λ be an activity pattern. The *neighborhood of agent $i \in N$ in activity pattern* Λ is denoted by $N_i(\Lambda)$. The *utility of agent i in activity pattern* Λ is denoted by $u_i(\Lambda)$. Furthermore, we say that agents $i \in N$ and $j \in N$ form a *blocking pair* if one of the conditions in Definition 3.4 is not satisfied with respect to these agents. Last, we introduce several relationships between activity patterns. We will say that a *blocking pair in activity pattern* Λ *is satisfied in activity pattern* Λ' if activity pattern Λ' is formed by satisfying the condition in Definition 3.4 that is violated in activity pattern Λ for a given blocking pair of agents i and j.

Let the activity pattern Λ' be formed by severing all links of agent i in activity pattern Λ and forming the autarky ii. Then the relationship between activity patterns Λ and Λ' will be denoted as $\Lambda' = \Lambda \cup \{ii\}$.

Let the activity pattern Λ' be formed by severing all links of two distinct agents i and j in activity pattern Λ with $j \notin N_i(\Lambda)$ and forming the matching ij. Then the relationship between activity patterns Λ and Λ' will be denoted as $\Lambda' = \Lambda \cup \{ij\}$.

Last, let the activity pattern Λ' be formed by severing all links of agent i in activity pattern Λ and forming the link between agents i and j with $j \notin N_i(\Lambda)$ such that agent j keeps all his links present in activity pattern Λ. Then the relationship between activity patterns Λ and Λ' will be denoted as $\Lambda' = \Lambda \oplus^j \{ij\}$ where $\oplus^j$ indicates that agent j acts as a convener and keeps all his links.

Note that for all agents $k \in N_i(\Lambda)$ such that $N_k(\Lambda) = \{i\}$, it will hold that $\{kk\} \subseteq \Lambda'$.

Below we present some preliminary results.

Lemma 2 *Let (N, Γ, u) be a standard relational economy such that the utility function u exhibits multiplicative size-based externalities with $\alpha_c > 0$ for all feasible conveners $c \in N$ in $\Gamma^3 = \Sigma(\Gamma^2)$. Then for any agent $i \in N$ and any two cooperatives G and $H \in \Gamma^3$ with $i \in N(G)$ and $i \in N(H)$ and $N^*(G) = N^*(H)$ and $N^*(G) \neq \{i\}$, it holds that*

(i) $u_i(G) = u_i(H)$ *if and only if* $\#N(G) = \#N(H)$

(ii) $u_i(G) < u_i(H)$ *if and only if* $\#N(G) < \#N(H)$.

The proof of Lemma 2 follows directly from the definitions and is therefore omitted.

Lemma 3 *Let (N, Γ, u) be a standard relational economy. Let Γ^2 be acyclic. Then there is at most one path between any two distinct agents in N.*

The proof of Lemma 3 follows immediately from the fact that Γ^2 is acyclic. As a corollary of Lemma 3, we know that for any agent $i \in N$ and any two distinct agents $j, k \in N_i(\Gamma^2)$, it holds that $jk \notin \Gamma^2$.

Lemma 4 *Let (N, Γ, u) be a standard relational economy and Γ^2 be acyclic. Let Λ and Λ' be two activity patterns in this relational economy such that Λ' is formed by satisfying a blocking pair between two agents $s, t \in N$. Consider an agent $j \in N \setminus \{s, t\}$ such that $p_{js} = (i_1, \dots, i_m)$ with $i_1 = j$ and $i_m = s$ and $t \notin p_{js}$ who does not form a blocking pair in Λ. Then:*

(i) If $j \in N_s(\Lambda)$ and $\Lambda' = \Lambda \oplus^s \{st\}$, j cannot form a blocking pair in Λ';

(ii) If $m > 4$, then agent j cannot form a blocking pair in Λ';

(iii) If $m \geq 3$ and $i_{m-1}s \notin \Lambda$, then agent j cannot form a blocking pair in Λ';

(iv) If $m = 2$ and $js \notin \Lambda$, then the only blocking pair agent j may form in Λ' is with agent s in which PS condition of Definition 3.4 is not satisfied and agent s acts as a convener;*

(v) If $m = 4$, then agent j may only form a blocking pair in Λ' with agent i_2 and only if $N_{i_2}(\Lambda) = N_s(\Lambda)$;

Proof. Consider a standard relational economy (N, Γ, u) with Γ^2 be acyclic. Let Λ and Λ' be two activity patterns such that Λ' is formed by satisfying a blocking pair between two agents $s, t \in N$. Consider an agent $j \in N \setminus \{s, t\}$ such that $p_{js} = (i_1, \dots, i_m)$ with $i_1 = j$ and $i_m = s$ and $t \notin p_{js}$ who does not form a blocking pair in Λ.

(i) Let $j \in N_s(\Lambda)$ and $\Lambda' = \Lambda \oplus^s \{js\}$. By Lemma 2, $u_j(\Lambda) < u_j(\Lambda')$ and by Lemma 3 for all $h \in N_j(\Gamma^2)$ with $j \neq s$ it holds that $N_h(\Lambda) = N_h(\Lambda')$ and $u_h(\Lambda) = u_h(\Lambda')$. Hence if agent j could form a blocking pair in Λ', he could form the same blocking pair in Λ.

(ii) Let $m > 4$. By Lemma 3, $m > 4$, and $t \notin p_{js}$ it follows that $N_j(\Gamma^2) \cap N_s(\Gamma^2) = \emptyset$ and $N_j(\Gamma^2) \cap N_t(\Gamma^2) = \emptyset$. Hence, for agent j it holds that $N_j(\Lambda) = N_j(\Lambda')$ and $u_j(\Lambda) = u_j(\Lambda')$. Moreover, since $m > 4$ for all agents $h \in N_j(\Gamma^2)$ it holds that $N_h(\Lambda) = N_h(\Lambda')$ and $u_h(\Lambda) = u_h(\Lambda')$. Since agent j can only form a blocking pair with an agent $h \in N_j(\Gamma^2)$, it follows that if j does not form a blocking pair in Λ, j cannot form a blocking pair in Λ' either.

(iii) If $m > 4$, the proof follows the proof of case (ii) above. Let $m = 3$ or $m = 4$ and $i_{m-1}s \notin \Lambda$. By $i_{m-1}s \notin \Lambda$ and using Lemma 3, it follows that $N_j(\Lambda) = N_j(\Lambda')$ and $u_j(\Lambda) = u_j(\Lambda')$ and that for all agents $h \in N_j(\Gamma^2)$ it holds that $N_h(\Lambda) = N_h(\Lambda')$ and $u_h(\Lambda) = u_h(\Lambda')$. Since agent j can only form a blocking pair with an agent $h \in N_j(\Gamma^2)$, it follows that if j does not form a blocking pair in Λ, j cannot form a blocking pair in Λ' either.

(iv) Let $m = 2$ and $js \notin \Lambda$. First suppose that agent j can form a blocking pair in Λ' with an agent $h \in N_j(\Gamma^2)$ with $h \neq s$. This is not possible due to case (iii) above.

Next, suppose that agents j and s form a blocking pair in Λ' because the PS condition of Definition 3.4 is not satisfied. Hence, it must be that $u_s(\Lambda') < u_s(js)$ and $u_j(\Lambda') < u_j(js)$. Since $u_j(\Lambda) = u_j(\Lambda')$ and $u_s(\Lambda) < u_s(\Lambda')$, agents j and s could form a blocking pair in Λ, which establishes a contradiction to the fact that that agents s and t form the only blocking pair in Λ.

Last, suppose that agents j and s form a blocking pair in Λ' becasue the PS* condition of Definition 3.4 is not satisfied and agent j acts as a convener. Hence it must be that $u_s(\Lambda') < u_s(js) + \alpha_j \# N_s(\Lambda')$ and $u_j(js) > -\alpha_j$. Since $u_s(\Lambda') > u_s(\Lambda)$ it follows that agents j and s could form a blocking pair in Λ, which establishes a contradiction to the fact that that agents s and t form the only blocking pair in Λ.

Hence the only blocking pair agents j and s can form in Λ' is if the PS* condition of Definition 3.4 is not satisfied with agent s acting as a convener.

(v) Let $m = 4$. Lemma 3, $m = 4$, and $t \notin p_{js}$ imply that $N_j(\Gamma^2) \cap N_s(\Gamma^2) = \emptyset$ and $N_j(\Gamma^2) \cap N_t(\Gamma^2) = \emptyset$. Hence, for agent j it holds that $N_j(\Lambda) = N_j(\Lambda')$ and $u_j(\Lambda) = u_j(\Lambda')$. Moreover, since $m = 4$ there is only one agent $k \in N_j(\Gamma^2)$ for whom it may hold that $u_k(\Lambda) > u_k(\Lambda')$ and it can only hold if $N_k(\Lambda) = N_s(\Lambda)$: for all other agents $h \in N_j(\Gamma^2) \setminus \{k\}$ it holds that $N_h(\Lambda) = N_h(\Lambda')$ and $u_h(\Lambda) = u_h(\Lambda')$. Therefore, if j does not form a blocking pair in Λ, the only blocking pair he can form in Λ' is with agent k.

■

Proof of Theorem 5.5

Let $\mathbb{E} = (N, \Gamma, u)$ be a standard relational economy such that u exhibits multiplicative size-based externalities such that $\alpha_c > 0$ for all potential conveners $c \in N^*(\Gamma^3)$. Suppose Γ^2 is acyclic.

Suppose, that $\mathbb{E}$ does not admit a stable activity pattern. Therefore there exists a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \ldots, L_r)$ with Λ_{k+1} constructed by satisfying a blocking pair in Λ_k for $k = 1, \ldots, r-1$ such that $\Lambda_r = \Lambda_1$. If not, due to the finite number of activity patterns, we can construct a stable activity pattern by satisfying blocking pairs sequentially.

Furthermore, all activity patterns have a blocking pair. Hence, starting from any sequence of activity patterns $\mathbf{\Lambda}' = (\Lambda'_1, \ldots, \Lambda'_r)$ with $r \geq 4$ such that any activity pattern $\Lambda'_f \subseteq \mathbf{\Lambda}'$ is formed by satisfying a blocking pair in the preceding activity pattern Λ'_{f-1} for $f = 1, \ldots, r-1$ contains an activity pattern $\Lambda'_k \subseteq \mathbf{\Lambda}$ such that $\Lambda'_r = \Lambda'_k$. Otherwise, due to the finite number of possible activity pattern, we can construct stable activity pattern by satisfying blocking pairs sequentially.

Without loss of generality, suppose that there is exactly one such sequence $\mathbf{\Lambda} = (\Lambda_1, \ldots, \Lambda_r)$ with $r \geq 4$ such that any activity pattern $\Lambda_k \subseteq \mathbf{\Lambda}$ is formed by satisfying a blocking pair in the preceding activity pattern Λ_{k-1} for $k = 1, \ldots, r-1$ with $\Lambda_r = \Lambda_1$. Hence starting from any activity pattern Λ by satisfying blocking pairs we reach some activity pattern $\Lambda_k \subseteq \mathbf{\Lambda}$. Moreover, each activity pattern $\Lambda_1, \ldots, \Lambda_r \subseteq \mathbf{\Lambda}$ has exactly one blocking pair, otherwise, there are other sequences of activity patterns $(\Lambda_1', \ldots, \Lambda_r')$ with $\Lambda_r' = \Lambda_1'$.

We will discuss all possible types of blocking pairs in Λ_1 and show that it cannot be that $\Lambda_r = \Lambda_1$.

Case I: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ with $\{ii\} \subseteq \Lambda_1$ and $\{jj\} \subseteq \Lambda_1$ such that agents i and j form a blocking pair. Hence $u_i(ii) < u_i(ij)$ and $u_j(jj) < u_j(ij)$. Since $\Lambda_r = \Lambda_1$, there must be an activity pattern $\Lambda_q \subseteq \mathbf{\Lambda}$ with $1 < q < r$ such that either agent i or agent j forms a blocking pair that requires him to delete the link with the other agent.

Without loss of genreality, suppose agent i deletes the link with agent j. For agent i to delete this link there must be an agent $t \in N_i(\Gamma^2)$ with $t \neq j$ such that $u_t(\Lambda_1) \neq u_t(\Lambda_q)$, so that agents t and i form a blocking pair in Λ_q but not in Λ_1. For $u_t(\Lambda_q) \neq u_t(\Lambda_1)$ it must be that agent t forms a blocking pair in some activity pattern Λ_k with $1 < k < q$. By Lemmas 3 and 4 cases (ii) and (iii) it follows that no agent $h \notin N_i(\Gamma^2) \cup N_j(\Gamma^2)$ may form a blocking pair before forming a blocking pair with agent i or j. Hence, agent t must form a blocking pair with agent i in Λ_k and by Lemma 4 case (iii), it follows that the agents i acts as convener in that blocking pair. Hence it must be that $u_i(it) > -\alpha_i$ and $u_t(\Lambda_1) < u_t(it) + \alpha_i \# N_i(\Lambda_k)$. By Lemmas 3 and 4 cases (i) and (iii), it follows that agent j will thus not form a blocking pair that requires him to delete the link with agent i in any activity pattern $\Lambda_{k+1}, \ldots, \Lambda_q$.

Since agents i and t form a blocking pair in Λ_q, they are not linked in Λ_q and since agent i cannot delete a link with agent t without deleting a link with agent j, there must be another activity pattern Λ_m with $k < m < q$ in which agent t forms a blocking pair that requires him to sever his link with agent i.

Because agent t forms a blocking pair in Λ_m by deleting the link with agent i, by Lemma 4 cases (i) and (iii), it must be that $\# N_i(\Lambda_q) = \# N_i(\Lambda_k)$. Since there is only one blocking pair in Λ_q and it requires agent i to delete its links, it must be that $u_t(\Lambda_q) > u_t(\Lambda_m) = u_t(it) + \alpha_i \# N_i(\Lambda_k) > u_t(it)$, otherwise agents i and t could form a blocking pair when i acts as a convener. Therefore, it cannot be that agents i and t form a blocking pair because the PS condition of Definition 3.4 is not satisfied. So it must be that agents i and t form a blocking pair in Λ_q because the PS* condition of Definition 3.4 is not satisfied and agent t acts as a convener. Hence $u_t(it) > -\alpha_t$. If agents i and t did not form a blocking pair in Λ_1 it must be that either agent t could not act as a convener in Λ_1, or $\# N_t(\Lambda_1) < \# N_t(\Lambda_q)$.

First suppose agents i and t cannot form a blocking pair in Λ_1 because agent t cannot act as a convener.

1. Suppose $\{tt\} \in \Lambda_1$. By Lemmas 3 and 4, we know that $N_l(\Lambda_1) = N_l(\Lambda_m)$ or all $h \in N_t(\Gamma^2) \setminus \{i\}$, thus, $u_h(\Lambda_1) = u_h(\Lambda_m)$. If agent t forms a blocking pair in Λ_m, such that he deletes the link with i, t could have formed a blocking pair in Λ_1 because $u_t(\Lambda_1) < u_t(\Lambda_m)$ by Lemma 2 and the fact that agent i cannot delete a link without deleting all its links. Thus establishing a contradiction that there is only one blocking pair in Λ_1 and it involves agents $i \neq t$ and $j \neq t$.

2. Suppose $st \in (\Lambda_1)$ with $s \in N^*(\Lambda_1)$. If agents i and t form a blocking pair in Λ_k such that agent i acts as a convener, it must be that $u_t(\Lambda_k) < u_t(it) + \alpha_i \# N_i(\Lambda_k)$. By Lemmas 3 and 4 if agent t forms a blocking pair in Λ_m that requires him to delete the link with agent i,

it must be to form a blocking pair with agent s as for all $h \in N_t(\Gamma^2)$ with $h \neq s$ and $h \neq i$, $N_h(\Lambda_1) = N_h(\Lambda_k) = N_h(\Lambda_m)$. If agents s and t form a blocking pair in Λ_m, it must be that $\#N_s(\Lambda_m) > \#N_s(\Lambda_1)$ and agent s acts as a convener with agent t. Hence agent t cannot form a blocking pair with agent i as t cannot act as a convener. Moreover, by Lemmas 3 and 4 and the fact that there is only one blocking pair in each activity pattern, then agent t could only form a blocking pair if agent s deletes all the links agent t will be autarkic and since $u_t(tt) < u_t(it) + \alpha_i \#N_i(\Lambda_k)$ the only blocking pair he could form is with agent i acting as a convener.

Second, suppose that agents i and t did not form a blocking pair in Λ_1 because $\#N_t(\Lambda_1) < \#N_t(\Lambda_q)$ and agent t could form a blocking pair when acting as a convener in Λ_1. Since there is no activity pattern $\Lambda' \subseteq \mathbf{\Lambda}$ such that t forms a blocking pair when acting as a convener with an agent $p \notin N_t(\Lambda_1)$ unless $\#N_t(\Lambda') > \#N_t(\Lambda_1)$, otherwise agent t could form this blocking pair in Λ_k, there is no activity pattern Λ_q in which agent i deletes the link with j to join agent t as a convener.
Thus we have shown that agents i and j will not delete their link, hence $\Lambda_r \neq \Lambda_1$.

Therefore, a blocking pair when the PS condition of Definition 3.4 is not satisfied for two autarkic agents cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case II Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{jj\} \in \Lambda_1$ and $\{ih\} \in \Lambda_1$ with $i \neq h$ such that agents i and j form a blocking pair because the PS* condition in Definition 3.4 is not satisfied. Hence $u_j(jj) < u_i(ij) + \alpha_i$ and $u_i(ij) > -\alpha_i$. Consider activity pattern $\Lambda_2 = \Lambda_1 \oplus^i \{ij\}$. Since $\Lambda_r = \Lambda_1$, there must be an activity pattern $\Lambda_m \subseteq \mathbf{\Lambda}$ with $1 < m < r$ such that either agent i or agent j forms a blocking pair that requires him to delete the link with the other agent.

Since there is no stable activity pattern, there must be a blocking pair in activity pattern Λ_2. By Lemma 4 cases (i), (ii), and (iii) the blocking pair must involve agent i or j.
Suppose the blocking pair involves agent i. By Lemmas 3 and 4 and the fact that there is only one blocking pair in Λ_1 any activity pattern formed by satisfying a blocking pair that involves an agent l with $i \in p_{jl}$ agent j or agent h will not form a blocking pair, unless satisfying the blocking pair does not require for agent i to delete simultaneously his links with agents j and h. So, for $\Lambda_1 = \Lambda_r$, there must be an activity pattern in which agent i deletes his links with agents j and h. To find a contradiction we can follow the reasoning in Case I.
Suppose instead the blocking pair involves agent j. It must be that it requires from agent j to sever his link with agent i. By Lemma 3 and Lemma 4 there is no such agent with whom j can form a blocking pair, otherwise j could form an alternative blocking pair in Λ_1.

Therefore, a blocking pair in which the PS* condition of Definition 3.4 is not satisfied for an autarkic agent and an agent in a matching cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case III Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{jj\} \in \Lambda_1$ and $i \in N^*(\Lambda_k)$ such that agents i and j form a blocking pair because the PS* condition in Definition 3.4 is not satisfied. Hence $u_j(jj) < u_i(ij) + \alpha_i \#N_i(\Lambda_1)$ and $u_i(ij) > -\alpha_i$. Consider activity pattern $\Lambda_2 = \Lambda_1 \oplus^i \{ij\}$. Since $\Lambda_r = \Lambda_1$, there must be an activity pattern $\Lambda_m \subseteq \mathbf{\Lambda}$ with $1 < m < r$ such that either agent i or agent j forms a blocking pair that requires him to delete the link with the other agent.

Following the reasoning of Cases I and II, we can show a contradiction.

Therefore, a blocking pair in which the PS* condition of Definition 3.4 is not satisfied for an autarkic agent and a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case IV: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$. Let agent i form a blocking pair because the IR condition of Definition 3.4 is not satisfied. Hence $u_i(ii) > u_i(ij)$.
Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{ii\}$. Since there is no stable activity pattern, there is a blocking pair in Λ_2. Since there is no other blocking pair in Λ_1, and for all $l \in N \setminus \{i, j\}$, $N_l(\Lambda_1) = N_l(\Lambda_2)$ and $u_l(\Lambda_1) = u_l(\Lambda_2)$ a blocking pair in Λ_2 must involve either agent i or j and an agent $h \in N_i(\Gamma^2) \cup N_j(\Gamma^2)$. By Lemma 4 case (iv) this is not possible as neither agent i nor j can act as a convener in a blocking pair.

Therefore, a blocking pair in which the IR condition of Definition 3.4 is not satisfied for an agent in a matching cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case V: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $N_i(\Lambda_1) = \{j\}$ and $j \in N^*(\Lambda_1)$. Let agent i form a blocking pair because the IR condition of Definition 3.4 is not satisfied. Hence $u_i(ii) > u_i(ij) + \alpha_j[\#N_j(\Lambda_1) - 1]$.
Consider activity pattern $\Lambda_2 = \Lambda_k \cup \{ii\}$. Since there is no stable activity pattern, there is a blocking pair in Λ_2. Since there is no other blocking pair in Λ_1, and for all $l \in N \setminus \{i, j\}$, $N_l(\Lambda_1) = N_l(\Lambda_2)$, and for all agents $l \in N \setminus \{i, j, N_j(\Lambda_2)\}$, $u_l(\Lambda_1) = u_l(\Lambda_2)$ a blocking pair in Λ_2 must involve either agent i or j or an agent $h \in N_j(\Lambda_2)$.
Following the analysis in Case IV, we can show that agent i does not form a blocking pair before he forms a blocking pair with agent j. Moreover, if there is an activity pattern such that $\#N_j(\Lambda_m) \geq \#N_j(\Lambda_1)$, then it must be that $\{ii\} \in \Lambda_m$ and agents i and j form blocking pair in which agent j acts as a convener and agent i is autarkic. By Case III we know that a blocking pair between an autarkic agent and an agent who is acting as a convener cannot be part of a sequence of activity patterns such that $(\Lambda_1, \ldots, \Lambda_r)$ with $\Lambda_1 = \Lambda_r$, and hence it cannot be that i and j form a blocking pair. Hence $\Lambda_r \neq \Lambda_1$.

Therefore, a blocking pair in which the IR condition of Definition 3.4 is not satisfied for an agent in a cooperative who is not convener of the cooperative cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case VI: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ii\} \in \Lambda_1$ and $\{js\} \in \Lambda_1$ with $j \neq s$. Let agents i and j form a blocking pair because the PS condition of Definition 3.4 is not satisfied. Hence $u_i(ii) < u_i(ij)$ and $u_j(ij) > u_j(js)$. Since this is the only blocking pair and $\alpha_j > 0$, it must also be that $u_j(ij) < -\alpha_j$, otherwise agents i and j could form a blocking pair because the PS* condition of Definition 3.4 is not satisfied. We will show that there cannot be an activity pattern $\Lambda_q \subseteq \mathbf{\Lambda}$ with $\{js\} \in \Lambda_q$.
Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{ij\}$. Since there is no stable activity pattern, there is a blocking pair in Λ_2. Since there is no other blocking pair in Λ_1, and for all $l \in N \setminus \{i, j, s\}$, $N_l(\Lambda_1) = N_l(\Lambda_2)$, and for all agents $l \in N \setminus \{i, j, s\}$ it must be that $u_l(\Lambda_1) = u_l(\Lambda_2)$ a blocking pair in Λ_2 must involve either agent i, j or s.
Suppose the blocking pair in Λ_2 involves agent i, then using the analysis for agent i in Case I, it can be shown that agent i will not delete the link with agent j. And by Lemmas 3 and 4, if agents j and s do not form a blocking pair in Λ_2, they will not form a blocking pair.

Next, suppose that the blocking pair in Λ_2 involves agent j. Since $u_j(js) < u_j(ij)$ and $u_j(ij) < -\alpha_j$ agent j will not form a blocking pair with agent s when acting as a convener. Since $u_j(ij) < -\alpha_j$ agent j will not form a blocking pair with an agent $h \in N_j(\Gamma^2)$ with $h \notin N_j(\Lambda_1)$ otherwise agent j could form another blocking pair in Λ_1.
It must be that agent s forms a blocking pair in Λ_2. Hence, by Lemmas 3 and 4 and the fact that there is only one blocking pair in each activity pattern in $\mathbf{\Lambda}$ the first blocking pair agent j is with agent s. Suppose agent j and s form a blocking pair in some activity pattern $\Lambda_k \subseteq \mathbf{\Lambda}$ with $2 < k < q$. By the above discussion, it follows that j and s form a blocking pair because the PS* condition of Definition 3.4 is not satisfied and agent s acts as a convener. Hence agent i is autarkic in Λ_{k+1} and does not form a blocking pair unless it is with agent j. In addition, there is at least one agent $h \in N_s(\Gamma^2)$ with $s \neq j$ such that $h \in N_s(\Lambda_k)$. Note that by Lemmas 3 and 4 and the fact that there is only one blocking pair, h does not form a blocking pair until agent s does not delete the link and agent s cannot delete the link with agent h without deleting the link with agent j as well. Hence, $\{js\}$ cannot be an element of an activity pattern unless agent s deletes all his links as a convener.

Suppose there is an activity pattern Λ_m with $k < m < r$ such that agent s deletes all his links as a convener and thus agent j is autarkic. Since $u_j(jj) < u_j(ij)$ (otherwise agent j could form a different blocking pair in Λ_1), it must be that the only blocking pair in Λ_m must be by agent j and i because the PS condition of Definition 3.4 is not satisfied. Since the blocking pair of agents i and j entails two autarkic agents who form a blocking pair and by Case I, we know that such blocking pair cannot be part of a sequence of activity patterns $(\Lambda_1, \ldots, \Lambda_r)$ with $\Lambda_r = \Lambda_1$, thus, we have established a contradiction.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for an autarkic agent and an agent in a matching cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case VII: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ii\} \in \Lambda_1$ and $\{j\} \in N^*(\Lambda_1)$. Let agents i and j form a blocking pair because the PS condition of Definition 3.4 is not satisfied. Hence $u_i(ii) < u_i(ij)$ and $u_j(ij) > \sum_{h \in N_j(\Lambda_1)} u_j(jh) + \alpha_j[\#N_j(\Lambda_1) - 1]$. Since this is the only blocking pair and $\alpha_j > 0$, it must also be that $u_j(ij) < -\alpha_j$, otherwise agents i and j can form a blocking pair because the PS* condition of Definition 3.4 is not satisfied. We will show that there cannot be an activity pattern $\Lambda_q \subseteq \mathbf{\Lambda}$ with $N_j(\Lambda_1) = N_j(\Lambda_q)$.
Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{ij\}$. Since there is no stable activity pattern, there is a blocking pair in Λ_2. Since there is no other blocking pair in Λ_1, and for all $l \in N \setminus \{i, j, N_j(\Lambda_1)\}$, $N_l(\Lambda_1) = N_l(\Lambda_2)$, and $u_l(\Lambda_1) = u_l(\Lambda_2)$ a blocking pair in Λ_2 must involve either agent i, j or an agent $s \in N_j(\Lambda_1)$.
Suppose the blocking pair in Λ_2 involves agent i, then using the analysis for agent i in Case I, it can be shown that agent i will not delete the link with agent j. And by Lemmas 3 and 4, if agents j and any agent $s \in N_j(\Lambda_1)$ do not form a blocking pair in Λ_2, they will not form a blocking pair unless agent i deletes his link with j.
Next, suppose the blocking pair in Λ_2 involves agent j and no agent $s \in N_j(\Lambda_1)$. Since $u_j(ij) < -\alpha_j$ agent j will not form a blocking pair with an agent $h \in N_j(\Gamma^2)$ with $h \notin N_j(\Lambda_1)$ otherwise agent j could form another blocking pair in Λ_1.
Suppose agent j forms a blocking pair in Λ_2 with an agent $s \in N_j(\Lambda_1)$ with $\{ss\} \in \Lambda_2$. This, however, contradicts either Case II or Case VI.
Last suppose that an agent $s \in N_j(\Lambda_1)$ forms a blocking pair in Λ_2 with an agent $f \in N_s(\Gamma^2)$

with $f \neq j$. Hence if there is an activity pattern Λ_q with $N_j(\Lambda_q) = N_j(\Lambda_k)$ agent j must form a blocking pair and the first blocking pair j can make by Lemmas 3 and 4 is with agent s. Let the activity pattern in which agents j and s form a blocking pair is Λ_k. It must be that j and s form a blocking pair because the PS* condition of Definition 3.4 such that agent s acts as a convener is not satisfied otherwise j and s could form a blocking pair in Λ_2. Hence by Lemmas 3 and 4 and the fact that there is only one blocking pair in each activity pattern agent j cannot form a blocking pair with an agent $h \in N_j(\Lambda_1)$ with $h \neq s$, unless agent s deletes all his links. If agent s deletes all his links, agent j will be autarkic, and hence, must form a blocking pair with agent i, otherwise agents i and j could not form a blocking pair in Λ_1. Since j is autarkic and i is autarkic when making a blocking pair with i, we know by Case I that this blocking pair cannot be part of a sequence of activity patterns $(\Lambda_1, \ldots, \Lambda_r)$ with $\Lambda_r = \Lambda_1$ and thus we have established a contradiction.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for an autarkic agent and a convener of a cooperative cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case VIII: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ii\} \in \Lambda_1$ and $N_j(\Lambda_1) = \{s\}$ and $s \in N^*(\Lambda_k)$. Let agents i and j form a blocking pair because the PS condition of Definition 3.4 is not satisfied. Hence $u_i(ii) < u_i(ij)$ and $u_j(ij) > u_j(js) + \alpha_s[\#N_s(\Lambda_1) - 1]$. We will show that there cannot be an activity pattern $\Lambda_q \subseteq \mathbf{\Lambda}$ with $N_s(\Lambda_q) = N_s(\Lambda_1)$.

For $N_s(\Lambda_q) = N_s(\Lambda_k)$, agents j and s must form a blocking pair. Agents j and s can form a blocking pair if and only if one of them acts as a convener.

Suppose agents j and s form a blocking pair in $\Lambda_k \subseteq \mathbf{\Lambda}$ and agent j acts as a convener. For $N_s(\Lambda_q) = N_s(\Lambda_1)$ agent s must be able to form blocking pairs as a convener, hence, there must be an activity pattern $\Lambda_m \subseteq \mathbf{\Lambda}$ with $k < m < q$ such that either $\{js\} \in \Lambda_m$ or s severs his link with j. By Lemma 4 case (i) if agent j acts as a convener, agent i will not sever his link with him, otherwise there could be another blocking pair in Λ_1. Hence, it must be that s severs his link with j, which implies that j must join agent s as a convener, and the analysis below will hold.

Suppose agents j and s form a blocking pair in $\Lambda_k \subseteq \mathbf{\Lambda}$ and agent s acts as a convener. For agent j to sever his link with i to join s as a convener, it must be that $N_s(\Lambda_k) > N_s(\Lambda_1) - 1$. Hence, there is an agent $h \in N_s(\Gamma^2)$ with $h \in N_s(\Lambda_k)$ and $h \notin N_s(\Lambda_1)$. So, $\Lambda_{k+1} = \Lambda_k \oplus^s \{js\}$. Hence by Lemma 4 no agent $h \in N_s(\Lambda_k)$ will form a blocking pair unless agent s severs all his links. Suppose there is activity pattern $\Lambda_m \subseteq \mathbf{\Lambda}$ with $k+1 < m < q$ agent s severs all his links, then agent j will be autarkic and must form a blocking pair with agent i as another autarkic agent or as i acting as a convener, otherwise agents i and j could not form a blocking pair in Λ_1 and by Case I, II, and III such blocking pair cannot be part of a sequence of activity patterns $(\Lambda_1, \ldots, \Lambda_r)$ with $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for an autarkic agent and an agent linked in a cooperative but not acting as the convener of the cooperative cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case IX: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$. Let agent i form a blocking pair because the IR condition of Definition 3.4 is not satisfied. Hence $\Lambda_2 = \Lambda_1 \cup \{ii\}$. Hence $\sum_{h \in N_i(\Lambda_1)} u_i(ih) + \alpha_i[\#N_i(\Lambda_1 - 1)] < u_i(ii)$.

Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{ii\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and all agents $h \in N_i(\Lambda_1)$) form blocking pairs in some activity patterns. Note that $\{ii\} \in \Lambda_2$ and $\{hh\} \in \Lambda_2$ for all $h \in N_i(\Lambda_1)$ and thus agent i and each agent $h \in N_i(\Lambda_1)$ must form a blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair when the IR condition of Definition 3.4 is not satisfied for a convener of a cooperative cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case X: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{st\} \in \Lambda_1$ with $s \in N_j(\Gamma^2)$ and $s \neq i$. Let agents j and s form a blocking pair because the PS condition of Definition 3.4 is not satisfied. Hence $u_j(ij) < u_j(js)$ and $u_s(st) < u_s(js)$.
Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and t form blocking pairs in some activity patterns. Note that $\{ii\} \in \Lambda_2$ and $\{tt\} \in \Lambda_2$ and thus i and t form blocking pairs as autarkic agents. As proven in Cases I, II, III, VI, VII, and VIII, activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for two agents linked in matchings cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XI: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{st\} \in \Lambda_1$ with $s \in N_j(\Gamma^2)$ and $s \neq i$. Let agents j and s form a blocking pair because the PS* condition of Definition 3.4 is not satisfied. Hence $u_j(ij) < u_j(js) + \alpha_S$ and $u_s(js) > -\alpha_s$.
Consider activity pattern $\Lambda_2 = \Lambda_1 \oplus^s \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agent i and j form blocking pairs in some activity pattern. Note that $\{ii\} \in \Lambda_2$ and thus i must form a blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS* condition of Definition 3.4 is not satisfied for two agents linked in matchings cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XII: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{s\} \in N^*(\Lambda_k)$. Let agents j and s form a blocking pair because the PS condition of Definition 3.4 is not satisfied. Hence $u_j(ij) < u_j(js)$ and $\sum_{h \in N_s(\Lambda_1)} u_s(hs) + \alpha_s[\#N_s(\Lambda_1) - 1] < u_s(js)$.
Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and $h \in N_s(\Lambda_1)$ form blocking pairs in some activity patterns. Note that $\{ii\} \in \Lambda_2$ and $\{hh\} \in \Lambda_2$ for all $h \in N_s(\Lambda_1)$ and thus i and each $h \in N_s(\Lambda_1)$ must form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \ldots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for an agent linked in a matchings and a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XIII: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{s\} \in N^*(\Lambda_1)$. Let agents j and s form a blocking pair because the PS* condition of Definition 3.4 is not satisfied and agent s acts as a convener. Hence $u_s(js) > -\alpha_s$ and $u_j(ij) < u_j(js) + \alpha_s \#N_s(\Lambda_1)$.

Consider activity pattern $\Lambda_2 = \Lambda_1 \oplus^s \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and j form blocking pairs in some activity patterns. Note that $\{ii\} \in \Lambda_2$ and thus agent i must form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \dots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS* condition of Definition 3.4 is not satisfied for an agent linked in a matchings and a convener such that the agent in the matching acts as a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XIV: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\} \in \Lambda_1$ and $\{s\} \in N^*(\Lambda_1)$. Let agents j and s form a blocking pair because the PS* condition of Definition 3.4 is not satisfied and agent j acts as a convener. Hence $\sum_{h \in N_s \Lambda_1} u_s(hs) + \alpha_s[\#N_s(\Lambda_1) - 1] < u_s(js) + \alpha_j$ and $u_j(js) > -\alpha_j$. Consider activity pattern $\Lambda_2 = \Lambda_1 \oplus^j \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents $h \in N_s(\Lambda_1)$ and s form blocking pairs in some activity patterns. Note that $\{hh\} \in \Lambda_2$ for all $h \in N_s(\Lambda_1)$ and thus each $h \in N_s(\Lambda_1)$ must form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \dots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS* condition of Definition 3.4 is not satisfied for an agent linked in a matchings and a convener such that the convener of the cooperative matching acts as a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XV: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$ and $j \in N^*(\Lambda_1)$. Let agents i and j form a blocking pair because the PS condition of Definition 3.4 is not satisfied. Hence $\sum_{h \in N_i(\Lambda_1)} u_i(ih) + \alpha_i[\#N_i(\Lambda_1) - 1] < u_i(ij)$ and $\sum_{f \in N_j(\Lambda_1)} u_j(jf) + \alpha_j[\#N_j(\Lambda_1) - 1] < u_j(ij)$. Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{ij\}$. For $\Lambda_r = \Lambda_1$ it must be that agents $h \in N_i(\Lambda_1)$ form blocking pairs with agent i and agents $f \in N_j(\Lambda_1)$ form blocking pairs with agent j in some activity patterns. Note that $\{hh\} \in \Lambda_2$ for all $h \in N_i(\Lambda_1)$ and $\{ff\} \in \Lambda_2$ for all $f \in N_j(\Lambda_1)$ and each $h \in N_i(\Lambda_1)$ and each $f \in N_j(\Lambda_1)$ must form at least one blocking pair as an autarkic agent. As proven in Cases II, III, IV, VII, VIII, and XI, activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \dots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for two conveners cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XVI: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$ and $j \in N^*(\Lambda_1)$. Let agents i and j form a blocking pair because the PS* condition of Definition 3.4 is not satisfied. Without loss of generality let $\sum_{h \in N_i(\Lambda_1)} u_i(ih) + \alpha_i[\#N_i(\Lambda_1) - 1] < u_i(ij) + \alpha_j \#N_j(\Lambda_1)$ and $u_j(ij) > -\alpha_j$. Consider activity pattern $\Lambda_2 = \Lambda_1 \oplus^j \{ij\}$. For $\Lambda_r = \Lambda_1$ it must be that agents $h \in N_i(\Lambda_1)$ form blocking pairs with agent i in some activity patterns. Note that $\{hh\} \in \Lambda_2$ for all $h \in N_i(\Lambda_1)$ and thus each $h \in N_i(\Lambda_1)$ must form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \dots . \Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS* condition of Definition 3.4 is not satisfied for two conveners cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XVII: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\}$ and $N_s(\Lambda_1) = \{t\}$ with $t \in N^*(\Lambda_1$. Let agents j and s form a blocking pair because the PS condition of Definition 3.4 is not satisfied.

Hence $u_j(ij) < u_j(js)$ and $u_s(st) + \alpha_t[\#N_t(\Lambda_1) - 1] < u_s(js)$.
Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{js\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and j form a blocking pair in some activity patterns. Note that $\{ii\} \in \Lambda_2$ and thus agent i form at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \ldots .\Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for an agent in a matching and an agent in a cooperative who does not acts as a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XVIII: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $\{ij\}$ and $N_s(\Lambda_1) = \{t\}$ with $t \in N^*(\Lambda_1)$. Let agents j and s form a blocking pair because the PS* condition of Definition 3.4 is not satisfied and j acts as a convener. Hence $u_j(js) > -\alpha_j$ and $u_s(st)+\alpha_t[\#N_t(\Lambda_1)-1] < u_s(js)+\alpha_j$. We will show that there is no activity patter $\Lambda_q \subseteq \mathbf{\Lambda}$ such that $N_t(\Lambda_q) = N_t(\Lambda_1)$.
A contradiction can be established following the same analysis as in Case VIII.

Therefore, a blocking pair in which the PS* condition of Definition 3.4 is not satisfied for an agent in a matching and an agent in a cooperative who does not acts as a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XIX: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$ and $N_j(\Lambda_1) = \{s\}$ with $s \in N^*(\Lambda_1)$ and $i \neq s$. Let agents i and j form a blocking pair because the PS condition of Definition 3.4 is not satisfied. Hence $\sum_{h \in N_i(\Lambda_1)} u_i(ih) + \alpha_i[\#N_i(\Lambda_1) - 1] < u_i(ij)$ and $u_j(js) + \alpha_s[\#N_s(\Lambda_1) - 1] < u_j(ij)$.
Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{ij\}$. For $\Lambda_r = \Lambda_1$ it must be that agent i and an agent $h \in N_i(\Lambda_1)$ form a blocking pair in some activity patterns. Note that $\{hh\} \in \Lambda_2$ for all $h \in N_i(\Lambda_1)$ and thus agent each agent $h \in N_i(\Lambda_1)$ forms at least one blocking pair as an autarkic agent. As proven in Cases I, II, III, VI, VII, and VIII, activity patterns in which autarkic agents form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \ldots .\Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for a convener and an agent in a cooperative who does not acts as a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XX: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $i \in N^*(\Lambda_1)$ and $N_j(\Lambda_1) = \{s\}$ with $s \in N^*(\Lambda_1)$ and $i \neq s$. Let agents i and j form a blocking pair because the PS* condition of Definition 3.4 is not satisfied. Hence $u_i(ij) > -\alpha_i$ and $u_j(js) + \alpha_s[\#N_s(\Lambda_1) - 1] < u_j(ij) + \alpha_i \#N_i(\Lambda_1)$. We will shows that there cannot be an activity pattern $\Lambda_q \subseteq \mathbf{\Lambda}$ such that $N_s(\Lambda_q) = N_s(\Lambda_1)$.
A contradiction can be established following the same analysis as in Case VIII.

Therefore, a blocking pair in which the PS* condition of Definition 3.4 is not satisfied for a convener and an agent in a cooperative who does not acts as a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

Case XXI: Consider activity pattern $\Lambda_1 \subseteq \mathbf{\Lambda}$ such that $N_i(\Lambda_1) = s$ with $s \in N^*(\Lambda_1)$ and $N_j(\Lambda_1) = \{t\}$ with $t \in N^*(\Lambda_1)$ and $i \neq j$. Let agents i and j form a blocking pair because the PS condition of Definition 3.4 is not satisfied. Hence $u_i(is)+\alpha_s[N_s(\Lambda_1)-1] < u_i(ij)$ and $u_j(jt)+\alpha_t[\#N_t(\Lambda_1)-1] < u_j(ij)$.

Consider activity pattern $\Lambda_2 = \Lambda_1 \cup \{ij\}$. For $\Lambda_r = \Lambda_1$ it must be that agents i and s form a blocking pair in some activity patterns and agents j and t forma a blocking pair in some activity pattern. Note that $\{ij\} \in \Lambda_2$ and thus at least one of agents i and j forms at least one blocking pair as an agent in a matching. As proven in Cases II, IV, VI, X, XI, XII, XIII, XIV, XVII, and XVIII activity patterns in which agents in a matching form blocking pairs cannot be part of a sequence of activity patterns $\mathbf{\Lambda} = (\Lambda_1, \ldots .\Lambda_r)$ such that $\Lambda_r = \Lambda_1$.

Therefore, a blocking pair in which the PS condition of Definition 3.4 is not satisfied for two agents linked in a cooperatives none of whom acts as a convener cannot be part of the sequence of activity patterns $\mathbf{\Lambda}$.

This completes the proof of Theorem 5.5.